New Mathematics for Elementary School

6

TOKYO SHOSEKI

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About the "D" Symbol

Available in Japanese only.

- Sections marked with this symbol have additional materials available online.
- Tell your teacher or parent before you use the Internet.

To Study Online

Use the URL below or QR code on the right to access online contents.

https://tsho.jp/02p/m6/

Notice to Teachers and Parents

Use of the online contents marked with the "D" Symbol is free of charge, but you need if you have Internet access.

Prior learning

5th Grade
Volume of Cubes and Cuboids/ Prisms and Cylinders

6th Grade
⑦ Area of Circles
⑧ Volume of Prisms and Cylinders

5th Grade
Proportional Relationship/ Investigating Changes

5th Grade
Average/ Percentage Bar Graphs and Pie Charts

1st through 6th Grades
Mathematics Study Key Viewpoints and Ways of Thinking

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proporcionalidad

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Later learning

⑨ Approximate Area and Volume
7th Grade
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7th Grade
Direct and Inverse Proportional Relationships

7th and 8th Grades
Probability

7th Grade
Data Analysis and Utilization

Mathematics for 7th Grade and Beyond

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Notice to Teachers and Parents

"New Mathematics 6 Plus" is an optional learning material for students who want or need to work on it.
Not all students need to use "New Mathematics 6 Plus."

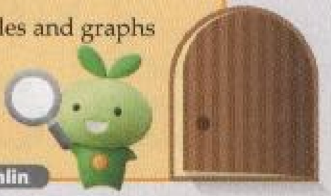
What if you use many different types of calculations and their systems? What if you carefully look at the roles of math sentences...

Determine what elements of geometric figures to focus on, such as lengths of the sides, positional relationships, and relationships between geometric figures, then...

Focus on how quantities change and the relationships between quantities, then...

To solve problems around you, organize data into tables and graphs and grasp the characteristics of the data, then...

Based on what you studied up through fifth grade, further explore the world of mathematics in sixth grade.



Mathlin

Mathlin gives you some hints as you study.

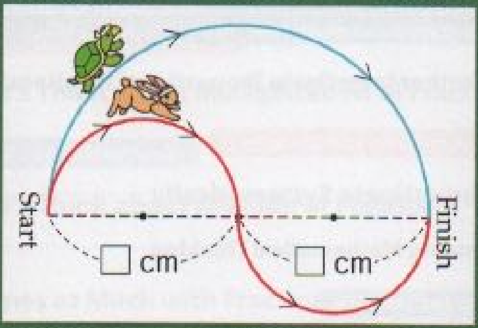
Door ways to Learning

Grasp the problem.

- What problem are we going to work on today?

1

A hare and a tortoise travel along the paths that are made up by parts of circumferences as shown below. Compare the two paths in length.



Circumference = Diameter × Pi

- What idea may be useful to solve the problem?
- Is there anything you have learned before that you can use to solve this problem?

① Plan how to find the answer.

Misaki: We don't know the radius of the big circle.

Riku: The same number goes in both of the □ s.

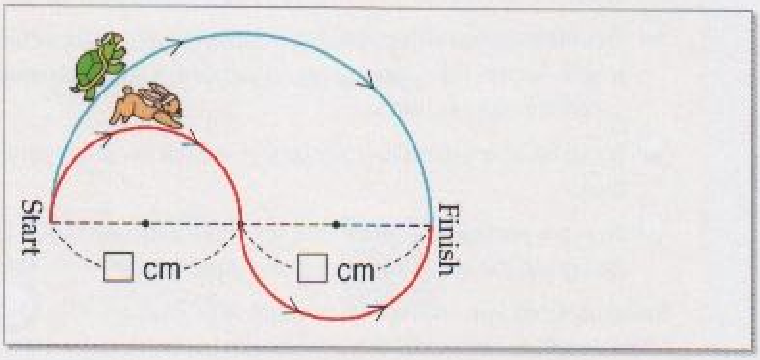
Ami: If we assume that □ is 10...

Let's think about how to compare the two paths in length.

Write down your ideas.

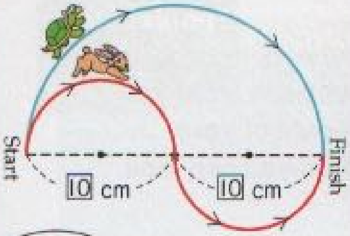
- Is your idea clear to others?

② Write a diagram or a math sentence to show your idea.



Misaki and Kota are explaining their classmates' ideas.

Ami



Tortoise

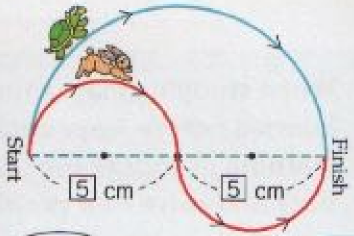
$(10\times2)\times3.14\div2$
$=31.4$

Hare

$(10\times3.14\div2)\times2$
$=31.4$

The two paths are the same in length.

Haruto



Tortoise

$(5\times2)\times3.14\div2$
$=15.7$

Hare

$(5\times3.14\div2)\times2$
$=15.7$

The two paths are the same in length.

While Ami assumed that the radius of the big circle was 10 cm, Haruto assumed that it was 5 cm...

Misaki

③ Look at the math sentence Ami and Haruto wrote. Explain their ideas.

④ Explain Riku's idea.

Riku

In Ami's idea,
$(10\underline{\times2})\times3.14\underline{\div2}=10\times3.14$
$(10\times3.14\underline{\div2})\underline{\times2}=10\times3.14$
The lengths of both paths are calculated by multiplying the radius of the big circle, 10 cm, by...

What about Haruto's idea?

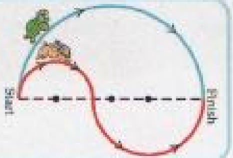
⑤ Look back and summarize today's lesson.

Summary

We can tell that the two paths are the same length either **by placing a number in the □ to find their lengths or by examining the math sentences carefully.**

Kota

What if the hare's path does not pass through the center of the big circle...



Ami

The next page shows a photocopy of my notebook page.

Learn with your classmates.

- Can you understand your classmates' ideas based on their diagrams and math sentences?
- What is common and what is different about your own idea and your classmates' ideas?
- What are the good points in your classmates' ideas?

Look back and summarize today's lesson.

- What did you learn from today's investigation?
- Which way of thinking was useful?
- What do you want to investigate next?

Keep a Math Notebook

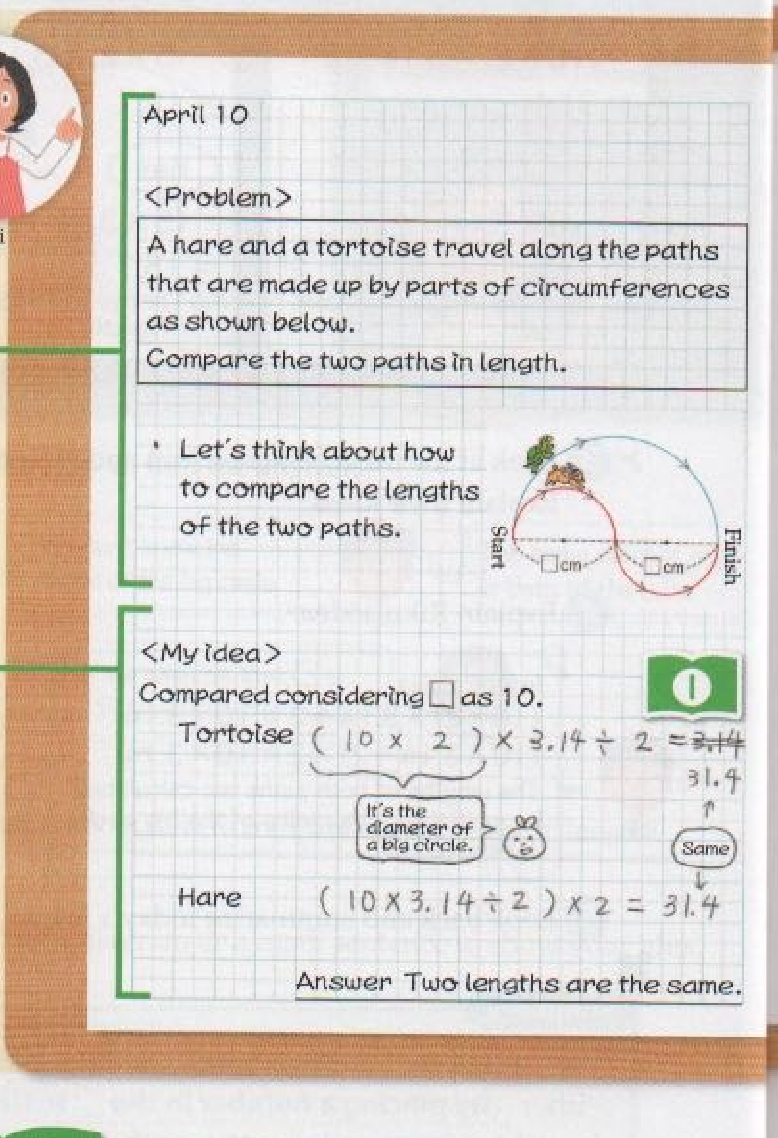
When studying mathematics, you use what you learned before. Keep a good record of your learning in your notebook so that you can look back and solve new problems.

I wonder what Ami wrote in her notebook.

Ami

Write down today's date and math problem, and grasp the problem.

Write down your idea.



April 10

<Problem>

A hare and a tortoise travel along the paths that are made up by parts of circumferences as shown below.
Compare the two paths in length.

- Let's think about how to compare the lengths of the two paths.

<My idea>

Compared considering □ as 10. ①

Tortoise $\underbrace{(10 \times 2)} \times 3.14 \div 2 = ~~3.14~~$ 31.4

It's the diameter of a big circle.

Same

Hare $(10 \times 3.14 \div 2) \times 2 = 31.4$

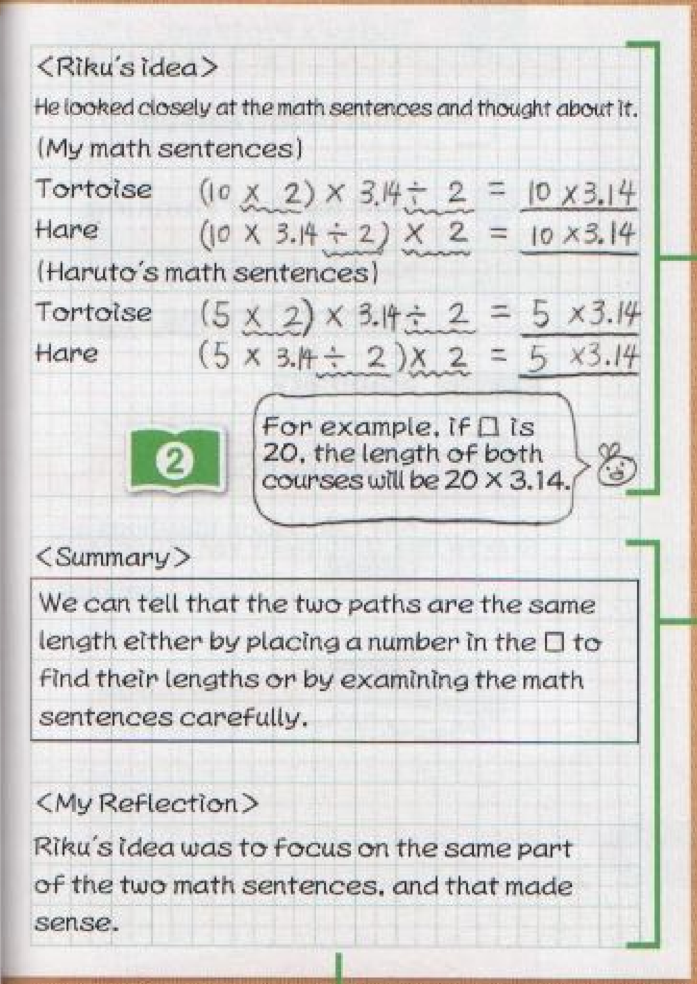
Answer Two lengths are the same.

Note Taking Tip ①

If she made a mistake, she did not erase it. she drew a line ═ through it.

Note Taking Tip ②

In a balloon, she wrote about what she needed to be careful of what she noticed during the lesson.



<Riku's idea>

He looked closely at the math sentences and thought about it.

(My math sentences)

Tortoise $(10 \underline{\times 2}) \times 3.14 \underline{\div 2} = \underline{10 \times 3.14}$

Hare $(10 \times 3.14 \underline{\div 2}) \underline{\times 2} = \underline{10 \times 3.14}$

(Haruto's math sentences)

Tortoise $(5 \underline{\times 2}) \times 3.14 \underline{\div 2} = \underline{5 \times 3.14}$

Hare $(5 \times 3.14 \underline{\div 2}) \underline{\times 2} = \underline{5 \times 3.14}$

②

For example, if □ is 20, the length of both courses will be 20 × 3.14.

<Summary>

We can tell that the two paths are the same length either by placing a number in the □ to find their lengths or by examining the math sentences carefully.

<My Reflection>

Riku's idea was to focus on the same part of the two math sentences, and that made sense.

Write down your classmates' ideas that you thought were useful.

Look back and summarize today's lesson.
Write down your reflections on today's lesson.

In ⟨My Reflection⟩, write down ideas like those listed below so that you can check your progress:

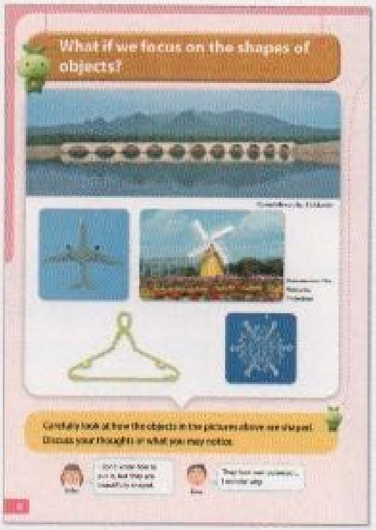
- What you noticed
- What you have learned to do
- What you want to investigate next
- What you thought as you listened to your classmates' ideas

Doorways to Learning

A Guide to Learning with This Textbook

Step to Learning

Entry to Learning


What if we focus on the shapes of objects?

Carefully look at how the objects in the pictures above are shaped. Discuss your thoughts on what you may notice.

Discuss the following with your classmates and set a goal of the lesson:

- **Math in your daily life**
- **What you have studied so far**

Today's Learning

1 **Today's Problem**

Goal of the Lesson

① **Hint for Your Thinking**

Key Viewpoints and Ways of Thinking

Summary **Summary**

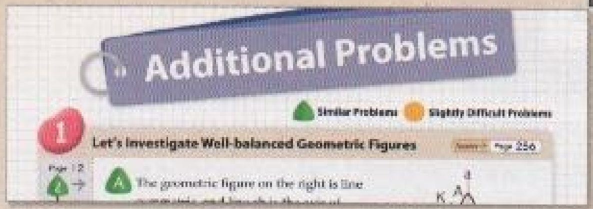
1 **Practice Problem**

Key calculation questions are colored.

Additional Problems
→Page246 A

Properties of Multiplication
Page 272①

New Mathematics Plus (Additional Material)


Additional Problems

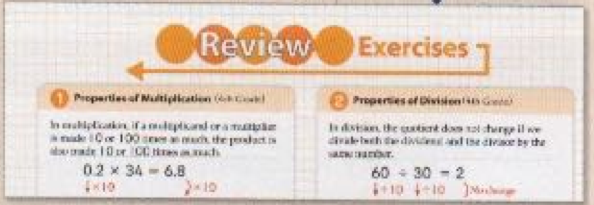
Similar Problems Slightly Difficult Problems

1 Let's Investigate Well-balanced Geometric Figures Page 256

Page 12

A The geometric figure on the right is line

Need more practice? Try these!
The answers are toward the back of the textbook so that you can check them yourself.


Review Exercises

① Properties of Multiplication

In multiplication, if a multiplicand or a multiplier is made 10 or 100 times as much, the product is also made 10 or 100 times as much.

$0.2 \times 34 = 6.8$

×10 ×10

② Properties of Division

In division, the quotient does not change if we divide both the dividend and the divisor by the same number.

$60 \div 30 = 2$

÷10 ÷10 No change

Use this section when you want to review what you studied before.


"New Mathematics Plus" is useful if you want to study more.

Wrapping Up Your Learning

Wrapping Up Your Learning: Symmetric Figures

Use What You Have Learned

Put what you just studied into use.

Wrapping Up Your Learning: Symmetric Figures

Check Your Understanding

Review what you just studied and practice.

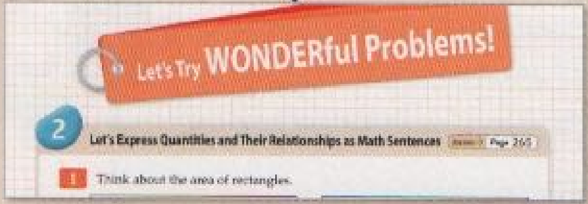
Wrapping Up Your Learning: Symmetric Figures

Grow Your "Eyes for Math" —Key Viewpoints and Ways of Thinking

Summarize key viewpoints and ways of thinking.

Challenge Yourself

→ Page 258



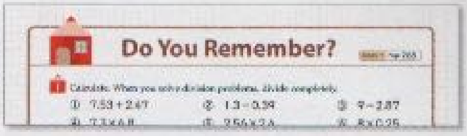
Let's Try WONDERful Problems!

2 Let's Express Quantities and Their Relationships as Math Sentences Page 265

1 Think about the area of rectangles.

Use this section to explore wider and dig deeper. The answers are toward the back of the textbook so that you can check them yourself.

Other Pages



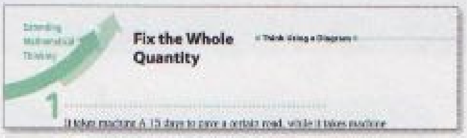
Do You Remember?

1 Calculate. When you solve division problems, divide completely.

① 7.53+2.47 ② 1.3−0.39 ③ 9−2.87

④ 7.3×4.8 ⑤ 2.56×2.4 ⑥ 8×0.25

Review what you studied before. The answers are toward the back of the textbook so that you can check them yourself.



Fix the Whole Quantity

Think using a diagram or a table.

Let's Think about Track and Field Records

Find information in graphs and tables and use it to solve problems.

Share What You Think!

Riku

First...then...

Ami

I think... because...

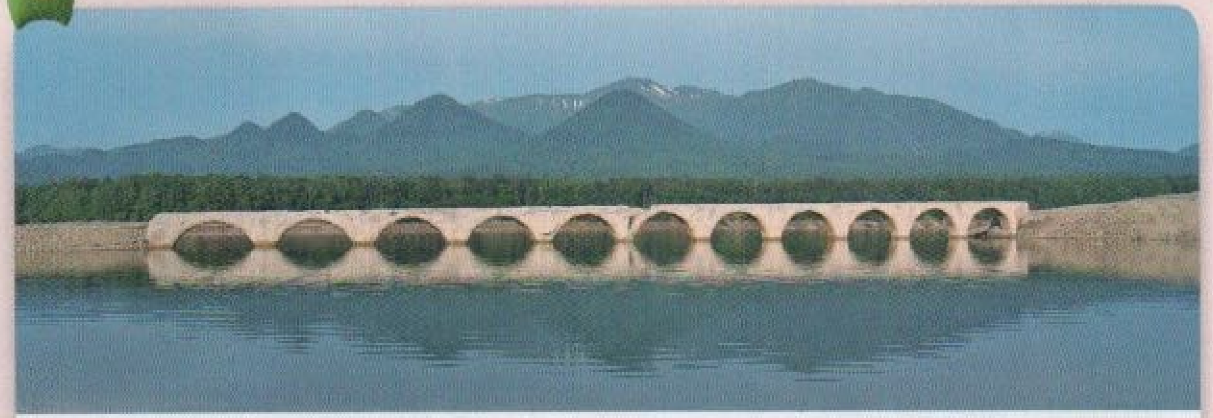
Haruto

You can show that in a diagram or a math sentence like this...

Notice to Teachers and Parents

- Sections marked with Advanced include something not covered by the Japanese government course guidelines for 6th grade. Not all students need to work on these sections.
- **"Do You Remember?"** sections are not for studying new topics and not included in the number of class hours. Use these sections for self-study and home learning.

What if we focus on the shapes of objects?



Kamishihoro-cho, Hokkaido

Hamamatsu City, Shizuoka Prefecture

Carefully look at how the objects in the pictures above are shaped. Discuss your thoughts or what you may notice.

Shiho

I don't know how to put it, but they are beautifully shaped.

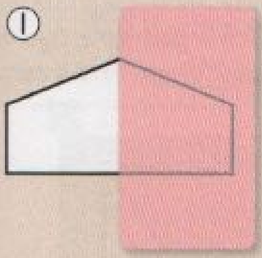
Kota

They look well-balanced... I wonder why.

1 Symmetric Figures

Let's Investigate Well-balanced Geometric Figures

Each of the well-balanced geometric figures is half covered as shown below. Guess the whole shapes from the visible halves.

①


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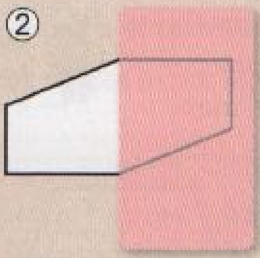

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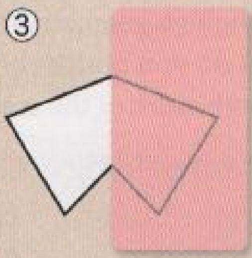

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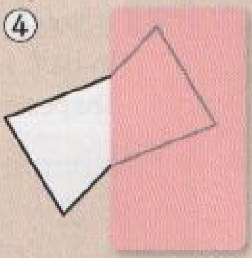

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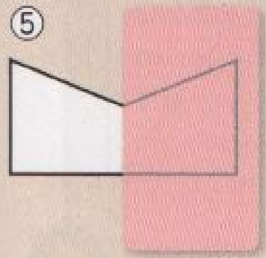

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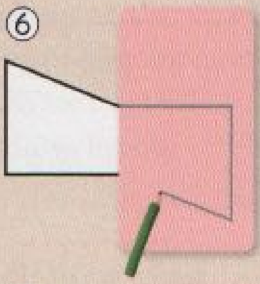

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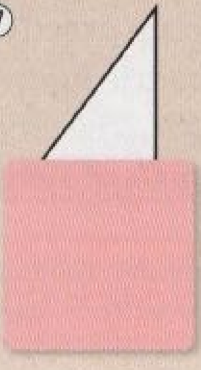

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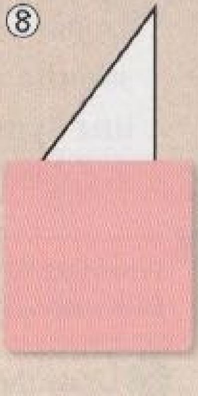

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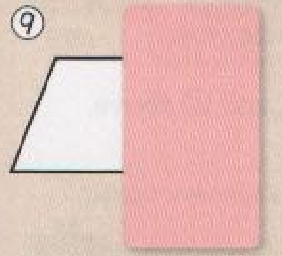
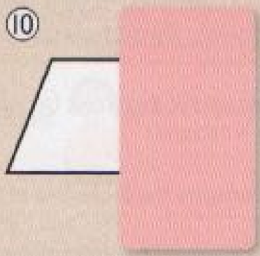


⑩


The whole shapes are on pages 10 and 14.

1 Line Symmetry

1 Put the geometric figures you have completed above into two groups depending on their shapes.

Think about how Riku's sorting continues.

Riku

Ⓐ


Ⓑ

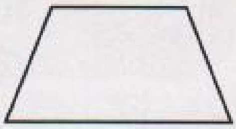

Riku put the following five geometric figures into group Ⓐ.

ⓐ


ⓑ


ⓒ


ⓓ


ⓔ


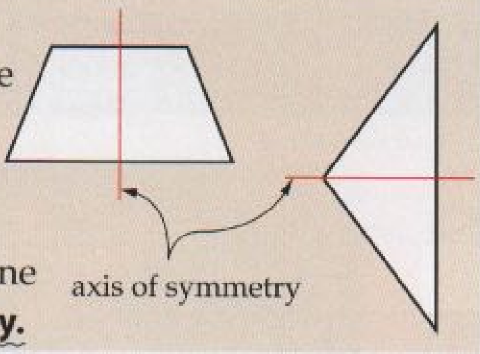
Let's investigate the characteristics of the shapes of the geometric figures above.

(Move)

① If you fold the five geometric figures above in half, what do you notice about the shapes on both sides of the crease line?

Cut out the geometric figures on page 279 and find it out.

When a geometric figure is folded along a line, and if the shapes on both sides of the crease match up exactly, we say the geometric figure is **line symmetric**. The crease line is called the **axis of symmetry.**



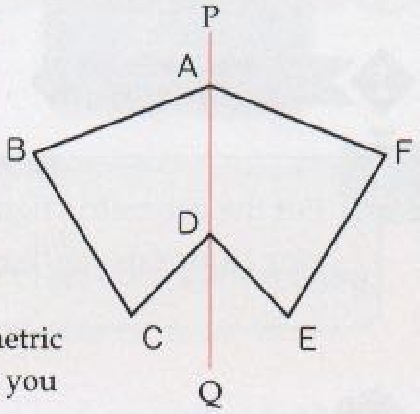
When each of the geometric figures above is **folded in half, the shapes on both sides match up exactly.** So, these geometric figures are line symmetric.

② Draw the axis of symmetry in geometric figures ⓐ, ⓑ, and ⓒ above.

I want to investigate the properties of line symmetric figures.

2

The geometric figure on the right is line symmetric, and line PQ is the axis of symmetry.
Using the figure on the right, investigate the properties of line symmetric figures.



③ To investigate the properties of line symmetric figures, what elements of these figures do you need to focus on?

Let's make the properties of line symmetric figures clear by focusing on the lengths of sides and the measures of angles.

In line symmetric figures, the matching sides, angles, and points are called corresponding sides, angles, and points.

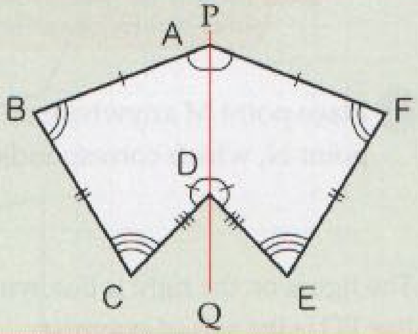
We also used the word "corresponding" when we talked about congruent figures, didn't we?

Congruent Figures
Page 274 ⑱

④ Examine the lengths of corresponding sides and the measures of corresponding angles in the figure in 2.

Summary

- In line symmetric figures, **the lengths of corresponding sides and the measures of corresponding angles are equal.**
- The two figures separated by the axis of symmetry are congruent.



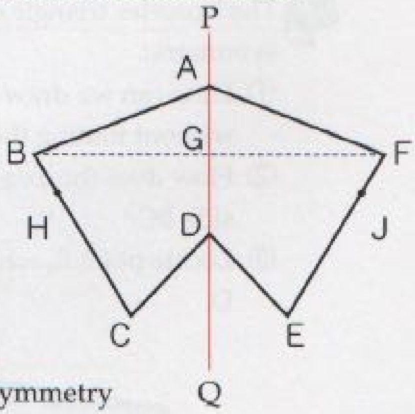
When we focused on the lengths of corresponding sides and the measures of corresponding angles, we were able to make properties of line symmetric figures clear, just like we did with congruent figures.

I wonder if there is any other property of line symmetric figures.

3 Let's investigate the properties of line symmetric figures in more detail.

① How does line BF, which connects two corresponding vertices, intersect the axis of symmetry, PQ?

② Find the lengths of line BG and line FG.



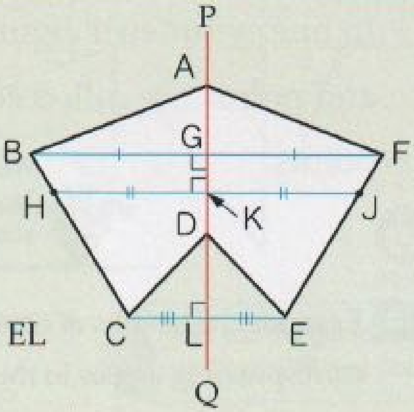
What about the other corresponding points?

Let's investigate the relationship between the axis of symmetry and the lines connecting two corresponding points.

Summary

In line symmetric figures, the lines connecting two corresponding points are perpendicular to the axis of symmetry. Moreover, the lengths from the point of intersection to the two corresponding points are equal.

BG = FG   HK = JK   CL = EL

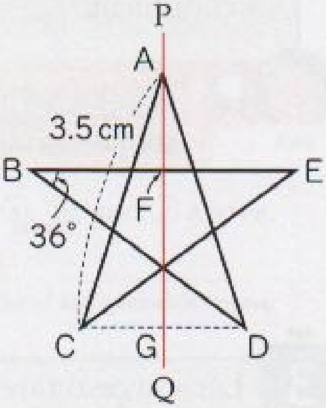


We picked a few combinations of two corresponding points and examined the way the lines connecting the points intersected the axis of symmetry. In all of these cases, the above held true.

③ Place point M anywhere on side BC in the figure above. Then, locate point N, which corresponds to point M.

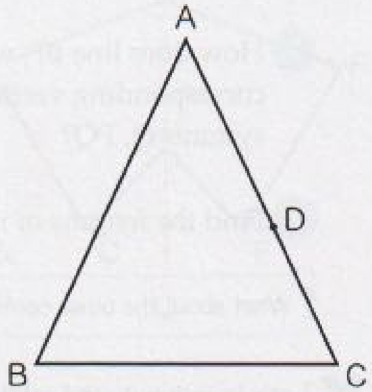
1 The figure on the right is line symmetric and line PQ is the axis of symmetry.

① How many cm is line AD?
② What is the measure of angle E?
③ Which lines are equal in length to line BF and DG, respectively?
④ Other than PQ, how many other axes of symmetry are there?



2 The isosceles triangle on the right is line symmetric.

① How can we draw the axis of symmetry without folding the triangle in half?
② How does the axis of symmetry intersect side BC?
③ Locate point E, which corresponds to point D.



Additional Problems
→Page246 A


Kota: I want to draw a line symmetric figure.

4 Draw a line symmetric figure.

Misaki: Using the properties of line symmetric figures...

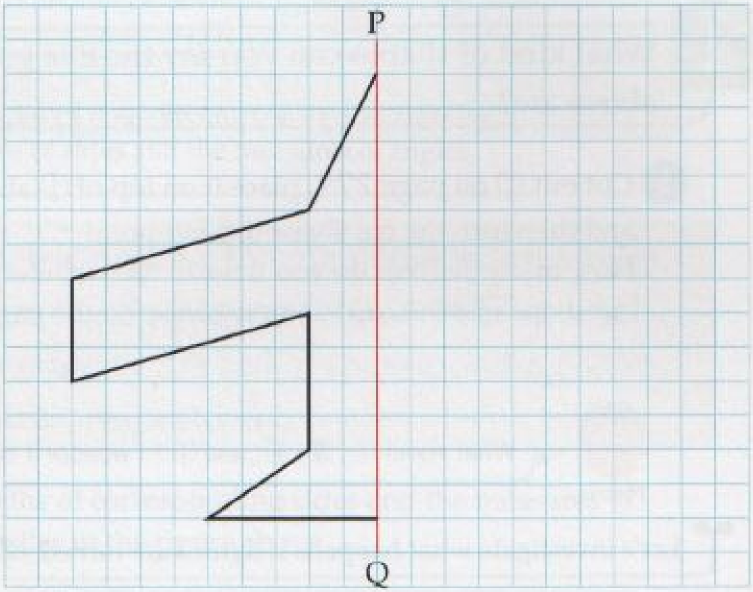
Let's think about how to draw a line symmetric figure using the properties of line symmetric figures.

① On the grid below, draw a figure that will be line symmetric around the axis PQ.

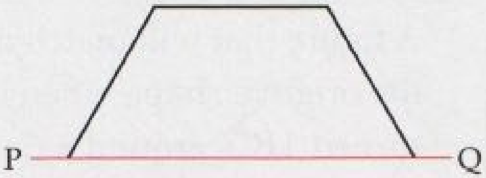
Explain what property you used.

We should **use the relationship between the axis of symmetry and the lines connecting two corresponding points.**

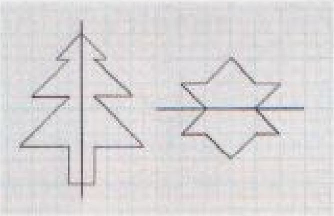
Riku



3 Draw a figure that is line symmetric around the axis PQ. What is the figure called?

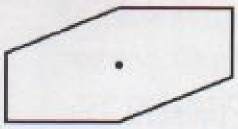


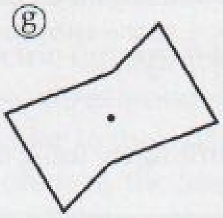
4 Draw your own line symmetric figures in your notebook by first drawing the axis of symmetry.

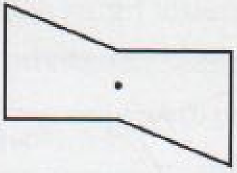


Shiho: I also want to investigate the geometric figures we put into group Ⓑ on page 9.

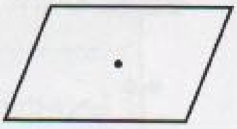
On page 9, Riku put the following five geometric figures into group Ⓑ.

ⓕ


ⓖ


ⓗ


ⓘ

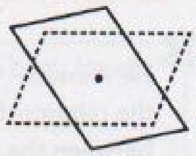

ⓙ


(Move)

2 Point Symmetry

1 What kind of shapes can you say the five geometric figures above are?

① Cut out ⓙ on page 279, place it on top of ⓙ above, and then turn the cut shape around point •. How many degrees do you have to turn it before the cut shape matches up with the shape on the page?



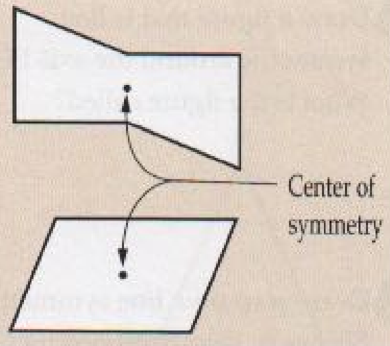

Haruto

What about ⓕ, ⓖ, ⓗ, and ⓘ? I wonder if the same thing as ⓙ happens.

Let's investigate what happens if figures are turned 180° around a point.

② Turn ⓕ, ⓖ, ⓗ, and ⓘ 180° around point •.

A figure that will match up the original shape when it is turned 180° around a point is called a **point symmetric** figure. This point is called the **center of symmetry.**



ⓕ, ⓖ, ⓗ, and ⓘ, too, are point symmetric because they **match up the original shape when they are turned 180° around •.**

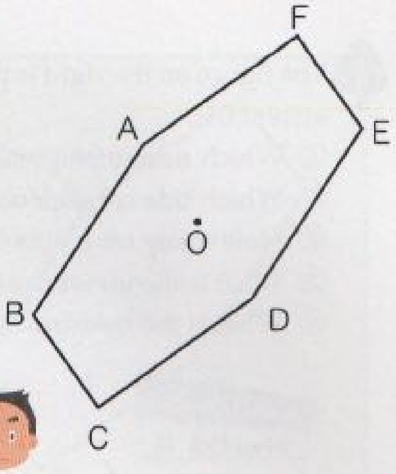

Misaki


Riku

I want to investigate the properties of point symmetric figures.

2

The figure on the right is point symmetric. Point O is the center of symmetry.
Using the figure on the right, investigate the properties of point symmetric figures.



When we investigated line symmetric figures, we focused on...


Kota

Let's make the properties of point symmetric figures clear by focusing on the lengths of sides and the measures of angles.

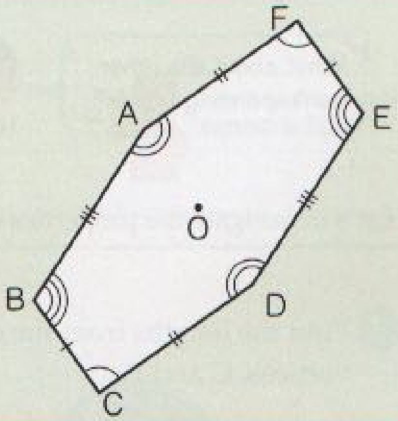
In point symmetric figures, the matching sides, angles, and points when the figures are rotated 180° around the center of symmetry are called corresponding sides, angles, and points, respectively.

① Examine the lengths of corresponding sides and the measures of corresponding angles in the figure above.

② Split the figure below in half with a line that passes through the center of symmetry. What is the relationship between the halves made?

Summary

- In point symmetric figures, **the lengths of corresponding sides and the measures of corresponding angles are equal.**
- The halves separated by a line that passes through the center of symmetry are congruent.



When we focused on the lengths of corresponding sides and the measures of corresponding angles, we were able to make properties of point symmetric figures clear, just like we were able to do with line symmetric figures.

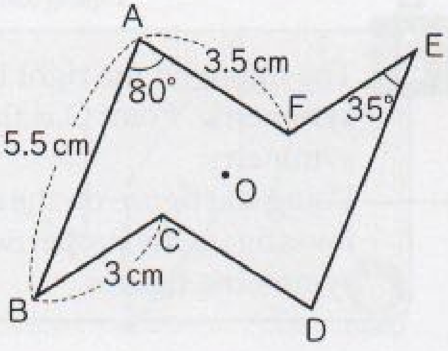
1 The figure on the right is point symmetric.

① Which side corresponds to side AB?
Which side corresponds to side EF?

② How many cm is side CD?

③ What is the measure of angle B?

④ What is the measure of angle D?



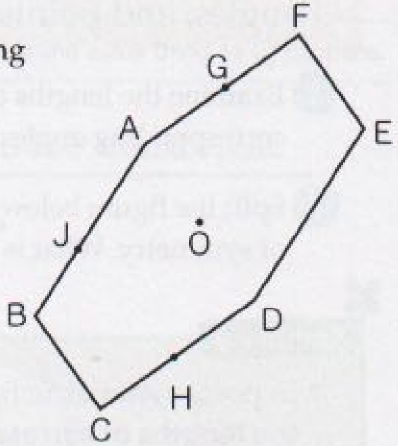
Additional Problems
→Page246 B

When we studied line symmetric figures, we examined the segments connecting corresponding points. I want to examine such segments in point symmetric figures, too.

3 Investigate the properties of point symmetric figures in more detail.

① Lines AD and BE connect two corresponding vertices. Where do these lines intersect?

② Find the lengths from the center of symmetry, O, to the corresponding two vertices, A and D.



What about the other corresponding points?

Let's investigate the properties of lines connecting two corresponding points.

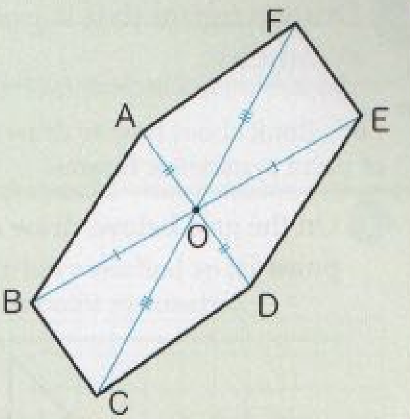
③ Find the lengths from the center of symmetry to the corresponding two vertices, C and F.

④ Point H corresponds to point G.
Find the lengths from the center of symmetry to H and G.

Summary

In a point symmetric figure, the lines connecting two corresponding points will pass through the center of symmetry.
Moreover, the lengths from the center to two corresponding points are equal.

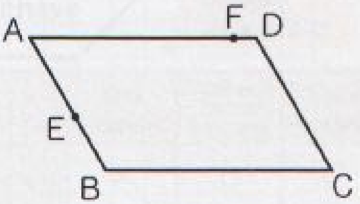
AO = DO BO = EO CO = FO



We picked a few combinations of lines connecting two corresponding points and examined the way the lines intersected each other. In all of these cases, the above held true.

5 In the figure in 3, locate point K, which corresponds to point J.

2 The parallelogram on the right is point symmetric.
① Locate the center of symmetry, O.
② Locate point G, which corresponds to point E. Locate point H, which corresponds to point F.



3 In the pictures on page 8, find line symmetric shapes and point symmetric shapes.

Using what you have studied, explain the properties of well-balanced shapes.

Shiho: You can find symmetric shapes everywhere, such as in map symbols and prefecture logos.

Kota: I want to draw a point symmetric figure.

Mathlin Newsletter

A Point Symmetric Figure?

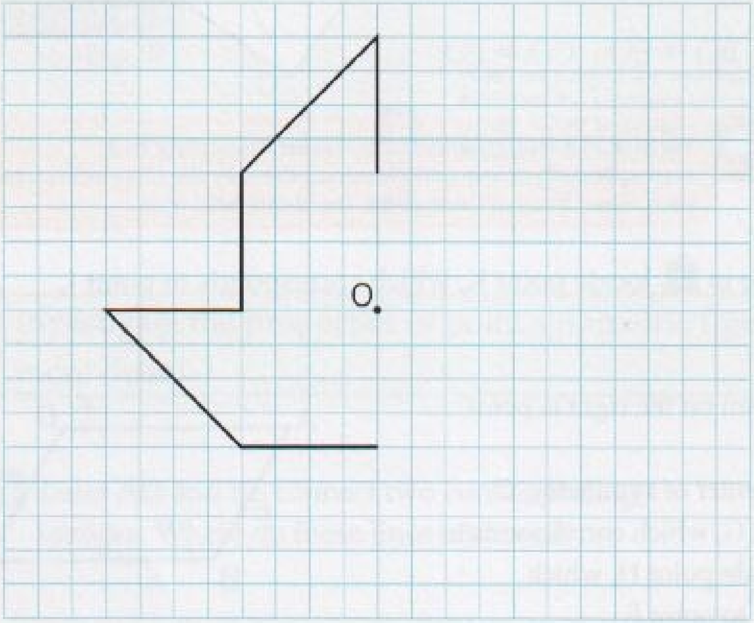
Is the figure on the right point symmetric?
What do you think?

4 Draw a figure that is point symmetric.

Riku: Using the properties of point symmetric figures...

Let's think about how to draw a point symmetric figure using the properties of point symmetric figures.

① On the grid below, draw a figure that is point symmetric around point O.

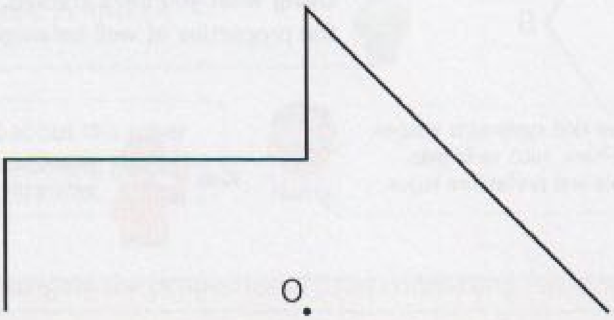


Explain what property you used.

You should **use the property that a segment connecting two corresponding points passes through the center of symmetry.**

Ami

4 Draw a figure that is point symmetric around point O.



Shiho: When you draw a geometric figure, you should use the properties of the geometric figure.

3 Polygons and Symmetry

1

Investigate to see if polygons that we have studied so far are line symmetric or point symmetric.

Let's re-examine the geometric figures that we have studied so far by focusing on whether they are line symmetric or point symmetric.

Quadrilaterals Re-examine the following quadrilaterals.

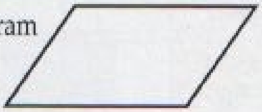
① Which quadrilaterals are line symmetric? Draw all the axes of symmetry.

② Which geometric figures are point symmetric? Draw the center of symmetry.

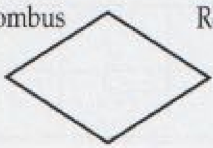
③ Which geometric figures are line symmetric and have diagonals that are the axes of symmetry?
Which line symmetric quadrilateral does not have diagonals that are the axis of symmetry?

Diagonal
Page 274 ⑰

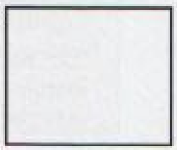
Parallelogram



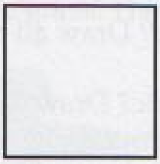
Rhombus



Rectangle



Square



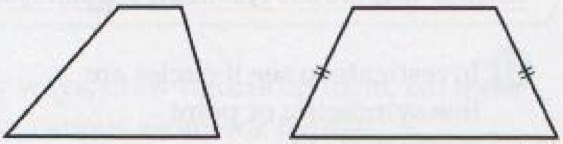
	Line Symmetric	Number of Axes of Symmetry	Point Symmetric
Parallelogram	×	0	○
Rhombus			
Rectangle			
Square			

Various Quadrilaterals
Page 274 ⑯

Organize your findings into a table.

④ Look at the figures and the table above and share what you notice.

⑤ Examine the trapezoids on the right in the same way as above.



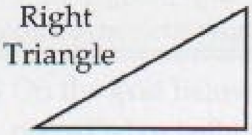
(Isosceles trapezoid)

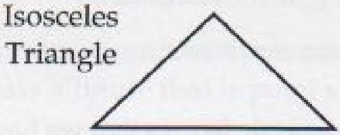
Haruto

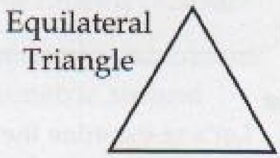
What about the other polygons?

Triangle Re-examine the following triangles.

Different Kinds of Triangles
Page 274 ⑤


Right Triangle

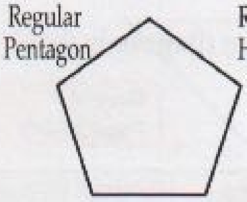

Isosceles Triangle

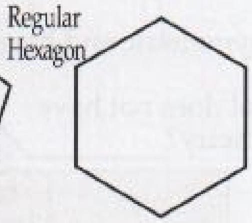

Equilateral Triangle

⑥ Which triangles are line symmetric? Draw all the axes of symmetry.

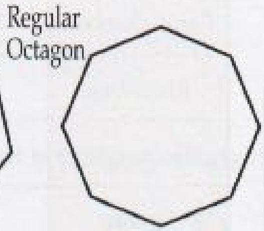
⑦ Is there any point symmetric figure here?

Regular Polygons Re-examine various regular polygons.


Regular Pentagon


Regular Hexagon


Regular Heptagon


Regular Octagon

	Line Symmetric	Number of Axes of Symmetry	Point Symmetric
Equilateral Triangle	○	3	×
Square			
Regular Pentagon			
Regular Hexagon			
Regular Heptagon			
Regular Octagon			

⑧ Which regular polygons are line symmetric? Draw all the axes of symmetry.

⑨ Which regular polygons are point symmetric? Draw the center of symmetry.

⑩ Look at the figures and the table above and share what you notice.

We can find additional properties and relationships of the geometric figures we have studied so far if we focus on whether they are line symmetric or point symmetric.


Riku

⑪ Investigate to see if circles are line symmetric or point symmetric.

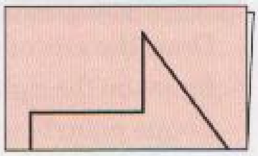
Additional Problems
→Page 246 C

Wrapping Up Your Learning: Symmetric Figures

Use What You Have Learned

Cut paper to make various geometric figures.

① On a piece of paper that is folded in two, draw the shape on the right. Then, cut out the shape and unfold it.



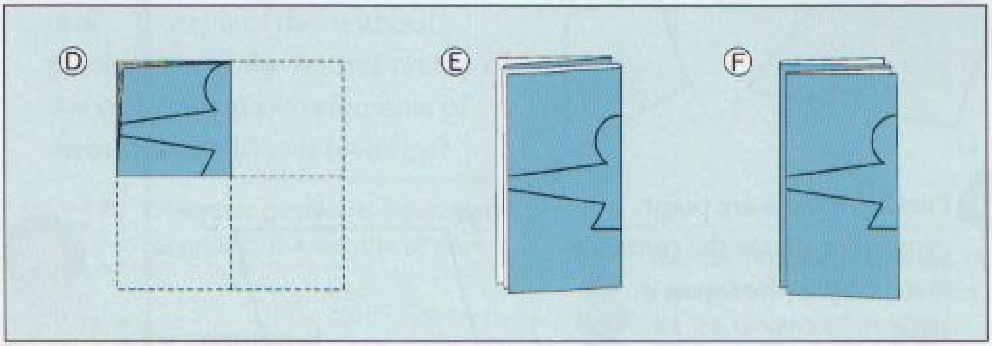
Before unfolding it, guess what shape will appear.

② Like you did in ①, cut out different shapes from pieces of paper that are folded in two, and then unfold the shapes to make various line symmetric figures.

Haruto

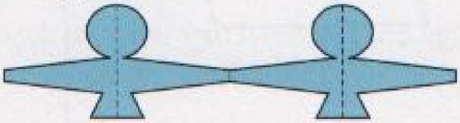
We are using a property of line symmetric figures.

③ If you cut out the shapes that are drawn on pieces of paper Ⓓ, Ⓔ, and Ⓕ and unfold the shapes, which geometric figures below will appear?

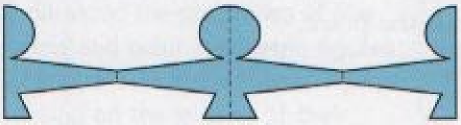


First, take a guess. Then, cut out the pieces of paper and see if you guessed correctly.

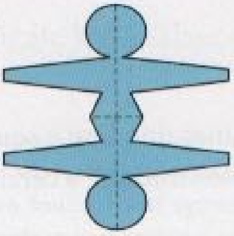
Ⓐ



Ⓑ



Ⓒ

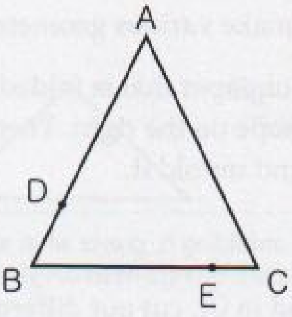


④ Fold pieces of paper in many ways, draw figures on them, cut them out, and unfold them to make various geometric figures.

Wrapping Up Your Learning: Symmetric Figures

Check Your Understanding

☐ **1** Isosceles triangles are line symmetric.
Draw the axis of symmetry in the figure on the right.
Locate point F, which corresponds to point D. Locate point G, which corresponds to point E.

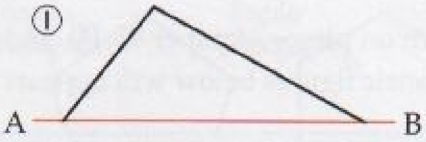


Can you locate the axis of symmetry and corresponding points of a line symmetric figure?

Page 11 **3**

☐ **2** Draw a figure that will be line symmetric around axis AB below. Draw a figure that is point symmetric around point O.

①

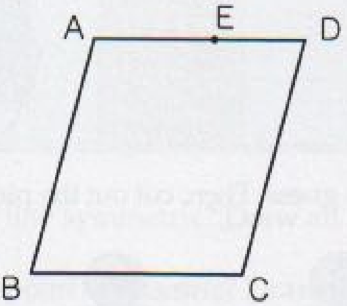


②

Can you draw line symmetric and point symmetric figures?

① Page 13 **4**
② Page 18 **4**

☐ **3** Parallelograms are point symmetric. Draw the center of symmetry in the figure on the right.
Also, locate point F, which corresponds to point E.



Can you locate the center of symmetry and corresponding points of a point symmetric figure?

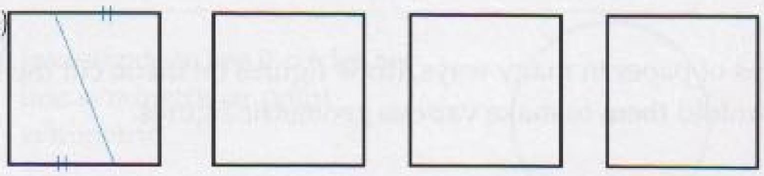
Page 16 **3**

☐ **4** Lines dividing a square into two congruent figures always pass through a certain point. Locate the point.

Can you focus on symmetry and tell the properties of squares?

Page 19 **1**

(Example)

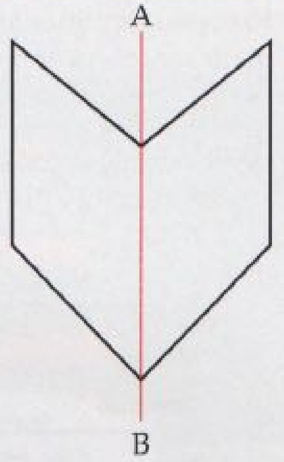


Wrapping Up Your Learning: Symmetric Figures

Grow Your "Eyes for Math" – Key Viewpoints and Ways of Thinking

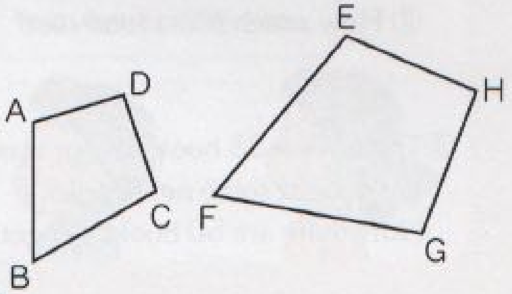
Focus on the Lengths of Sides and the Measures of Angles and Think about the Properties of Geometric Figures

① The geometric figure on the right is line symmetric around axis AB.
Riku is explaining the properties of the geometric figure on the right. What elements of the figure is he focusing on?



The lengths of their ⓐ☐ ⓑ☐ and the measures of their ⓐ☐ ⓒ☐ are equal.

② The two geometric figures on the right are not congruent to each other. To explain this without placing one of the figures on top of the other, what two elements of these figures do you focus on?



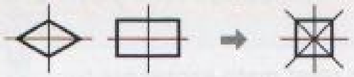
When two geometric figures are congruent, the lengths of their ⓕ☐ ⓖ☐ are...
the measures of ⓕ☐ ⓗ☐ are...

You can find the properties of a geometric figure if you find out the lengths of its sides and the measures of its angles.

Look back on what you have learned in "Let's Investigate Well-balanced Geometric Figures" and discuss.

We understood the properties of line symmetric and point symmetric figures well when we investigated these figures by focusing on the lengths of their sides and the measures of their angles.

I found it interesting that squares have four axes of symmetry while rhombus and rectangles have two.



Junior High School

In junior high school, you will dig deeper into symmetric figures and study in detail what happens when you move geometric figures.

Let's review math sentences

Express quantities and their relationships in situations Ⓐ and Ⓑ below as math sentences and think about them.

ⒶWe packed 3 cookies that cost 150 yen per piece in a box that cost 100 yen.

① Write a math sentence to express the total cost.

Math Sentence ▶

② How much is the total cost?

ⒷThere were 38 books in our class library.
We bought some new books.
Now there are 50 books altogether.

① Use □ as the number of new books we bought and write a math sentence representing the relationship among quantities.

Math Sentence ▶

② What is the number that fits in □ ?

The number of books there are now is the sum of 38 and □, and it is 50.

In our previous study, we used math sentences in what kinds of situations?
Discuss with your classmates.

We used math sentences to find answers to questions and to express our ideas clearly.

Ami

We used □ as an unknown number.
When we investigated changes, we used □ and ○ to express many cases as one math sentence.

Riku

I want to study more about math sentences.

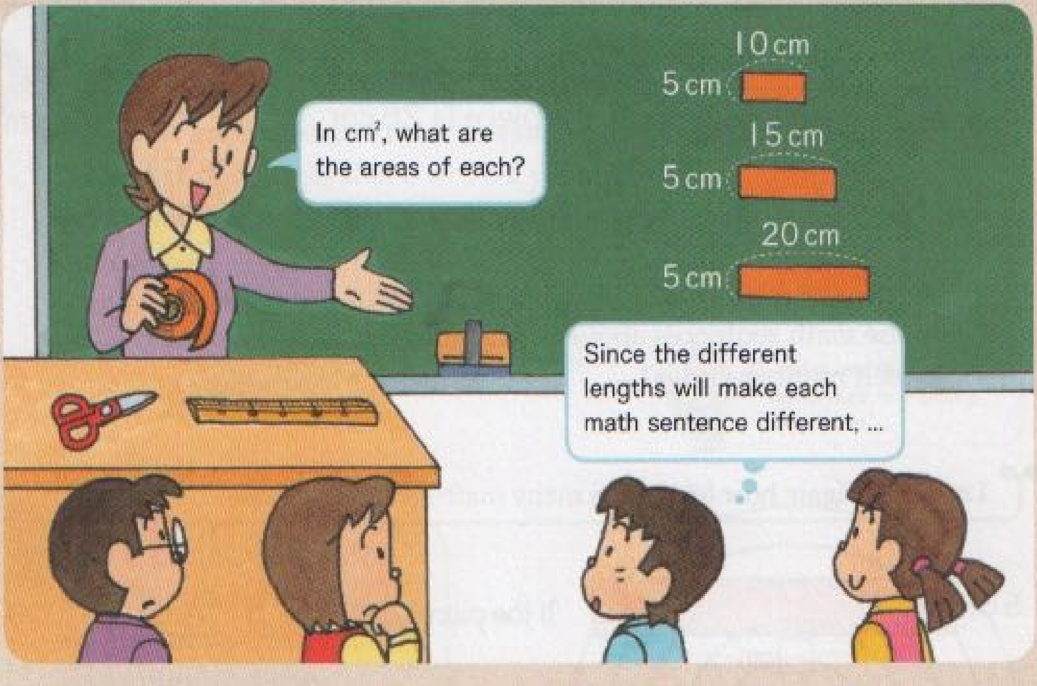
Haruto

Letters and Math Sentences

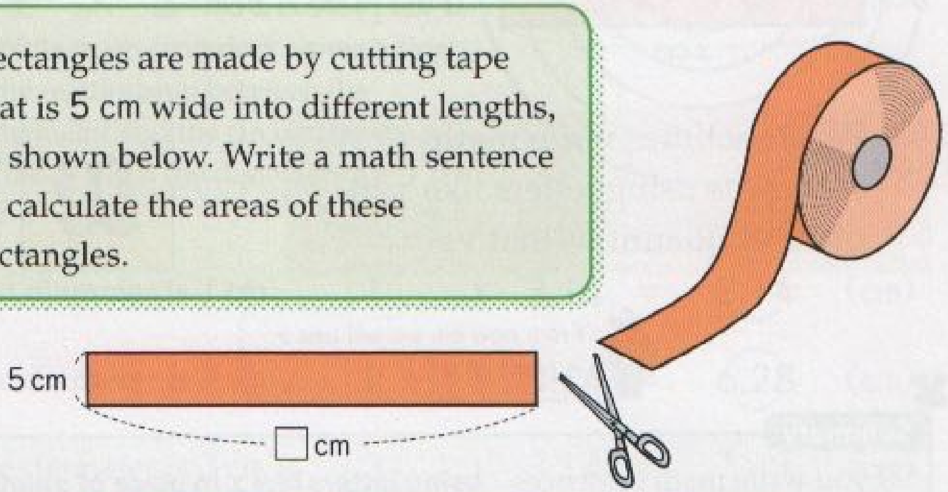
2

Let's Express Quantities and Their Relationships as Math Sentences

A tape that is 5 cm wide is cut into several pieces of different lengths.

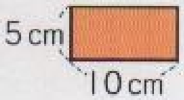


1 Rectangles are made by cutting tape that is 5 cm wide into different lengths, as shown below. Write a math sentence to calculate the areas of these rectangles.



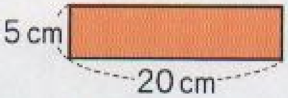
① Write math sentences to calculate the areas if the lengths are 10 cm, 15 cm, 20 cm, 25 cm, ...

Length × Width



If the piece is 10 cm $5 \times \boxed{10}$ (cm^2)

If the piece is 15 cm $5 \times \boxed{15}$ (cm^2)



If the piece is 20 cm $5 \times \boxed{20}$ (cm^2)

If the piece is 25 cm $5 \times \boxed{25}$ (cm^2)

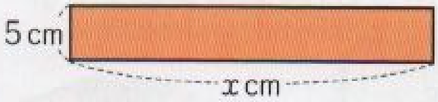
⋮ ⋮ ⋮

② In the math sentences above, which number stays constant? Which number varies?

Let's investigate how to express many math sentences as one.

If the piece is □ cm $5 \times \square$ (cm^2)

↓ ↓ ↓



If the piece is x cm $5 \times x$ (cm^2)

We sometimes write math sentences using letters like x in place of quantities that vary.

From now on, we will use x in place of a □.

Summary

If you write math sentences **using letters like x in place of quantities that vary**, you can summarize many math sentences as one.

③ We are going to figure out the areas of the rectangles when their lengths are 26 cm, 27 cm and 28 cm. Put 26, 27 and 28 in x in the math sentence $5 \times x$, calculate the areas of the rectangles.

④ Using the math sentence $5 \times x$, calculate the area of the rectangle when the number used for x is 7.5.

A decimal number can go into x, can't it?

1 Yuri is going to buy some oranges to give as presents.

① Write a math sentence to represent the total cost when she buys x number of oranges that cost 180 yen each and puts them into a basket that costs 250 yen.

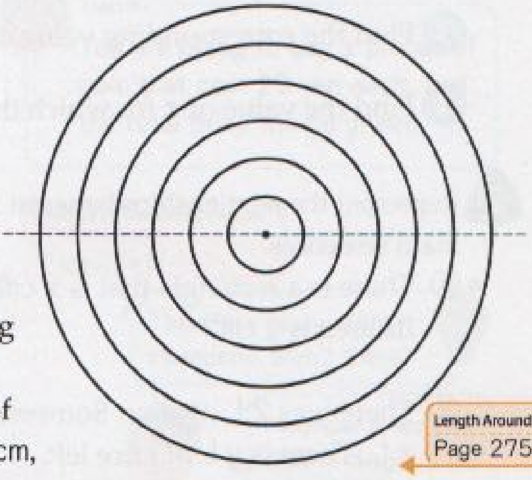
② Find the cost for each when she buys 5 oranges and 12 oranges in situation ①.

Additional Problems
→Page246 D

Riku

Today, we studied math sentences expressing quantities.

2 Write a math sentence representing the relationship between the diameter and the circumference of a circle.



Length Around Circles
Page 275 ㉒

① Write math sentences representing the relationship between the diameter and the circumference of circles when the diameters are 1 cm, 2 cm, 3 cm, ...

Diameter × Pi = Circumference

If the diameter is 1 cm $\boxed{1} \times 3.14 = 3.14$ (cm)

If the diameter is 2 cm $\boxed{2} \times 3.14 = 6.28$ (cm)

If the diameter is 3 cm $\boxed{3} \times 3.14 =$ ○ (cm)

⋮ ⋮ ⋮

Let's investigate how to express many math sentences that represent the relationship between quantities in one math sentence.

If the diameter is $\square$ cm $\square \times 3.14 = \bigcirc$ (cm)

↓ ↓ ↓

If the diameter is x cm $x \times 3.14 = y$ (cm)

The circumference is the product of x and 3.14, and that's represented by y.

Summary

If you use letters such as x and y, you can express the relationship between quantities as one math sentence.

② What number does y represent in the math sentence $x \times 3.14 = y$, when the numbers 10, 15, and 20 are used for x?

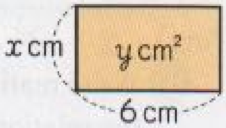
> The 10 that was used for x is called the value of x. The resulting number that y represents, 31.4, is called the corresponding value of y when the value of x is 10.

③ Find the corresponding value of y when the value of x is 2.5.

④ Find the value of x for which the corresponding value of y is 47.1.

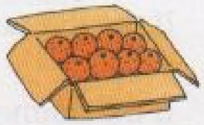
2 Represent the relationships between x and y in the following situations using math sentences.

① There is a rectangle that is x cm wide and 6 cm long. Its area is y cm^2.



② There was 2 L of juice. Someone drank x L. There is y L of juice left.

③ x kg of oranges are put into a box that weighs 0.6 kg. The total weight is y kg.



④ A student is planning to read an x-page book in 10 days. He must read an average of y pages a day.

Average
Page 273 ⑫

Additional Problems
→Page247 E

Kota

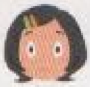
$x \times 3.14 = y$. When the value of x is determined, the value of y is determined, too.

Shiho

Today, we studied math sentences representing relationships between quantities.

3 Come up with a situation where a relationship between quantities is represented by each of the following math sentences.

(1) $20+x=y$ (2) $20-x=y$ (3) $20\times x=y$ (4) $20\div x=y$


Ami

I wonder what situations can be made.

In (1), the sum of 20 and x is y, so...


Riku

Let's think about what we have learned so far and come up with the situations that fit the math sentences.


Kota

I am buying candy for 20 yen and a juice x yen.
The total price is y yen.


Shiho

A rectangle has an area of 20 cm^2. Its length is x cm and its width is y cm.


Ami

There are 20 pieces of origami paper. We are going to use x pieces, and y pieces will be left.


Haruto

You are going to buy x pieces of gum that cost 20 yen each, and the total price will be y yen.


Misaki

I drew a diagram to represent the situation.

Parallelogram

Formula for Calculating the Area of a Parallelogram
Page 275 ㉓

There are many other situations, aren't there?


Riku

I wonder if we can draw a diagram to show Shiho's situation.

① Which student created a situation for math sentence (1)? How about (2) through (4)?

You can create many situations out of math sentences. Math sentences are the "language of math".


Misaki

3 In the math sentences (1) through (4) above, change 20 to a different number and create different situations.

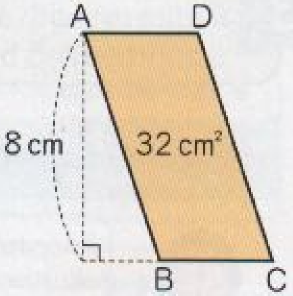
Share the situations you created with others.


Haruto

It looks like we can also create many different situations out of math sentences other than (1) through (4).

4

Look at the parallelogram on the right. If you consider side BC as the base, the height is 8 cm, and the area is 32 cm².
How many cm is side BC?




Misaki

What was the formula for calculating the area of a parallelogram...?

We don't know the length of side BC.


Haruto

Let's think about how to express a relationship between quantities when there is an unknown quantity.

① Let x cm be the length of side BC, and express the relationship between the quantities in a multiplication math sentence.

Math Sentence ▸ $x\times$

② Find the value of x.


Kota

$x\times8=32$

$2\times8=16$

$3\times8=24$

⋮

$x\times8=32$

$x=32\div\cdots$


Shiho

Answer ☐ cm

Summary

If you replace an unknown quantity by a letter like x, you can express the relationship between quantities as a math sentence.

You can express the relationship between quantities with a math sentence just as it is described in text.

4 A car traveling at a certain speed traveled 120 km in 3 hours.
How many km per hour did this car travel?

Let x km be the distance the car travels per hour, and express the relationship between the quantities as a multiplication math sentence.

Speed
Page 273 ①

Mathlin Newsletter

Letters That Can Be Many Numbers

The properties of operations we have studied so far can be expressed with letters such as a, b, and c.

① $a \times b = b \times a$

② $(a \times b) \times c = a \times (b \times c)$

③ $(a + b) \times c = a \times c + b \times c$

④ $(a - b) \times c = a \times c - b \times c$

■×●=●×■

⇩

$a \times b = b \times a$


Kota

Put in numbers for a, b, and c and check that the equal sign holds. In a math sentence, the same letters represent the same numbers.

Letters can also be used in the following cases, for example.

Relationship between division and fractions

$$a \div b = \frac{a}{b}$$


Ami

$5 \div 4 = \frac{5}{4}$, $4 \div 5 = \frac{4}{5}$

(a represents whole numbers, and b represents whole numbers other than 0.)

Property of Fractions

$$\frac{b}{a} = \frac{b \times c}{a \times c}$$

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$


Riku

This math sentence shows the property that multiplying both the denominator and the numerator of a fraction by the same number does not change the size of the fraction.


Shiho

Another property is that dividing both the denominator and the numerator of a fraction, $\frac{b}{a}$, by the same number other than 0, c, does not change the size of the fraction. If you use these letters to express this property, it goes like this: $\frac{b}{a} = \ldots$

Wrapping Up Your Learning: Letters and Math Sentences

Check Your Understanding

☐ **1** Express the following situations as math sentences.

① The amount of tea left if you have 1.2 L of tea and drink x L of it

② x m of tape was shared among 5 people equally. Each person got y m of the tape.

Do you understand how to express quantities and their relationships by using letters?

① Page 25 **1**
② Page 27 **2**

☐ **2** Which relationships among Ⓐ to Ⓒ do math sentences ① to ③ represent? Write the symbol that matches each math sentence.

① $24+x=y$ ② $24-x=y$ ③ $24\times x=y$

Ⓐ You read x pages of a 24-page book. You have y pages to read.

Ⓑ There are x boxes of cookies, and each box contains 24 cookies.
There are y cookies altogether.

Ⓒ There are 24 children and x adults.
There are y people altogether.

Can you understand the situation a math sentence represents?

Page 29 **3**

☐ **3** The length around the rhombus below is 28 cm.

Rhombus
Page 274 ⑯

① Let x cm be the length of each side and express a relationship between quantities as a multiplication math sentence.

② Find the value of x.

Can you write a math sentence using a letter for unknown numbers and find the numbers?

Page 30 **4**

Look back on what you have learned in "Let's Express Quantities and Their Relationships as Math Sentences" and discuss.

Now I know how to write math sentences using letters such as x for numbers that vary or for unknown numbers.

Except that x and y are used instead of □ and ○, what we studied today was the same as before. From now on, I want to use letters in place of symbols.

Junior High School In junior high school, you will study math sentences using letters in detail. Let's get used to using letters little by little.

Challenge Yourself
→ Page 258

Do You Remember?

Answers → Page 268

1 Calculate. When you solve division problems, divide completely.

① 7.53+2.47　② 1.3−0.39　③ 9−2.87

④ 7.3×6.8　⑤ 2.56×2.4　⑥ 8×0.25

⑦ 6.97÷3.4　⑧ 13.4÷5.36　⑨ 30.4÷0.8

⑩ 9.8+4×2.5

Multiplication Algorithm of Decimal Numbers
Division Algorithm of Decimal Numbers
Page 273 ⑦⑨

Warm-up

2 Fill in the $\boxed{}$ with the appropriate numbers.

① $\frac{3}{5}$ is made of $\boxed{}$ pieces of $\frac{1}{5}$

② $1\frac{1}{4}$ is made of $\boxed{}$ pieces of $\frac{1}{4}$

③ $\frac{7}{9} = \boxed{} \div 9$

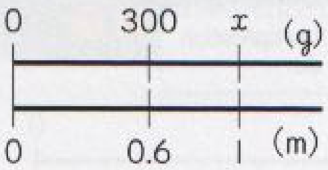
④ $11 \div 8 = \frac{11}{\boxed{}}$

Warm-up

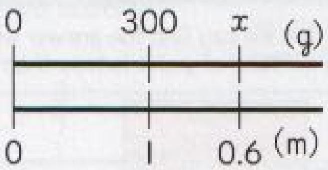
3 There is a hose that weighs 300 g per m. We are going to find the weight of 0.6 m of this hose.

① Let x g be the weight of 0.6 m of this hose. Which number line diagram represents the relationship between quantities correctly?

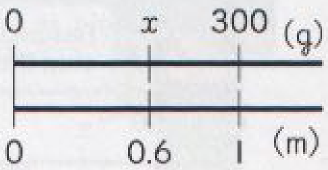
Ⓐ



Ⓑ



Ⓒ



② Write math sentences, then find the answer.

Playing with Numbers and Calculations

Strange Calculations

Calculate Ⓐ and Ⓑ and compare the answers.

① Ⓐ 12×63
Ⓑ 36×21

② Ⓐ 23×64
Ⓑ 46×32

③ Ⓐ 4.8×4.2
Ⓑ 2.4×8.4

In Ⓑ, the order of numerals is reversed from Ⓐ.

Ⓐ 12×63
Ⓑ 36×21

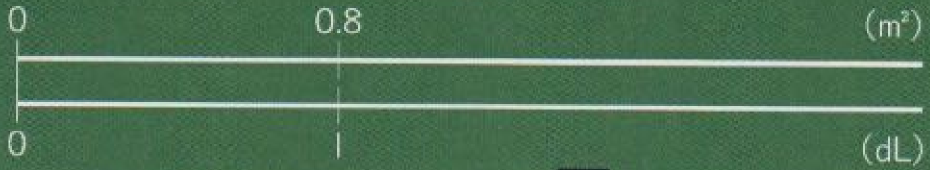
12×63

If you multiply each pair of digits in the same place of the multiplicand and the multiplier, the products will be...

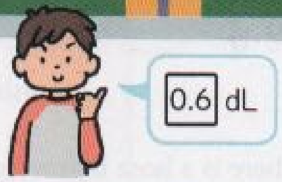
Let's look back on multiplication of decimal numbers

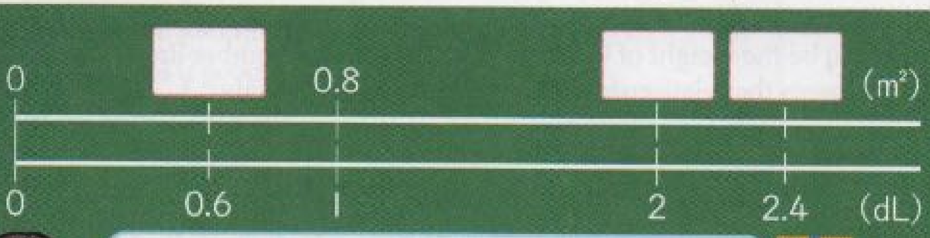
Recall what you learned about multiplication of decimal numbers in 5th grade.

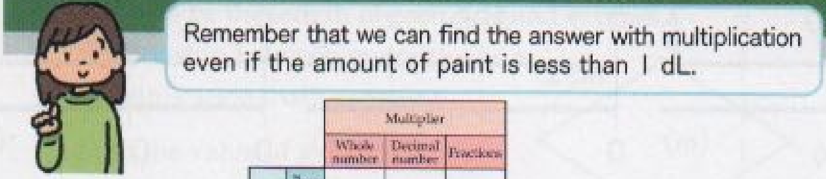
With 1 dL of paint, we can paint 0.8 m² of boards.



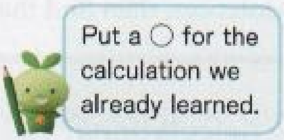
How many m² of boards can we paint with ☐ dL of this paint?







		Multiplier		
		Whole number	Decimal number	Fractions
Multiplicand	Whole number			
	Decimal number			
	Fractions			

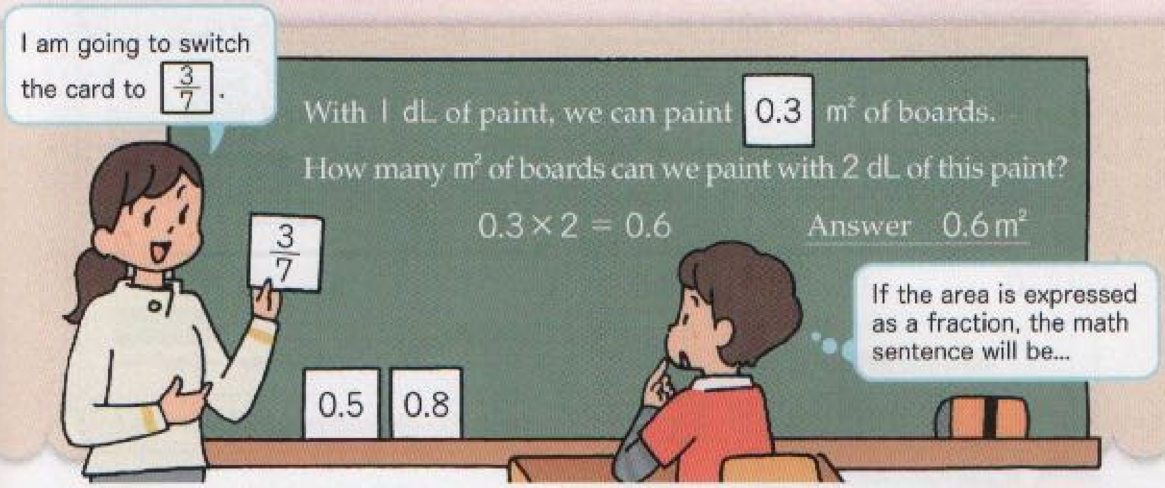


Look at the math problem and table above and discuss the types of calculation we have not studied yet.

3

Multiplication of Fractions

Let's Think about Multiplication of Fractions



With 1 dL of paint, we can paint $\boxed{0.3}$ m² of boards.

How many m² of boards can we paint with 2 dL of this paint?

$0.3 \times 2 = 0.6$ Answer $0.6\,\text{m}^2$

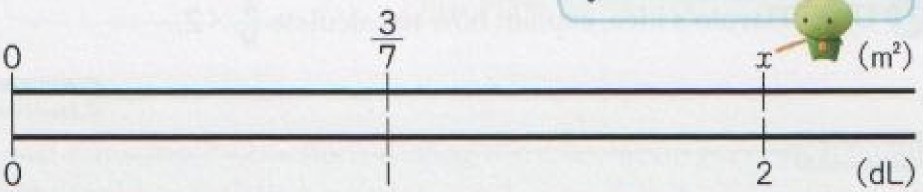
1 Multiplication and Division of Fractions

Fracción propia

1 With 1 dL of paint, we can paint $\frac{3}{7}$ m² of boards.
How many m² of boards can we paint with 2 dL of this paint?

① What math sentence do we need to write?

From now on, let's use letters such as x in place of symbols like □.



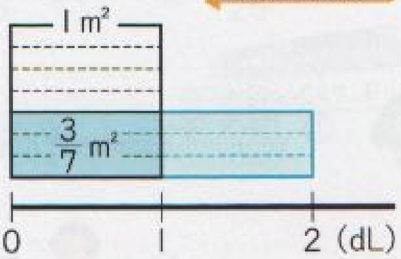
Kota: As the amount of paint becomes 2, 3... times, the area we can be paint also becomes 2, 3... times.

For how to draw and examine the diagram above, see page 270.

Proportional Relationship Page 273 ⑬

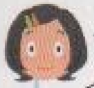
Math Sentence

Explain the reason for your math sentence.

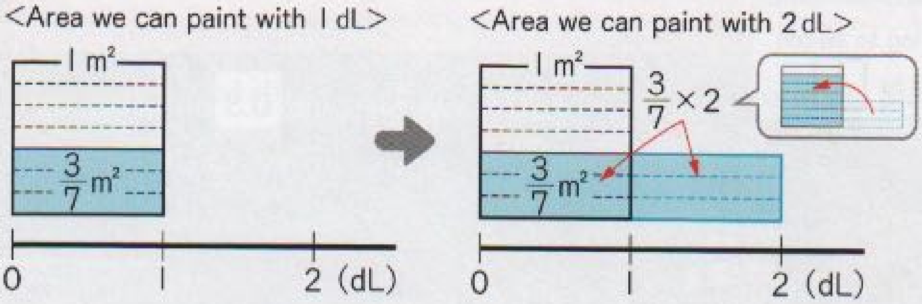


Let's think about how to multiply a fraction by a whole number.

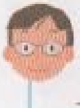
② Explain these 2 students' ideas.



Ami



Answer $\frac{6}{7}$ m²



Haruto

$\frac{3}{7}$ is made of three pieces of $\frac{1}{7}$.

So, $\frac{3}{7}\times 2$ is

(3×2) pieces of $\frac{1}{7}$.

Answer $\frac{6}{7}$ m²

$$\frac{3}{7}\times 2=\frac{3\times 2}{7}$$
$$=\frac{6}{7}$$

Answer $\frac{6}{7}$ m²

③ Using Haruto's idea, explain how to calculate $\frac{4}{9}\times 2$.

Summary

To multiply a fraction by a whole number while keeping the denominator the same, multiply the numerator by the whole number.

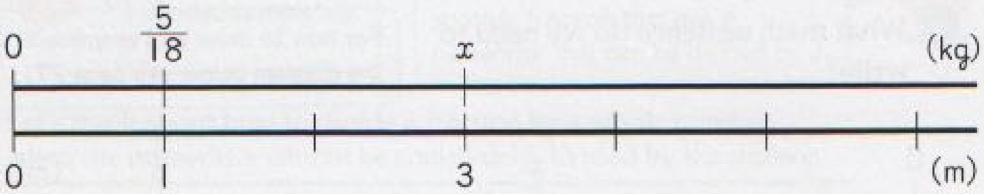
$$\frac{b}{a}\times c=\frac{b\times c}{a}$$

We thought about how many pieces of $\frac{1}{7}$ or $\frac{1}{9}$ a number was made of.

1 ① $\frac{2}{7}\times 3$ ② $\frac{3}{13}\times 4$ ③ $\frac{5}{2}\times 3$ ④ $\frac{1}{7}\times 5$

Misaki: The way we thought about how many pieces of the base amount a number is made of was useful for today's study, too.

2 There is a 1 m hose that weighs $\frac{5}{18}$ kg.
How much will 3 m of this hose weigh?



Math Sentence ▶

How is it different from how to calculate $\frac{3}{7}\times2$?

Let's think about how to calculate.

① Explain these 2 students' ideas.

Haruto

$$\frac{5}{18}\times3=\frac{5\times3}{18}=\frac{\overset{5}{\cancel{15}}}{\underset{6}{\cancel{18}}}=\frac{5}{6}$$

Misaki

$$\frac{5}{18}\times3=\frac{5\times\overset{1}{\cancel{3}}}{\underset{6}{\cancel{18}}}=\frac{5}{6}$$

Simplifying Fractions Page 273 ⑧

Answer $\frac{5}{6}$ kg

Summary

If you can simplify fractions during the calculation process, it may make calculation simpler.

2
① $\frac{2}{9}\times3$ ② $\frac{7}{6}\times3$ ③ $\frac{1}{8}\times6$ ④ $\frac{7}{12}\times8$
⑤ $\frac{3}{8}\times18$ ⑥ $\frac{5}{7}\times7$ ⑦ $\frac{6}{5}\times15$ ⑧ $\frac{3}{25}\times100$

Aquí están invertidos el multiplicador y el multiplicando

Additional Problems → Page 247 F

3 How much will 6 m and 9 m of the hose in math problem **2** weigh?

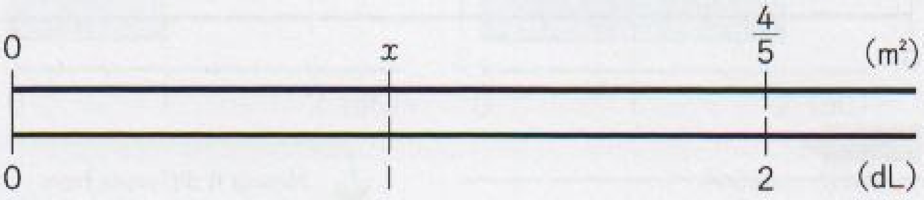
Kota: I also want to be able to divide a fraction by a whole number.

3

With 2 dL of paint, we can paint $\frac{4}{5}$ m² of boards.
How many m² of boards can we paint with 1 dL of this paint?

① What math sentence do we need to write?

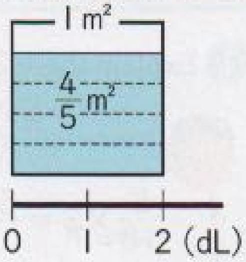
For how to draw and examine the diagram below, see page 271.



Math Sentence ______________________

Explain the reason for your math sentence.

<Area we can paint with 2 dL>

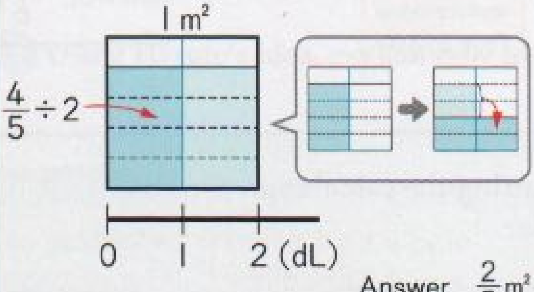


Let's think about how to divide a fraction by a whole number.

② Explain these 2 students' ideas.

Kota

<Area we can paint with 1 dL>



Answer $\frac{2}{5}$ m²

Shiho

$\frac{4}{5}$ is made of four pieces of $\frac{1}{5}$. So $\frac{4}{5} \div 2$ is $(4 \div 2)$ pieces of $\frac{1}{5}$.

Answer $\frac{2}{5}$ m²

$$\frac{4}{5} \div 2 = \frac{4 \div 2}{5}$$

$$= \frac{2}{5}$$

Answer $\frac{2}{5}$ m²

You should find **how many pieces of $\frac{1}{5}$** there are.
When you multiplied a fraction by a whole number, you multiplied the numerator by the whole number. So, when you divide a fraction by a whole number, you divide the numerator by the whole number.

Haruto

Ami $\frac{4}{5} \div 2$ is $\frac{1}{2}$ the size of $\frac{4}{5}$.

Riku What if the numerator is indivisible by the divisor?

4 Explain how to calculate $\frac{4}{5} \div 3$.


Kota

4 ÷ 3 cannot be divided completely, so ...

I wonder if we can change $\frac{4}{5}$ into another fraction that has a numerator that can be divided by 3.


Misaki

Let's think about how to divide a fraction by a whole number when the numerator cannot be completely divided by the divisor.

$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$\frac{8}{10} \downarrow \frac{4\times2}{5\times2} \qquad \frac{12}{15} \downarrow \frac{4\times3}{5\times3}$$

The value of a fraction doesn't change if we multiply both the denominator and the numerator by the same number, does it?


Misaki

$$\begin{aligned}\frac{4}{5} \div 3 &= \frac{4\times3}{5\times3} \div 3\\ &= \frac{4\times3\div3}{5\times3}\\ &= \frac{4}{5\times3}\\ &= \frac{4}{15}\end{aligned}$$

① Calculate $\frac{4}{5} \div 2$ from Problem **3** using Misaki's idea.

$$\frac{4}{5} \div 2 = \frac{4\times\square}{5\times\square} \div 2$$

$$=$$

Summary

To divide a fraction by a whole number while keeping the numerator the same, multiply the denominator by the whole number.

$$\frac{b}{a} \div c = \frac{b}{a \times c}$$

By using a property of fractions, we were able to figure out a way of dividing a fraction by a whole number.

4

① $\frac{2}{5} \div 3$ ② $\frac{3}{4} \div 7$ ③ $\frac{6}{7} \div 3$ ④ $\frac{5}{6} \div 5$

⑤ $\frac{8}{9} \div 6$ ⑥ $\frac{24}{25} \div 16$ ⑦ $\frac{12}{11} \div 8$ ⑧ $\frac{25}{3} \div 100$

Additional Problems
→ Page247 G


Shiho

It's interesting that you do multiplication in order to do division.

2 Practice

1 Calculate the following.

① $\frac{1}{5}\times 2$ ② $\frac{3}{7}\times 8$ ③ $\frac{5}{4}\times 6$ ④ $\frac{7}{8}\times 8$

⑤ $\frac{11}{20}\times 15$ ⑥ $\frac{17}{7}\times 14$ ⑦ $\frac{2}{3}\div 2$ ⑧ $\frac{7}{9}\div 9$

⑨ $\frac{16}{5}\div 7$ ⑩ $\frac{4}{7}\div 8$ ⑪ $\frac{100}{11}\div 25$ ⑫ $\frac{18}{5}\div 12$

2 To cook 3 kg of rice, you need $\frac{9}{2}$ L of water.

① How many L of water do you need to cook 1 kg of rice?

② How many L of water do you need to cook 6 kg of rice?

3 Make different problems by putting numbers 2-9 in the □ of Ⓐ and Ⓑ.
Answer the following questions.

Ⓐ $\frac{5}{4}\times$ □

Ⓑ $\frac{6}{7}\div$ □

① For which numbers will the answer of Ⓐ become whole numbers?

② What kinds of numbers will make the answers of Ⓐ whole numbers?

③ Is there any numbers that will make the answer of Ⓑ a whole number?

Warm-up

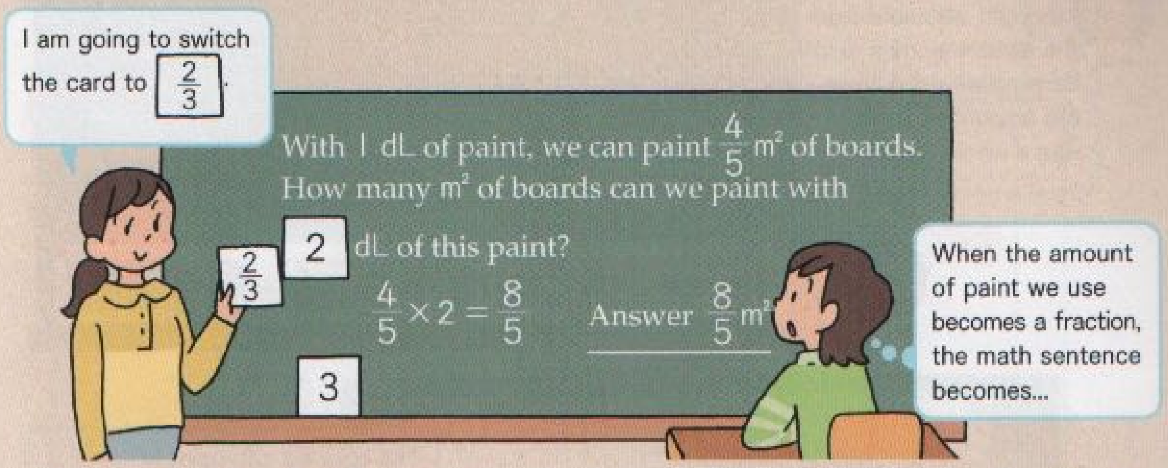
4 With 1 dL of paint, we can paint $\frac{4}{5}$ m² of boards. Let y m² be the area we can be paint with x dL of this paint. Is y proportional to x?

Proportional Relationship
Page 273 ⑬

Add ⌒ to the table and find it out.

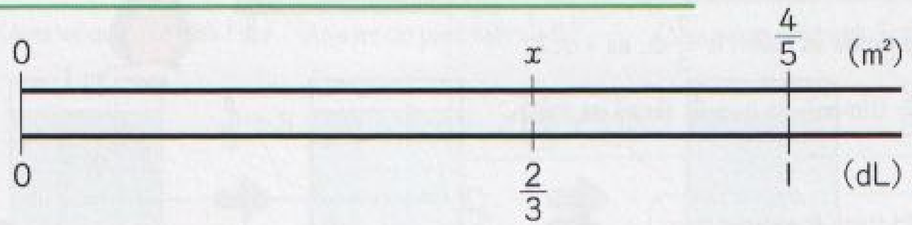
Amount of paint x(dL)	1	2	3	4	5	6	7	8
Area we can paint y(m²)	$\frac{4}{5}$	$\frac{8}{5}$	$\frac{12}{5}$	$\frac{16}{5}$	4	$\frac{24}{5}$	$\frac{28}{5}$	$\frac{32}{5}$

3 Multiplication of Fractions



1 With 1 dL of paint, we can paint $\frac{4}{5}$ m² of boards. How many m² of boards can we paint with $\frac{2}{3}$ dL of this paint?

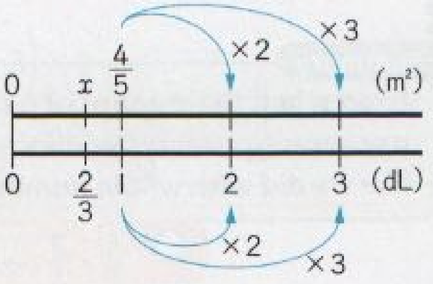
Let's think about what math sentence we should write.



Math Sentence

If the amount of paint we use is a whole number...

Since the area we can paint is proportional to the amount of paint we use...



① Explain the reason why you wrote that math sentence.

I thought we could use the same way of thinking we used when the amount of paint was a whole number.

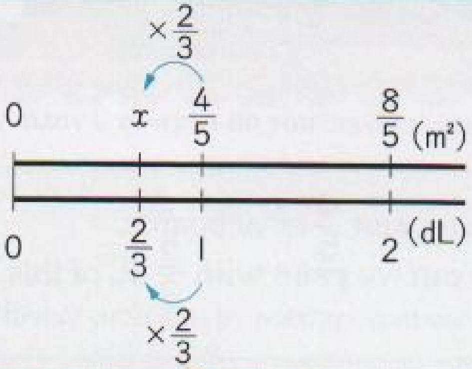

Riku

2dL …… $\frac{4}{5} \times 2 = \frac{8}{5}$

3dL …… $\frac{4}{5} \times 3 = \frac{12}{5}$

$\frac{2}{3}$dL …… $\frac{4}{5} \times \frac{2}{3} = x$

Area we can paint with 1 dL × Amount of paint(dL) = Area we can paint



$\frac{4}{5} \times \frac{2}{3} = x$

The area we can paint is proportional to the amount of paint we use. So, I thought that if the amount of paint we use was made $\frac{2}{3}$ times as much, the area we could paint would become $\frac{2}{3}$ times as much, and I thought that multiplication would work.


Ami

How many times as much is $\frac{2}{3}$ dL as 1 dL?

$\frac{2}{3} \div 1 = \frac{2}{3}$ (times), so it is $\frac{2}{3}$ times as much.

We could think the same way as we did when multiplying decimal numbers.


Misaki

What $\frac{4}{5} \times \frac{2}{3}$ means can be ... as multiplication of decimal numbers.


Kota

Summary

Even when the amount of paint we use is a fraction, we can still use a multiplication sentence to find the area we can paint **just like we did with whole numbers and decimal numbers.**

$\frac{4}{5} \times \frac{2}{3}$ can be said as a math sentence to find the area that corresponds to when $\frac{4}{5}$ m² is considered as 1.

$$\frac{4}{5} \times \frac{2}{3}$$

Haruto

I wonder how $\frac{4}{5} \times \frac{2}{3}$ is calculated.

Let's think about how to calculate when the multiplier is a fraction.


Ami

We know how to calculate when the multiplier or the divisor is a whole number, so...


Kota

Change $\frac{2}{3}$ into a whole number, and then calculate.

$$\frac{4}{5}\times\frac{2}{3}=\frac{4}{5}\times\left(\frac{2}{\underset{1}{\cancel{3}}}\times\overset{1}{\cancel{3}}\right)\div 3$$

$$=\frac{4}{5}\times 2\div 3$$

$$=\frac{\boxed{}\times\boxed{}}{\boxed{}\times\boxed{}}$$

$$=\boxed{}$$

$$\begin{tikzcd}
\frac{4}{5}\times\frac{2}{3} \arrow[d, "\times 3"] & = x \arrow[d, "\times 3"] \\
\frac{4}{5}\times\left(\frac{2}{\underset{1}{\cancel{3}}}\times\overset{1}{\cancel{3}}\right) & = \frac{4}{5}\times 2 \arrow[u, bend right, dashed, "\div 3"']
\end{tikzcd}$$

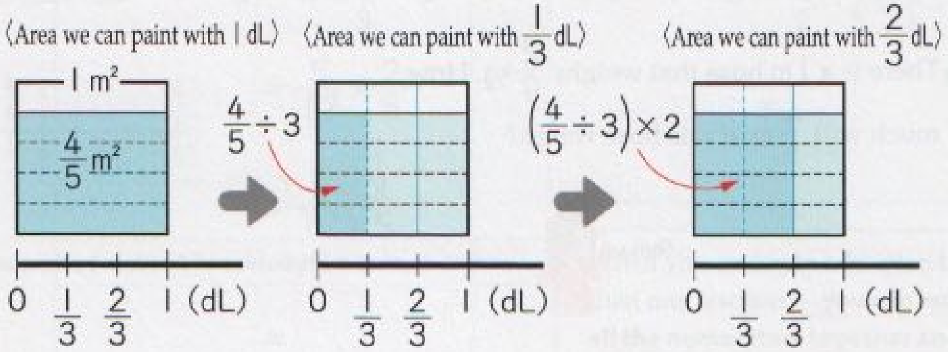
$$\begin{tikzcd}
80\times 2.3 \arrow[d, "\times 10"] & = 184 \arrow[d, "\times 10"] \\
80\times 23 & = 1{,}840 \arrow[u, bend right, dashed, "\div 10"']
\end{tikzcd}$$

This is the property of multiplication we used when the multiplier was a decimal number.

Properties of Multiplication
Page 272 ①


Shiho

First, find the area of boards you can paint with $\frac{1}{3}$ dL, and then double that amount.


〈Area we can paint with 1 dL〉 〈Area we can paint with $\frac{1}{3}$ dL〉 〈Area we can paint with $\frac{2}{3}$ dL〉

$$\frac{4}{5}\times\frac{2}{3}=\left(\frac{4}{5}\div 3\right)\times 2$$

$$=\frac{4}{5\times 3}\times 2$$

$$=\frac{\boxed{}\times\boxed{}}{\boxed{}\times\boxed{}}$$

$$=\boxed{}$$

$\frac{1}{3}$ of $\frac{4}{5}$ m² is made 2 times as much.

② Compare the last part of the math sentences of the two students on the previous page.

$$\frac{4}{5}\times\frac{2}{3}=\frac{4\times 2}{5\times 3}$$

$$=\frac{8}{15}$$

Answer $\frac{8}{15}$ m²

Summary

To multiply a fraction by another fraction, multiply the numerators together and the denominators together.

$$\frac{b}{a}\times\frac{d}{c}=\frac{b\times d}{a\times c}$$

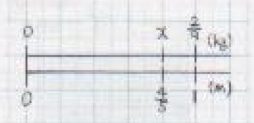
We focused on the properties of multiplication and the proportional relationship between the amount of paint we used and the area we could paint, and changed the calculation into multiplication and division by whole numbers

1 ① $\frac{1}{2}\times\frac{3}{4}$ ② $\frac{3}{5}\times\frac{2}{7}$ ③ $\frac{5}{6}\times\frac{5}{3}$

④ $\frac{4}{9}\times\frac{2}{3}$ ⑤ $\frac{3}{2}\times\frac{7}{5}$ ⑥ $\frac{9}{7}\times\frac{5}{8}$

Additional Problems
→Page248 H

2 There is a 1 m hose that weighs $\frac{2}{9}$ kg. How much will $\frac{4}{5}$ m of this hose weigh?

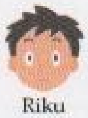


Shiho

I want to do more multiplication of fractions by fractions.

2 Explain how to calculate $\frac{8}{9}\times\frac{3}{10}$.

Let's think about ways to calculate.



Riku

It seems that these fractions can be simplified, so...

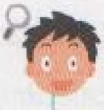
Simplifying Fractions
Page 273 ⑧

Ami

$$\frac{8}{9}\times\frac{3}{10}=\frac{8\times 3}{9\times 10}$$

$$=\frac{\overset{\boxed{}}{\overset{12}{\cancel{24}}}}{\underset{\boxed{}}{\underset{45}{\cancel{90}}}}$$

$$=\boxed{}$$



Riku

$$\frac{8}{9}\times\frac{3}{10}=\frac{\overset{4}{\cancel{8}}\times\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}\times\underset{5}{\cancel{10}}}$$

$$=\boxed{}$$

Haruto

If you can simplify fractions during the calculation process, it makes the calculation simpler, doesn't it?

① Explain how to calculate $\frac{3}{4}\times\frac{5}{9}\times\frac{2}{5}$.

Kota

$$\frac{3}{4}\times\frac{5}{9}\times\frac{2}{5}=\frac{\overset{1}{\cancel{3}}\times 5}{4\times\underset{3}{\cancel{9}}}\times\frac{2}{5}$$

$$=\frac{5}{12}\times\frac{2}{5}$$

$$=\frac{\overset{1}{\cancel{5}}\times\overset{1}{\cancel{2}}}{\underset{6}{\cancel{12}}\times\underset{1}{\cancel{5}}}$$

$$=\boxed{}$$

Shiho

$$\frac{3}{4}\times\frac{5}{9}\times\frac{2}{5}=\frac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{5}}\times\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}\times\underset{3}{\cancel{9}}\times\underset{1}{\cancel{5}}}$$

$$=\boxed{}$$

When you multiply a fraction by more than one fraction, **you can multiply all the numerators together and all the denominators together at a time.**

Misaki

3

① $\frac{4}{9}\times\frac{1}{12}$ ② $\frac{6}{7}\times\frac{1}{4}$ ③ $\frac{3}{2}\times\frac{4}{9}$ ④ $\frac{5}{12}\times\frac{9}{10}$

⑤ $\frac{3}{100}\times\frac{25}{9}$ ⑥ $\frac{8}{5}\times\frac{5}{2}$ ⑦ $\frac{3}{7}\times\frac{7}{3}$ ⑧ $\frac{4}{5}\times\frac{5}{6}\times\frac{2}{3}$

Additional Problems
→Page248 1

Kota

Look at the fractions carefully and don't forget to simplify them.

3 Explain how to calculate the following.

I wonder how we can use the ways of multiplying fractions by fractions that we have studied so far...

Let's think about how to calculate.

(1) $3\times\frac{2}{7}=\frac{3}{\boxed{}}\times\frac{2}{7}$

$=\boxed{}$

You should consider a whole number as a fraction with the denominator of 1.

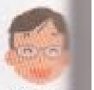
We can think like $3\times\frac{2}{7}=\frac{3\times2}{7}$, can't we?

(2) $1\frac{2}{3}\times\frac{3}{10}=\frac{5}{3}\times\frac{3}{10}$

$=\frac{\overset{1}{\cancel{5}}\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}\times\underset{2}{\cancel{10}}}$

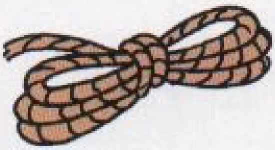
$=\boxed{}$

When multiplication sentences have mixed numbers, **you should change the mixed numbers into improper fractions, and then calculate.**

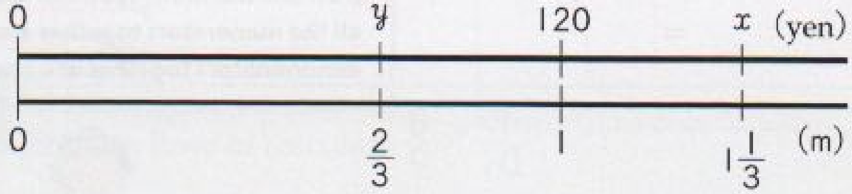


4 There is a rope that costs 120 yen per m. How much is $1\frac{1}{3}$ m of this rope? How much is $\frac{2}{3}$ m of this rope?

Additional Problems
→Page248 J



① Write math sentences, then find the answer.



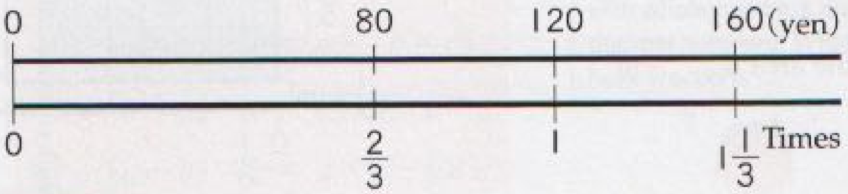
Ⓐ The price of $1\frac{1}{3}$ m Math Sentence ________ Answer $\boxed{}$ yen

Ⓑ The price of $\frac{2}{3}$ m Math Sentence ________ Answer $\boxed{}$ yen

The price of $\frac{2}{3}$ m of the rope is ... than 120 yen.

Let's think about the relationship between the size of the multiplier and the size of the product.

② When you consider 120 yen as 1, what amounts do math sentences Ⓐ and Ⓑ represent?



Summary

When the multiplier is less than 1, the product will be less than the multiplicand. This is true even when the multiplier is a fraction.

This is the same as when the multiplier is a decimal number. $400\times0.6<400$

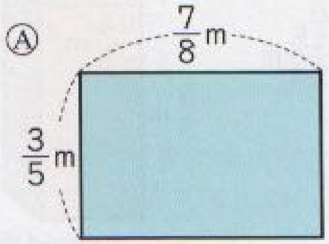
4 Write the appropriate inequality sign in each $\square$.

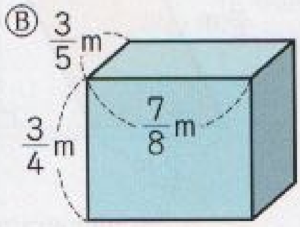
① $5\times1\frac{3}{5}\square5$ ② $\frac{3}{4}\times\frac{2}{3}\square\frac{3}{4}$ ③ $\frac{1}{2}\times\frac{7}{5}\square\frac{1}{2}$

Without actually calculating, you can tell whether the product will be greater or less than the multiplicand.

5

Find the area of rectangle Ⓐ and the volume of cuboid Ⓑ below.





When the lengths of the sides/edges were whole numbers or decimal numbers, we could use a formula, but...

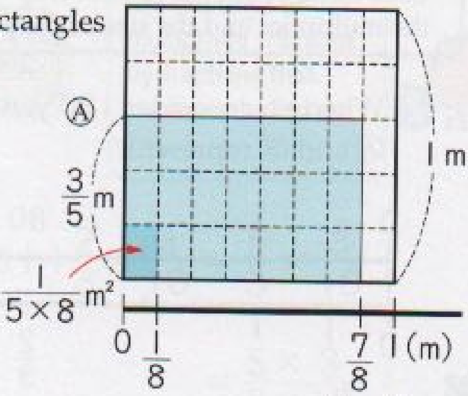
Let's investigate whether you can use the formula for area or volume even when the lengths of the sides/edges are fractions.

The Formulas for Calculating Volume of Cubes and Cuboids
Page 275 ㉕

① In the rectangle of Ⓐ, how many rectangles with an area of $\frac{1}{5\times8}$ m² are there?

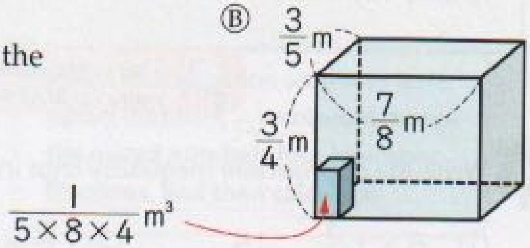
There are $\boxed{}$ $\frac{1}{5\times8}$ m² rectangles,

so the area is $\boxed{}$ m²



② See if the area of rectangle of Ⓐ can be found by calculating $\frac{3}{5}\times\frac{7}{8}$.

③ Find the volume of the cube of the rectangle of Ⓑ.

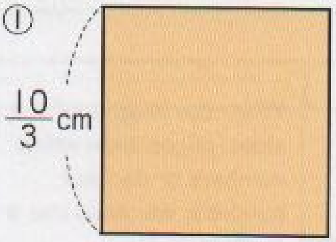


Summary

Even when the lengths of the sides/edges are fractions, you can calculate area or volume by using a formula **just like you did with whole numbers or decimal numbers.**

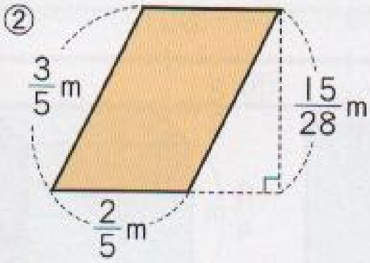
5 Find the areas of geometric figures ① and ② and the volumes of solid figures ③ and ④.

①



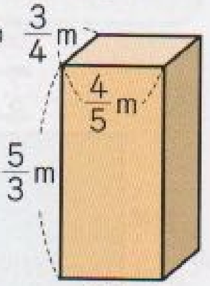
Square

②



Parallelogram

③



Cuboid

④ a cube with $\frac{5}{3}$ cm edges

The formula for the area of parallelogram
Page 275 ㉓

I wonder if there is any other thing about fractions that is the same as whole numbers and decimal numbers.

6 Think about the following properties of the operations.

Ⓐ $a\times b=b\times a$

Ⓑ $(a\times b)\times c=a\times(b\times c)$

Ⓒ $(a+b)\times c=a\times c+b\times c$

Ⓓ $(a-b)\times c=a\times c-b\times c$

These properties hold true with whole numbers and decimal numbers. What if we have fractions ...


Kota

Let's investigate to see if the properties of operations that hold true with whole numbers and decimal numbers are also true with fractions.

① Put $\frac{1}{2}$ in a, $\frac{1}{4}$ in b, and $\frac{1}{5}$ in c in Ⓐ to Ⓓ, and check to see if properties Ⓐ to Ⓓ are true.

In a, b, and c, put numbers other than those you did in ①, and see what happens.

② Explain how to calculate $\frac{2}{3}\times\frac{1}{4}+\frac{2}{3}\times\frac{3}{4}$.


Haruto

$$\frac{2}{3}\times\frac{1}{4}+\frac{2}{3}\times\frac{3}{4}=\frac{\overset{1}{\cancel{2}}\times 1}{3\times\underset{2}{\cancel{4}}}+\frac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}\times\underset{2}{\cancel{4}}}$$
$$=\frac{1}{6}+\frac{1}{2}$$
$$=\frac{1}{6}+\frac{3}{6}$$
$$=\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}}$$
$$=\frac{2}{3}$$


Misaki

$$\frac{2}{3}\times\frac{1}{4}+\frac{2}{3}\times\frac{3}{4}=\frac{2}{3}\times\left(\frac{1}{4}+\frac{3}{4}\right)$$
$$=\frac{2}{3}\times 1$$
$$=\frac{2}{3}$$

The properties of operations are also true **with fractions.** You can use the properties to make calculations simpler.


Riku

6 Using the properties of operations, think of ways to make the calculations simpler, and then calculate.

① $\left(\frac{7}{8}\times\frac{5}{6}\right)\times\frac{6}{5}$ ② $\left(\frac{2}{3}+\frac{1}{4}\right)\times 12$ ③ $\frac{3}{4}\times 5+\frac{3}{4}\times 7$

Additional Problems →Page248 K


Shiho

You can put whole numbers, decimal numbers, and fractions in a, b, and c in the math sentences representing the properties of operations.

7 $\frac{3}{4}\times\frac{4}{3}=1$. From the fractions in the ▭ below, pick two fractions that give the product of 1 and write the math sentence.

$\frac{3}{4}\times\frac{4}{3}=1$

$\frac{5}{6}$ $\frac{2}{9}$ $\frac{6}{5}$ $\frac{1}{4}$ $\frac{7}{8}$ $\frac{9}{8}$ $\frac{8}{7}$ $\frac{9}{2}$ 4

Compare the multipliers and multiplicands in math sentences that give the product of 1, and find what they have in common.

$\frac{5}{6}\times\frac{6}{5}=1$ $\frac{2}{9}\times\frac{9}{2}=1$ $\frac{7}{8}\times\frac{8}{7}=1$ $\frac{1}{4}\times 4=1$

If you carefully look at the denominators and the numerators...

$\frac{1}{4}\times 4=\frac{1}{4}\times\frac{4}{\square}$

Switching the numerator and the denominator of one fraction makes the other fraction. **When a proper or improper faction is multiplied by a fraction formed by switching the numerator and denominator of the multiplicand, the product is always 1.**

When the product of two numbers, such as $\frac{5}{6}$ and $\frac{6}{5}$ or $\frac{1}{4}$ and 4, is equal to 1, we say that one number is the **reciprocal** of the other. The reciprocal of a proper or an improper fraction is a fraction that is obtained by switching the numerator and the denominator of the original fraction. $\frac{b}{a} \rightarrow \frac{a}{b}$

The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

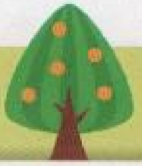
The reciprocal of $\frac{6}{5}$ is $\frac{5}{6}$.

① Write the reciprocal of 7 as a fraction.

7 Find the reciprocals of the following numbers.

$0.3=\frac{\square}{10}$

① $\frac{5}{7}$ ② $\frac{1}{3}$ ③ $\frac{13}{9}$ ④ 6 ⑤ 0.3 ⑥ 2.7



Wrapping Up Your Learning: Multiplication of Fractions

Check Your Understanding

☐ **1** With 1 dL of paint, we can paint $\frac{3}{5}$ m² of boards.

① With 3 dL of the same paint, what is the area of boards that we can paint?

② With $\frac{2}{3}$ dL of the same paint, what is the area of boards that we can paint?

Can you write a math sentence and then find the answer?

Page 35 **1**
Page 41 **1**

☐ **2** Which math sentences will have a product that is less than 3? Answer without calculating them.

Ⓐ $3\times\frac{2}{7}$ Ⓑ $3\times1\frac{1}{2}$ Ⓒ $3\times\frac{5}{4}$ Ⓓ $3\times\frac{14}{15}$

Do you understand the relationship between the size of the multiplier and the size of the product?

Page 46 **4**

☐ **3** Calculate the following.

① $\frac{5}{6}\times8$ ② $\frac{3}{7}\div6$ ③ $\frac{3}{7}\times\frac{1}{5}$

④ $\frac{9}{8}\times\frac{1}{4}$ ⑤ $\frac{3}{8}\times\frac{7}{12}$ ⑥ $\frac{7}{24}\times\frac{15}{14}$

⑦ $\frac{4}{9}\times\frac{9}{4}$ ⑧ $\frac{3}{4}\times\frac{7}{6}\times\frac{2}{3}$ ⑨ $3\times\frac{5}{6}$

⑩ $\left(\frac{5}{6}+\frac{3}{4}\right)\times12$ ⑪ $\left(\frac{3}{16}\times\frac{7}{12}\right)\times\frac{12}{7}$

Can you multiply fractions?

① Page 37 **2**
③④ Page 41 **1**
⑤～⑧ Page 44 **2**
⑨ Page 46 **3**
⑩⑪ Page 49 **6**

Can you divide fractions by whole numbers?

② Page 39 **4**

☐ **4** Find the reciprocals of the following numbers.

① $\frac{2}{7}$ ② $\frac{5}{8}$ ③ 8

④ 0.9 ⑤ 0.07 ⑥ 1.3

Can you find the reciprocals of numbers?

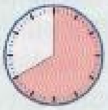
Page 50 **7**

☐ **5** It took 2 hours and 40 minutes for an airplane flying 600 km per hour to get to Naha Airport in Okinawa from Haneda Airport in Tokyo.

① How many hours is 2 hours and 40 minutes? Express the answer as a fraction.

② How many km is the flight path from Haneda Airport to Naha Airport?

How many hours is 40 minutes?



Can you solve math problems about speed by calculating with fractions?

Speed
Page 273 ⑪

Wrapping Up Your Learning: Multiplication of Fractions

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Properties of Operations and Think about How to Calculate

① Haruto is explaining why we multiply the denominators together and the numerators together when we multiply two fractions, as shown below. Write the appropriate number in each $\boxed{}$ to complete Haruto's explanation.

I used the fact that if the multiplier becomes *a* times as much, the product also becomes *a* times as much.
So, first, I made the multiplier 7 times as much...


Haruto

$$\frac{6}{5}\times\frac{2}{7}=\frac{6}{5}\times\left(\frac{2}{\underset{\boxed{}}{\cancel{7}}}\times\overset{\boxed{}}{\cancel{7}}\right)\div\boxed{}$$

$$=\frac{6}{5}\times 2\div\boxed{}$$

$$=\frac{6\times 2}{5\times 7}$$

② In 5th grade, we studied how to multiply by decimal numbers. Using Haruto's idea in ①, explain how to calculate 80 × 2.3.

First, make the multiplier $\boxed{}$ times as much...


Shiho

$$\begin{tikzcd}[column sep=tiny] 80\times & 2.3 \arrow[d, "\times\boxed{}"] & = & \boxed{} \arrow[d, "\times\boxed{}"] \\ 80\times & 23 & = & 1,840 \arrow[u, bend right, dashed, "\div\boxed{}"'] \end{tikzcd}$$


Kota

When we thought about how to calculate such math sentences, we used the properties of multiplication.

Look back on what you have learned in "Let's Think about Multiplication of Fractions" and discuss.


Ami

Now we know that the properties of multiplication and the formulas we studied work not only for whole numbers and decimal numbers but also fractions.


Riku

Looking at the numerators and denominators carefully, we were able to think about how we could make the calculations as simple as possible.

Challenge Yourself
→Page259

Do You Remember?

Answers → Page 268

1 Express the range of the following numbers by using the phrases "greater than or equal to" or "less than."

① Numbers that become 940,000 when rounded to the nearest ten thousand
② Numbers that become 35,000 when rounded to the second highest place

Approximate Numbers
Page 272 ⑤

2 Find the least common multiple of the numbers in each ( ).

① (4, 7)　② (6, 8)　③ (3, 6, 15)

3 Find the greatest common factor of the numbers in each ( ).

① (9, 27)　② (12, 18)　③ (24, 48, 60)

4 Represent the relationships between x and y in the following situations using math sentences.

① You read x pages of a 64-page book.
You have y pages to read.
② The length around a square with x cm sides is y cm.
③ You are going to divide 2 L of juice among x people.
Each person will get y L.

Warm-up

5 An iron pipe that is 1.2 m long weighs 3 kg.

① How many kg will 1 m of this iron pipe weigh?
② How many m long will 1 kg of this iron pipe be?

Playing with Numbers and Calculations

Divide 60 into Numbers

Put whole numbers greater than 2 in a, b, c, d, e, and f to make math sentences. The same letters represent the same numbers.

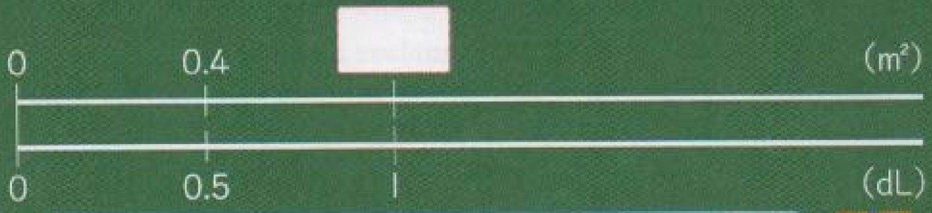
① $60 = a + a + a$
② $60 = b \times c$
③ $60 = d \times d \times e \times f$

Some of the sentences have more than one set of answers.

Let's look back on the division of decimal numbers and fractions we have studied

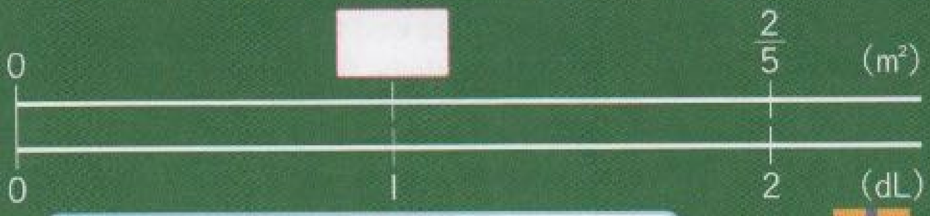
Recall division of decimal numbers by decimal numbers and fractions by whole numbers we have studied.

With 0.5 dL of paint, we could paint of 0.4 m² boards.
What is the area of boards that we can paint with 1 dL of this paint?



Remember than we can do division even when the divisor is less than 1.

With 2 dL of paint, we could paint of $\frac{2}{5}$ m² boards.
What is the area of boards that we can paint with 1 dL of this paint?



We also studied dividing fractions by whole numbers.

		Divisor		
		Whole number	Decimal number	Fractions
Divided	Whole number			
	Decimal number			
	Fractions			

Put a ○ for the calculation we already learned.

Look at the math problem and table above and discuss the types of calculation we have not studied.

When the divisor is a fraction...

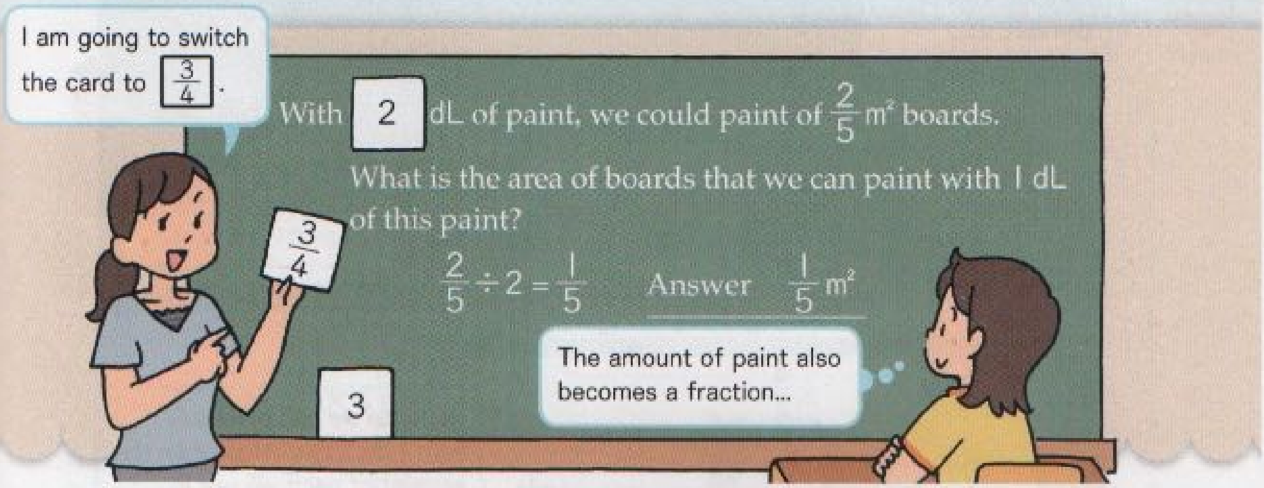
Shiho Haruto

When both the area we can paint and the amount of paint are fractions...

Division of Fractions

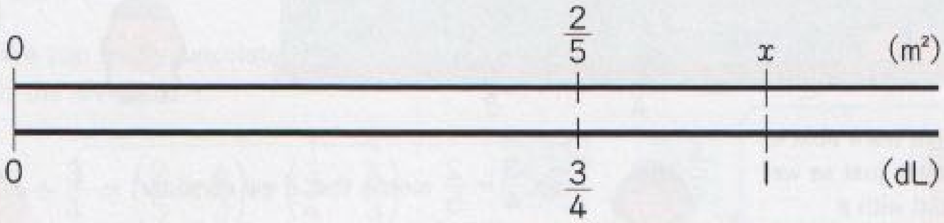
4

Let's Think about Division of Fractions

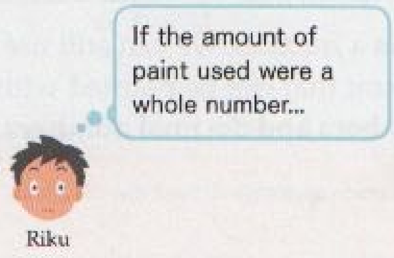


1 With $\frac{3}{4}$ dL of paint, we could paint of $\frac{2}{5}$ m² boards.
What is the area of boards that we can paint with 1 dL of this paint?

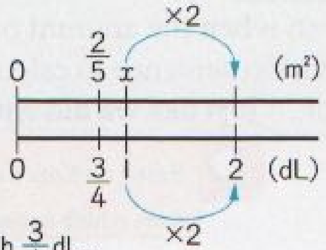
Let's think about what math sentence we should write.



Math Sentence



Let the area we can paint with 1 dL of paint be x m². With 2 dL, the area we can paint is 2 times as large.



With $\frac{3}{4}$ dL...

① Explain the reason for your math sentence.

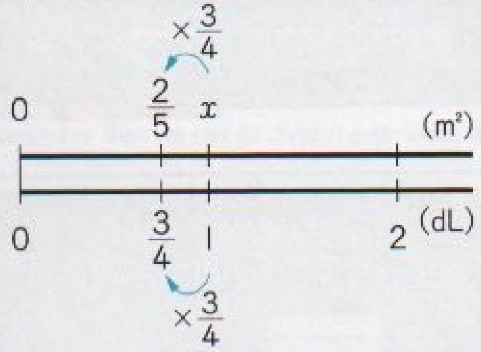
Suppose the amount of paint used was a whole number. This is division, so...

2 dL …… $\frac{2}{5} \div 2 = \frac{1}{5}$

3 dL …… $\frac{2}{5} \div 3 = \frac{2}{15}$

$\frac{3}{4}$ dL …… $\frac{2}{5} \div \frac{3}{4} = x$

Area Painted ÷ Amount of paint used (dL) = Area we can paint with 1 dL



$x \times \frac{3}{4} = \frac{2}{5}$

$x = \frac{2}{5} \div \frac{3}{4}$

Suppose we can paint x m² with 1 dL. I think that when the amount of paint we use is made $\frac{3}{4}$ as much, the area we can paint also becomes $\frac{3}{4}$ as much. So, we can say that $x \times \frac{3}{4} = \frac{2}{5}$. Since we are finding the value of x...

We were able to think just as we did with a decimal divisor.

$x \times \frac{3}{4} = \frac{2}{5}$ means that if we consider x m² as 1, $\frac{2}{5}$ m² corresponds to $\frac{3}{4}$.

Summary

Even when the amount of paint used is a fraction, we can still use a division sentence to calculate the amount that can be painted with 1 dL, **just like we did with whole numbers and decimal numbers.**

Based on Kota's idea, $\frac{2}{5} \div \frac{3}{4}$ is a math sentence to find the value of x m², which is considered as 1.

$\frac{2}{5} \div \frac{3}{4}$

I wonder how $\frac{2}{5} \div \frac{3}{4}$ is calculated.

Let's think about how to calculate when the divisor is a fraction.

I wonder if we can change $\frac{3}{4}$ into a whole number, just like we did when using a decimal divisor.

Haruto

Riku

Change $\frac{3}{4}$ into a whole number, and then calculate.

$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times 4\right) \div \left(\frac{3}{\underset{1}{\cancel{4}}} \times \overset{1}{\cancel{4}}\right)$$

$$= \left(\frac{2}{5} \times 4\right) \div 3$$

$$= \frac{2 \times 4}{5} \div 3$$

$$= \frac{\boxed{} \times \boxed{}}{\boxed{} \times \boxed{}}$$

$$= \boxed{}$$

$$\begin{array}{ccccc} \frac{2}{5} & \div & \frac{3}{4} & = & x \\ \downarrow \boxed{\times 4} & & \downarrow \boxed{\times 4} & & \\ \left(\frac{2}{5} \times 4\right) & \div & \left(\frac{3}{\underset{1}{\cancel{4}}} \times \overset{1}{\cancel{4}}\right) & = & \frac{2}{5} \times 4 \div 3 \end{array}$$

equal

$$\begin{array}{ccccc} 300 & \div & 2.5 & = & 120 \\ \downarrow \boxed{\times 10} & & \downarrow \boxed{\times 10} & & \\ 3{,}000 & \div & 25 & = & 120 \end{array}$$

equal

It's the same idea we used when we divided by a decimal number, isn't it?

Properties of Division
Page 272 ②

Ami

We can easily calculate if the divisor is 1.

$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times \frac{4}{3}\right) \div \left(\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}\right)$$

$$= \left(\frac{2}{5} \times \frac{4}{3}\right) \div 1$$

$$= \frac{2}{5} \times \frac{4}{3}$$

$$= \frac{\boxed{} \times \boxed{}}{\boxed{} \times \boxed{}}$$

$$= \boxed{}$$

$$\begin{array}{ccccc} \frac{2}{5} & \div & \frac{3}{4} & = & x \\ \downarrow \boxed{\times \frac{4}{3}} & & \downarrow \boxed{\times \frac{4}{3}} & & \\ \left(\frac{2}{5} \times \frac{4}{3}\right) & \div & \left(\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}\right) & = & \frac{2}{5} \times \frac{4}{3} \div 1 \end{array}$$

equal

② Compare the last part of the math sentences of these two students.

$$\frac{2}{5} \div \frac{3}{4} = \frac{2 \times 4}{5 \times 3}$$

$$= \frac{8}{15}$$

Answer $\frac{8}{15}$ m²

Summary

To divide by a fraction, we can multiply the dividend by the reciprocal of the divisor.

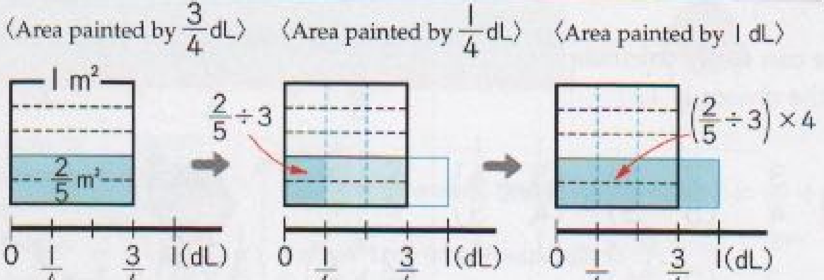
$$\frac{b}{a} \div \frac{d}{c} = \frac{b}{a} \times \frac{c}{d}$$

$$= \frac{b \times c}{a \times d}$$

We focused on the properties of division and changed the divisor from a fraction into a whole number.

Using a diagram like the one Shiho drew on page 43, you can explain as follows.

First, find how much area can be painted with $\frac{1}{4}$ dL, and then find 4 times as much as that number.



$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \div 3\right) \times 4$$

$$= \frac{2}{5 \times 3} \times 4$$

$$= \frac{2 \times 4}{5 \times 3}$$

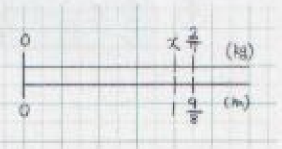
$$= \frac{8}{15}$$

1 ① $\frac{3}{8} \div \frac{2}{7}$ ② $\frac{8}{9} \div \frac{3}{4}$ ③ $\frac{3}{5} \div \frac{5}{4}$

④ $\frac{1}{7} \div \frac{2}{5}$ ⑤ $\frac{4}{9} \div \frac{3}{2}$ ⑥ $\frac{3}{2} \div \frac{1}{3}$

Additional Problems
→Page 249 L

2 $\frac{9}{8}$ m of a hose weighs $\frac{2}{7}$ kg.
How much will 1 m of this hose weigh?



2 Think about how to simplify the calculation of $\frac{9}{14} \div \frac{3}{4}$.

Let's think about ways to calculate.

$$\frac{9}{14} \div \frac{3}{4} = \frac{\overset{\square}{\cancel{9}} \times \overset{\square}{\cancel{4}}}{\underset{\square}{\cancel{14}} \times \underset{\square}{\cancel{3}}}$$

$$= \square$$

If you can simplify fractions during the calculation process, **it makes the calculation simpler** just like it does in multiplication, doesn't it?

Simplifying Fractions
Page 273 ⑧

① Think about how to calculate $\frac{3}{4} \div \frac{6}{5} \times \frac{1}{5}$.

$$\frac{3}{4} \div \frac{6}{5} \times \frac{1}{5} = \frac{3}{4} \times \frac{\square}{\square} \times \frac{1}{5}$$

$$= \frac{\overset{\square}{\cancel{3}} \times \overset{\square}{\cancel{5}} \times 1}{4 \times \underset{\square}{\cancel{6}} \times \underset{\square}{\cancel{5}}}$$

$$= \square$$

Calculations with fractions that have both multiplication and division **can be changed to calculations with only multiplication if you use the reciprocal of the divisor**, can't they?

3

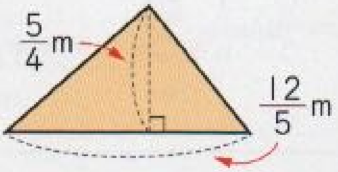
① $\frac{6}{7} \div \frac{3}{5}$ ② $\frac{9}{10} \div \frac{4}{5}$ ③ $\frac{12}{5} \div \frac{8}{15}$ ④ $\frac{7}{6} \div \frac{21}{8}$

⑤ $\frac{3}{8} \div \frac{9}{14}$ ⑥ $\frac{2}{15} \div \frac{6}{5}$ ⑦ $\frac{9}{100} \div \frac{3}{25}$ ⑧ $\frac{7}{2} \div \frac{7}{4}$

⑨ $\frac{2}{3} \times \frac{1}{8} \div \frac{7}{9}$ ⑩ $\frac{16}{7} \div 9 \times \frac{3}{8}$ ⑪ $\frac{2}{9} \div \frac{4}{7} \div \frac{5}{6}$

Additional Problems
→Page249 M

4 Find the area of the triangle on the right.



Formula for the area of the triangle
Page 275 ㉓

Shiho: Now I know that division of fractions can always be expressed as multiplication math sentences.

3 Explain how to calculate the following.

Riku: To use the ways of dividing fractions by fractions that we have studied so far...

Let's think about how to calculate.

① $4 \div \frac{9}{2} = \frac{4}{\square} \times \frac{2}{9}$

$= \square$

We can think like
$4 \div \frac{9}{2} = 4 \times \frac{2}{9}$
$= \frac{4 \times 2}{9}$,
can't we?

Haruto: **A whole number is considered as a fraction with the denominator of 1.**

② $\frac{2}{3} \div 3\frac{1}{5} = \frac{2}{3} \div \frac{16}{5}$

$= \frac{\overset{\square}{\cancel{2}} \times 5}{3 \times \underset{\square}{\cancel{16}}}$

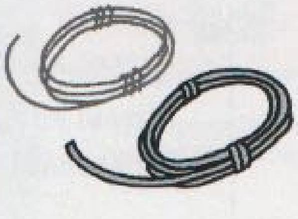
$= \square$

Ami: When division sentences have mixed numbers, **you should change the mixed numbers into improper fractions, and then calculate**, just like when multiplication sentences have mixed numbers.

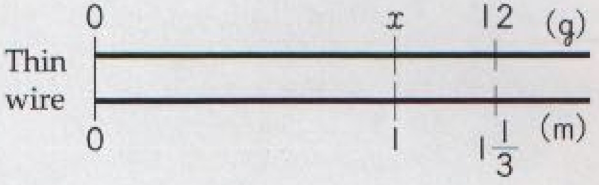
Additional Problems
→Page249 N

4

$1\frac{1}{3}$ m of a thin wire weighs 12 g. $\frac{2}{3}$ m of a thick wire weighs 12 g.
How many g does 1 m of the thin wire weigh?
How many g does 1 m of the thick wire weigh?

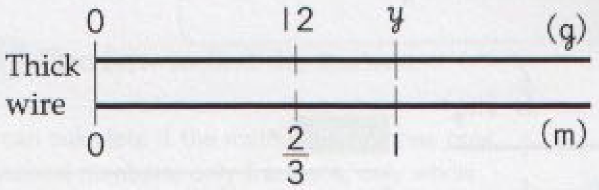


① Write math sentences, then find the answer.



Math Sentence

Answer ☐ g



Math Sentence

Answer ☐ g

② Which math sentence will have a quotient greater than the dividend, or 12?

Let's investigate the relationship between the size of the divisor and the size of the quotient.

③ Which math sentence has a divisor that is less than 1?

Summary

When the divisor is less than 1, the quotient will be greater than the dividend. This is true even when the divisor is a fraction.

This is the same as when the divisor is a decimal number. $240 \div 0.8 > 240$

5 Write the appropriate inequality sign in each ☐.

① $4 \div \frac{3}{7} \square 4$ ② $\frac{2}{5} \div \frac{5}{4} \square \frac{2}{5}$ ③ $6 \div 1\frac{2}{7} \square 6$

Without actually calculating, you can tell whether the quotient will be greater or less than the dividend.

5

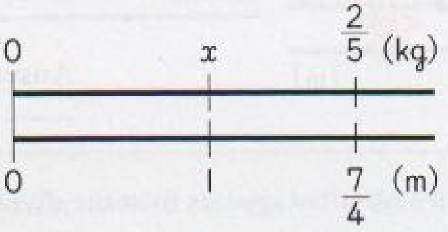
$\frac{7}{4}$ m of a hose weighs $\frac{2}{5}$ kg.
Misaki and Riku wrote word problems about this situation.
What math sentence represents each of their problems?

Let's think about each of the two students' problems by using a number line diagram.

① Write a math sentence for each of their problems.

Misaki

How much will 1 m of this hose weigh?



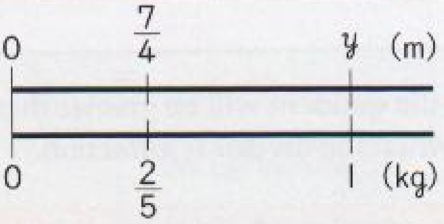
Math Sentence ▸ ______________________

Since we are finding the value of x by calculating $x \times \frac{7}{4} = \frac{2}{5}$ ...

Haruto

Riku

How long will 1 kg of this hose be?



Math Sentence ▸ ______________________

② Find the answer for each problem.

If you look at the number line diagram for Riku's problem, you can see that the answer is ... than $\frac{7}{4}$ m.

Shiho

If you use a number line diagram, it's easy to tell what math sentence can be written and to understand the relationship between quantities.

Kota

Ami

Number line diagrams are useful.

Deepen Your Learning

Let's review.

- $\boxed{\frac{3}{7}\times\frac{5}{6}} = \frac{\overset{1}{\cancel{3}}\times 5}{7\times\underset{2}{\cancel{6}}}$
  $= \frac{5}{14}$

- $\boxed{\frac{3}{10}\div\frac{2}{5}} = \frac{3\times\overset{1}{\cancel{5}}}{\underset{2}{\cancel{10}}\times 2}$
  $= \frac{3}{4}$

$$\boxed{0.3\div\frac{3}{2}\times 3}$$

6 Let's think about how to calculate $0.3\div\frac{3}{2}\times 3$.

① Plan how to find the answer.

I can calculate if the math sentence has only decimal numbers, only fractions, only whole numbers, just decimal numbers and whole numbers, or just fractions and whole numbers, but...


Ami

Let's think about how to calculate when a decimal number, a fraction, and a whole number are mixed in a multiplication or division math sentence.

② Put your idea into a math sentence.


Riku

Write $\frac{3}{2}$ as decimal numbers.

$\frac{3}{2} = 3\div 2$

$= \square$


Misaki

Write 0.3 as decimal numbers.

$0.3 = \frac{3}{\square}$

Relationships between Fractions and Decimal Numbers
Page 272 ⑥

$0.3\div\frac{3}{2}\times 3$

$=$

Think about how this math sentence continues.

Grasp the problem.

- What problem are we going to work on today?

- What idea may be useful to solve the problem?
- Is there anything you have learned before that you can use to solve this problem?

Write down your ideas.

- Is your idea clear to others?

Haruto and some other students are explaining their classmates' ideas.

Riku

$$
\begin{aligned}
&0.3 \div \frac{3}{2} \times 3\\
&= 0.3 \div 1.5 \times 3\\
&= 0.2 \times 3\\
&= 0.6
\end{aligned}
$$

Misaki

$$
\begin{aligned}
&0.3 \div \frac{3}{2} \times 3\\
&= \frac{3}{10} \div \frac{3}{2} \times 3\\
&= \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{5}{\cancel{10}} \times \underset{1}{\cancel{3}}} \times 3\\
&= \frac{1}{5} \times 3\\
&= \frac{3}{5}
\end{aligned}
$$

Haruto

Ami

Learn with your classmates.

- Can you understand your classmates' ideas based on their math sentences?
- What is common and what is different about your own idea and your classmates' ideas?
- What are the good points in your classmates' ideas?

③ Explain Riku's and Misaki's ideas.

④ Check to see if 0.6 and $\frac{3}{5}$ are equal.

⑤ There are errors in the following calculation. Explain them.

$$
\begin{aligned}
0.3 \div \frac{3}{2} \times 3 &= \frac{3}{10} \div \frac{3}{2} \times 3\\
&= \frac{3}{10} \div \frac{9}{2}\\
&= \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{5}{\cancel{10}} \times \underset{3}{\cancel{9}}}\\
&= \frac{1}{15}
\end{aligned}
$$

The order of operations should be...

Order of Operations
Page 272 ③

- What about the following math problem?

$$0.2 \div \frac{2}{3} \times 3$$

Shiho

$$\frac{2}{3} = 0.666\cdots$$

2 is not divisible by 3.

Kota

$$\begin{aligned} & 0.2 \div \frac{2}{3} \times 3 \\ = & \frac{2}{10} \div \frac{2}{3} \times 3 \\ = & \frac{\overset{1}{\cancel{2}} \times 3}{10 \times \underset{1}{\cancel{2}}} \times 3 \\ = & \frac{3}{10} \times 3 \\ = & \frac{9}{10} \end{aligned}$$

⑥ Let's think about how to calculate $0.2 \div \frac{2}{3} \times 3$.

⑦ Look back and summarize today's lesson.

Summary

When there is a combination of fractions, decimal numbers, and whole numbers in math sentences with multiplication and division, **you can always calculate them if you change the decimal numbers and whole numbers into fractions.**

$$0.2 \div \frac{2}{3} \times 3 = \frac{2}{10} \div \frac{2}{3} \times \frac{3}{1}$$

6 Change decimal numbers and whole numbers into fractions, then calculate.

① $2 \times \frac{3}{7} \div 0.9$

② $\frac{9}{10} \div 8 \div 2.7$

③ $0.21 \times 7 \div 4.2$

④ $4.2 \div 3 \div 0.35$

(Practice)

Additional Problems → Page 249 ○

Dig deeper into your study.

- Which idea can always be used?

Look back and summarize today's lesson.

- What did you learn from today's investigation?
- Which way of thinking was useful?

Put it into use.

- Can you use what you learned in a new problem?

Deepen **Your** Learning

Use Your Math Notebook for Better Learning

Look back at the ideas you used to solve problems.

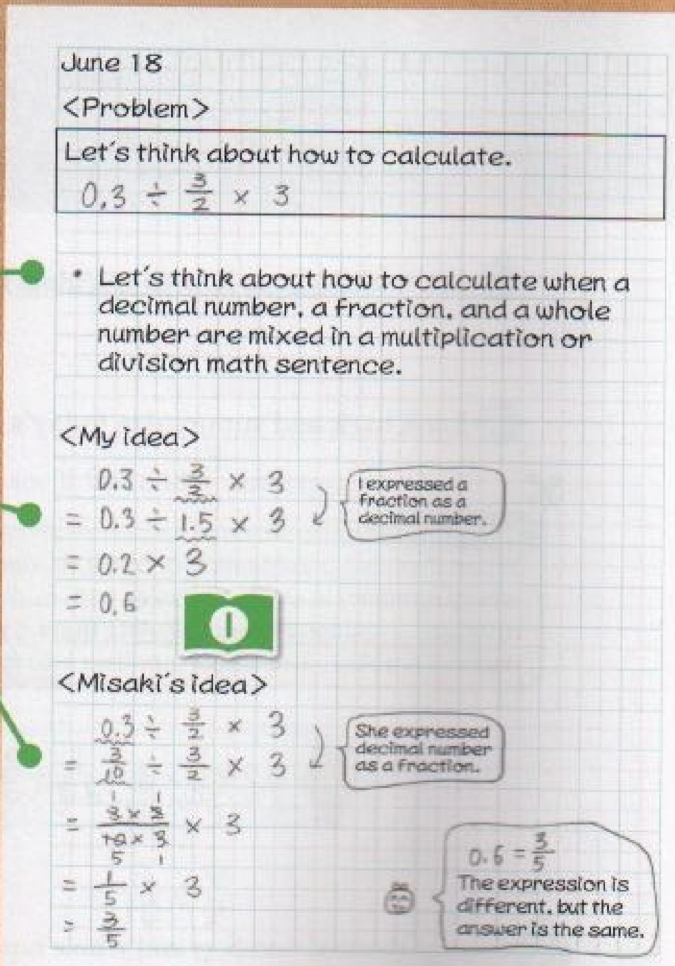
Riku

He thought about how today's calculation was different from previous ones.

He paid attention to how numbers were expressed, and changed a decimal number into a fraction or a fraction into a decimal number to calculate.

When you work on problems, not only write math sentences and answers but also consider using:

- **Diagrams**
- **Tables**
- **Graphs**



June 18

<Problem>

Let's think about how to calculate.

$0.3 \div \frac{3}{2} \times 3$

- Let's think about how to calculate when a decimal number, a fraction, and a whole number are mixed in a multiplication or division math sentence.

<My idea>

$0.3 \div \frac{3}{2} \times 3$

$= 0.3 \div 1.5 \times 3$

$= 0.2 \times 3$

$= 0.6$

I expressed a fraction as a decimal number.

<Misaki's idea>

$0.3 \div \frac{3}{2} \times 3$

$= \frac{3}{10} \div \frac{3}{2} \times 3$

$= \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{5}{\cancel{10}} \times \underset{1}{\cancel{3}}} \times 3$

$= \frac{1}{5} \times 3$

$= \frac{3}{5}$

She expressed decimal number as a fraction.

$0.6 = \frac{3}{5}$

The expression is different, but the answer is the same.

Classmates' reflections

Shiho

I thought changing all numbers to fractions was useful because it could be used at any time.

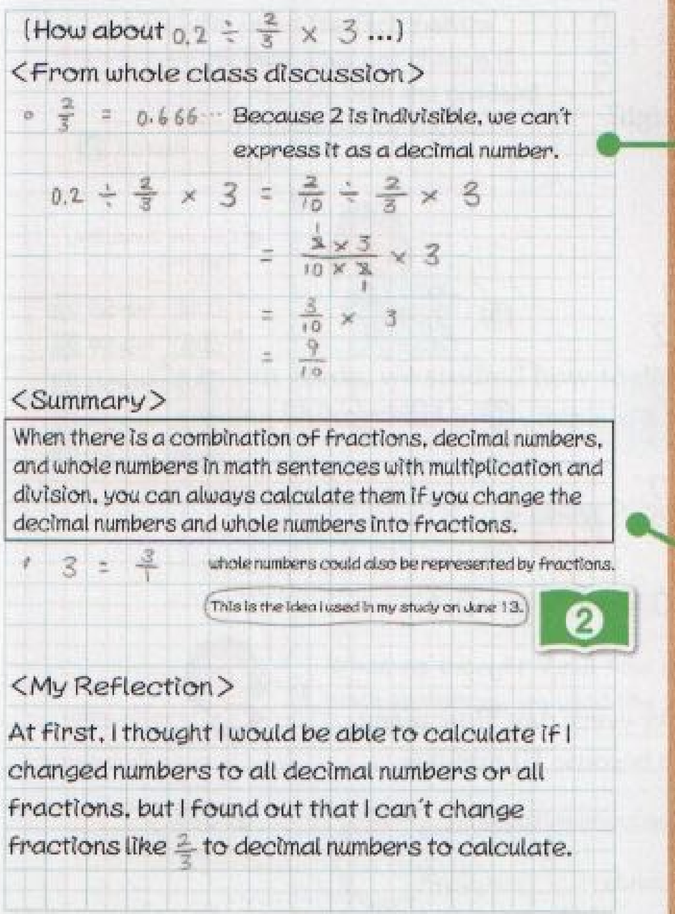
She wrote what she noticed and the reason why that is so.

Note Taking Tip ❶

When writing math sentences, line up the equal signs vertically to show your idea clearly.

Note Taking Tip ❷

Today's lesson was based on what he studied before. So, he added the date of the lesson so that he can go back to the page of his notebook.



(How about $0.2 \div \frac{2}{3} \times 3$ ...)

<From whole class discussion>

- $\frac{2}{3} = 0.666\cdots$ Because 2 is indivisible, we can't express it as a decimal number.

$$\begin{aligned} 0.2 \div \frac{2}{3} \times 3 &= \frac{2}{10} \div \frac{2}{3} \times 3 \\ &= \frac{\overset{1}{\cancel{2}} \times 3}{10 \times \underset{1}{\cancel{2}}} \times 3 \\ &= \frac{3}{10} \times 3 \\ &= \frac{9}{10} \end{aligned}$$

<Summary>

When there is a combination of fractions, decimal numbers, and whole numbers in math sentences with multiplication and division, you can always calculate them if you change the decimal numbers and whole numbers into fractions.

- $3 = \frac{3}{1}$ whole numbers could also be represented by fractions.

This is the idea I used in my study on June 13. ②

<My Reflection>

At first, I thought I would be able to calculate if I changed numbers to all decimal numbers or all fractions, but I found out that I can't change fractions like $\frac{2}{3}$ to decimal numbers to calculate.

As he dug deeper into my study, he realized that the idea of changing fractions into decimal numbers sometimes did not work.

He summarized a calculation method that always worked. To show it clearly, he gave an example as a specific math sentence.

Kota

Any complex calculations involving a mix of decimal numbers, fractions, and whole numbers can be calculated using fractions. I want to try various problems.

He wrote about further ideas he would like to investigate.

Wrapping Up Your Learning: Division of Fractions

Check Your Understanding

1 Which math sentences will have a quotient that is greater than 3?
Answer without calculating them.

Ⓐ $3 \div \frac{2}{7}$ Ⓑ $3 \div 1\frac{1}{2}$

Ⓒ $3 \div \frac{5}{4}$ Ⓓ $3 \div \frac{14}{15}$

Do you understand the relationship between the size of the divisor and the size of the quotient?

Page 61 **4**

2 $\frac{3}{5}$ L of rice weighs $\frac{1}{2}$ kg.
How much does 1 L of this rice weigh?

Can you write a math sentence and then find the answer?

Page 62 **5**

3 Calculate the following.

① $\frac{2}{9} \div \frac{3}{8}$ ② $\frac{6}{5} \div \frac{7}{12}$ ③ $\frac{2}{7} \div \frac{4}{9}$

④ $\frac{3}{4} \div \frac{9}{8}$ ⑤ $14 \div \frac{2}{15}$ ⑥ $12 \div \frac{7}{8}$

⑦ $\frac{3}{5} \div \frac{3}{4} \times \frac{5}{4}$ ⑧ $\frac{7}{3} \times \frac{1}{2} \div 14$

⑨ $0.3 \div \frac{9}{10} \times 3.6$ ⑩ $0.9 \div 8 \times 4 \div 2.1$

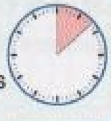
Can you divide the fractions?

①②	Page 55 **1**
③④	Page 59 **2**
⑤⑥	Page 60 **3**
⑦⑧	Page 59 **2**
⑨⑩	Page 63 **6**

4 ① A clock becomes 8 seconds faster every day. How many days will it take for this clock to become 10 minutes fast? Express 8 seconds using minutes as the unit to calculate.

60 seconds = 1 minute

8seconds = $\frac{8}{\square}$ minutes

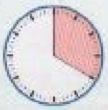


② Akira's brother completed a 42 km wheel chair marathon in 2 hours 20 minutes.

(1) How many hours is 2 hours 20 minutes? Express the answer as a fraction.

(2) What was the brother's speed per hour?

How much of an hour is 20 minutes?



Can you solve math problems about speed by using calculations of fractions?

Speed
Page 273 ⑪

Wrapping Up Your Learning: Division of Fractions

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Properties of Operations and Think about How to Calculate

① Shiho is trying to explain why we can divide by a fraction by "multiplying by the reciprocal of the divisor."
Write the appropriate number in each $\boxed{}$ and complete her explanation.

I'm using the fact that the quotient does not change if you multiply both the dividend and the divisor by the same number.


Shiho

$$\frac{7}{5} \div \frac{2}{3} = \left(\frac{7}{5} \times \boxed{}\right) \div \left(\frac{2}{\underset{\boxed{}}{\cancel{3}}} \times \overset{\boxed{}}{\cancel{3}}\right)$$

$$= \left(\frac{7}{5} \times \boxed{}\right) \div 2$$

$$= \frac{7}{5} \times \frac{3}{2}$$

② In 5th grade, we studied how to divide by decimal numbers.
Using Shiho's idea in ①, explain how to calculate 300 ÷ 2.5.

Make both the dividend and the divisor $\boxed{}$ times as much...


Riku

$$\begin{tikzcd} 300 \arrow[d, "\times 10"] & \div & 2.5 \arrow[d, "\times 10"] & = & \boxed{} \arrow[d, dash, bend left, "\text{Equal}"] \\ 3{,}000 & \div & 25 & = & 120 \end{tikzcd}$$


Ami

When we thought about how to calculate such math sentences, we used the properties of division.

Look back on what you have learned in "Let's Think about Division of Fractions" and discuss.


Kota

A number line diagram was helpful in telling what math sentence could be written even when we worked on division of fractions.


Misaki

Now we know that when a division math sentence contains a whole number, a decimal number, and a fraction, you can calculate it by expressing all of the numbers as fractions, then multiplying.

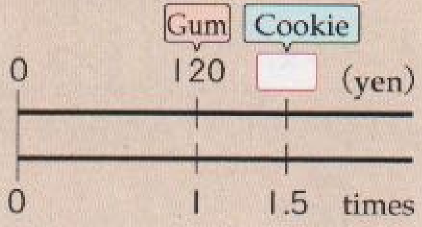
Junior High School In junior high school, you will explore further into the world of numbers and learn in more detail about numbers, calculations, and math sentences.

Challenge Yourself
→ Page 260

Times as Much with Fractions

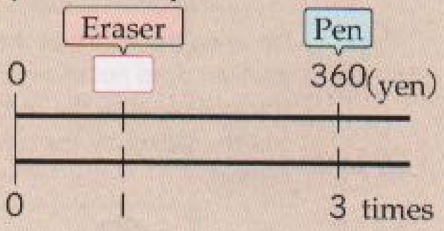
When we compare objects in size by finding how many times as large one is as the other, we consider a base amount as 1. Think about what numbers go into the □.

① **Relationship between the gum's price and the cookie's**



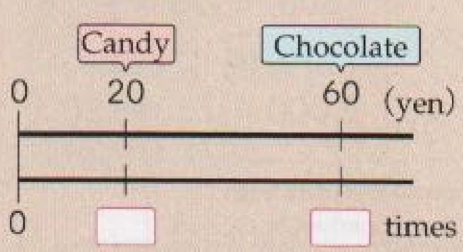
Since the cookie's price is 1.5 times as much as the gum's...

② **Relationship between the eraser's price and the pen's**



Since 3 times as much as the eraser's price equals to the pen's...

③ **Relationship between the candy's price and the chocolate's**



If you consider the candy's price, 20 yen, as 1, the chocolate's price, 60 yen, corresponds to...

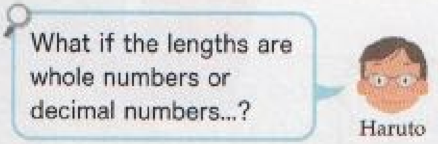
If you consider the chocolate's price as 1...

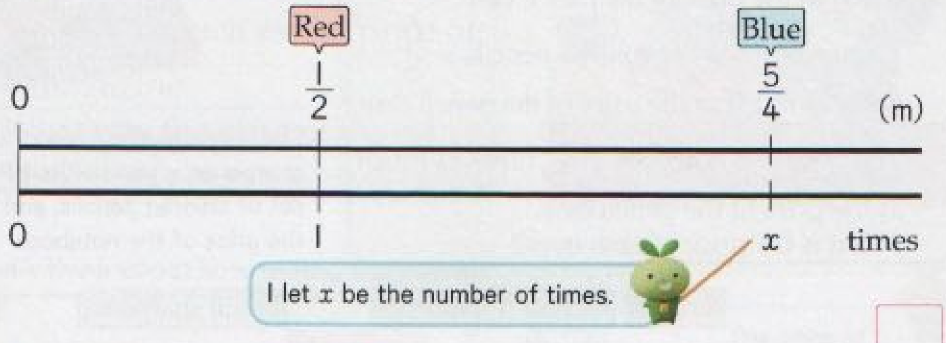
1 The lengths of 3 pieces of ribbon are shown in the table on the right. Compared to the red ribbon, how many times as long is the blue ribbon? How about the yellow ribbon?

	Length (m)
Red	$\frac{1}{2}$
Blue	$\frac{5}{4}$
Yellow	$\frac{3}{8}$

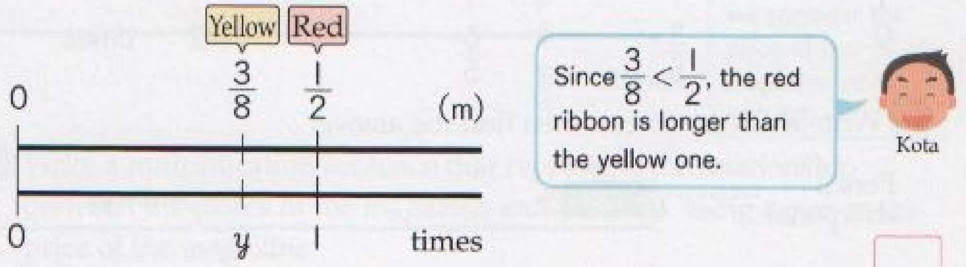
Let's think about which calculation we should use.

If we consider the red ribbon as 1...





Blue ribbon········ Math Sentence ▸ ________ Answer ☐ times



Yellow ribbon······ Math Sentence ▸ ________ Answer ☐ times

Summary

Even when fractions are involved, we use division to find out how many times as much something is as a base quantity.

$\frac{5}{2}$ times as much means if we consider $\frac{1}{2}$ m as 1, $\frac{5}{4}$ m corresponds $\frac{5}{2}$.

1 In problem 1, how many times as long are the red ribbon and the blue ribbon as the yellow ribbon?

2 Answer the following questions.

① If we consider $\frac{2}{3}$ kg as the base quantity, how many times as much is $\frac{5}{9}$ kg?

② If we consider $\frac{8}{9}$ L as 1, $\frac{5}{6}$ L will correspond to what?

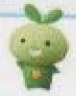
Additional Problems → Page 250 P

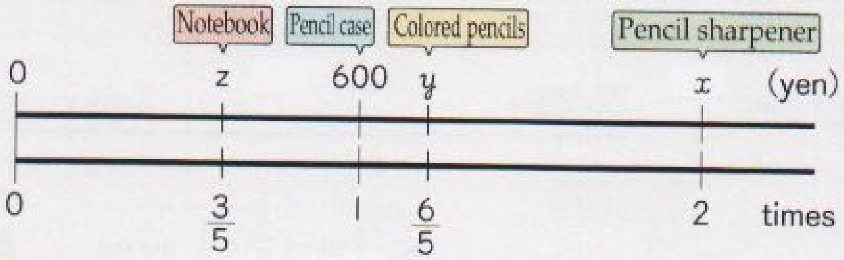
We thought about relationships between quantities when the lengths of ribbons or the number of times as much one quantity is as the other quantity were expressed as fractions.

2

The price of pencil case is 600 yen.
The price of a pencil sharpener is 2 times as much as the price of the pencil case.
The price of a set of colored pencils is $\frac{6}{5}$ times as much as the price of the pencil case.
The price of a notebook is $\frac{3}{5}$ times as much as the price of the pencil case.
What is the price of each item?

I let x yen be the price of the pencil sharpener, y yen be the price of the set of colored pencils, and z yen be the price of the notebook.





① Write math sentences, then find the answer.

Pencil sharpener ········· Math Sentence ________ Answer □ yen

Colored pencils ········· Math Sentence ________ Answer □ yen

Notebook ········· Math Sentence ________ Answer □ yen

Let's think about the meaning of the math sentences.

Explain using the diagram above.

② Write the appropriate number in each □.

Pencil sharpener ··· The math sentence, $600 \times 2 = 1{,}200$, means that if we consider 600 yen as 1, the price that corresponds to 2 is 1,200 yen.

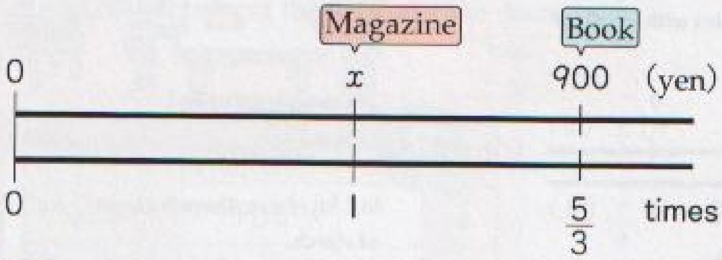
Colored pencils ··· The math sentence, $600 \times \frac{6}{5} = 720$, means that if we consider 600 yen as 1, the price that corresponds to □ is 720 yen.

Notebook··· The math sentence, $600 \times \frac{3}{5} = 360$, means that if we consider 600 yen as □, the price that corresponds to □ is 360 yen.

We thought about what the math sentences meant based on the number line diagram.

3 Hiromi bought a book for 900 yen. The price of this book was $\frac{5}{3}$ as much as the price of a magazine.
What was the price of the magazine?

Let's think about how to find the answer.



The price of the book can be considered as $\frac{5}{3}$ if we consider the price of the magazine as 1, isn't it?

① Write a multiplication sentence that represents the relationship between the prices of the magazine and the book using x yen as the price of the magazine.

Math Sentence $x \times \square = \square$

② Find the number for x.

$$x = \square \div \square$$
$$= \square$$

Answer $\square$ yen

Summary

When finding the base quantity, it may be easier to **write a multiplication sentence with x first**. This is true even when the math sentence contains a fraction.

Since $x \times \frac{5}{3} = 900$, $x = 900 \div \frac{5}{3}$.

3 There is some juice and milk. The amount of juice is $\frac{6}{5}$ L, which is $\frac{4}{3}$ times as much as the amount of milk.
How much milk is there?

Additional Problems
→Page250 Q

Ami

Even when a fraction is used to express the number of times as much one quantity is as the other quantity, you can calculate in the same way as when it is expressed with a whole number or a decimal number.

Which Operation Should We Use?

1

To walk to a supermarket from Nami's house, it takes $\frac{2}{7}$ hr.

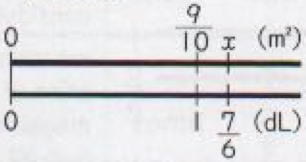
If she rides her bicycle, it takes $\frac{2}{25}$ hr.

How many times as long is the time it takes to walk to the supermarket compared to the time it takes to ride her bicycle?

2

With 1 dL of paint, you can paint $\frac{9}{10}$ m² of boards.

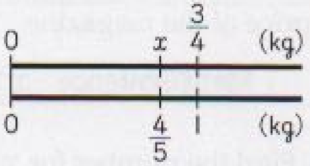
What is the area of boards you can paint with $\frac{7}{6}$ dL of this paint?



3

In 1 kg of rice, there is about $\frac{3}{4}$ kg of starch.

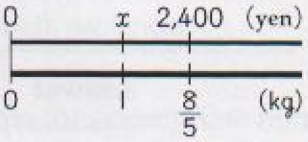
About how much starch is there in $\frac{4}{5}$ kg of rice?



4

When we bought $\frac{8}{5}$ kg of meat, the cost was 2,400 yen.

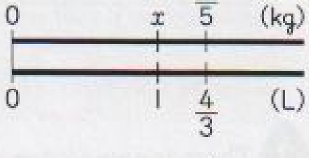
How much will 1 kg of this meat cost?



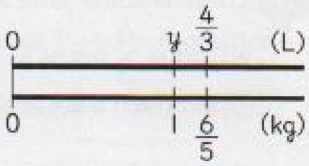
5

$\frac{4}{3}$ L of oil weighs $\frac{6}{5}$ kg.

How much will 1 L of this oil weigh? (kg)



How many L will 1 kg of this oil be?



Do You Remember?

Answers→ Page 268

1 ① $\frac{7}{12}\times\frac{3}{14}$ ② $\frac{8}{3}\times\frac{21}{20}$ ③ $\frac{2}{7}\div\frac{8}{21}$ ④ $16\div\frac{4}{5}$

⑤ $\left(\frac{1}{6}+\frac{1}{3}\right)\times\frac{4}{5}$ ⑥ $\frac{5}{6}-\frac{7}{9}\div 14$ ⑦ $\frac{5}{18}\div\frac{10}{9}\times\frac{6}{7}$

Order and Properties of Operations
Page 272 ③

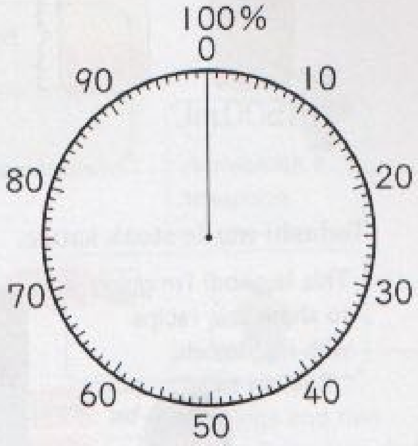
2 The table below shows the results of a survey of favorite sports Hideharu's elementary school conducted. Write the percentage of each sport in the table below. Then, represent the data on a pie chart.

Rates
Page 273 ⑭

Favorite Sports

Sports	Students	%
Soccer	108	
Baseball	102	
Basketball	48	
Dodge ball	18	
Other	24	
Total	300	

Favorite Sports



Warm-up

3 Answer the following questions.

① How many times as long is a $\frac{9}{8}$ m ribbon as a $\frac{3}{4}$ m ribbon?

② If we consider $\frac{4}{3}$ L as 1, in liters, how much liquid will be considered as $\frac{2}{3}$?

③ What is the rate of $\frac{3}{5}$ m^2 to $\frac{21}{20}$ m^2?

Playing with Numbers and Calculations

Let's Complete the Math Sentences

Complete the following math sentences by placing six different cards from $\boxed{4}$ to $\boxed{9}$ in $\square$.

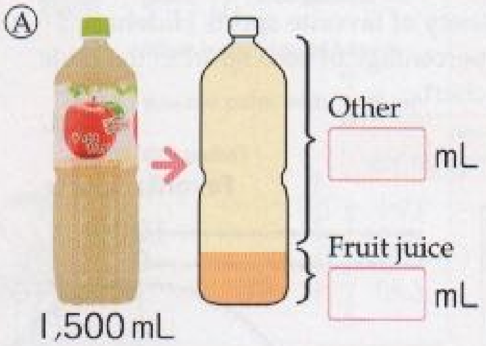
① $\frac{2}{3}\times 6=\square\times 0.8$ ② $\frac{18}{7}\div\square=12\div 42$ ③ $\frac{1}{2}\div 5=\square\div 40$

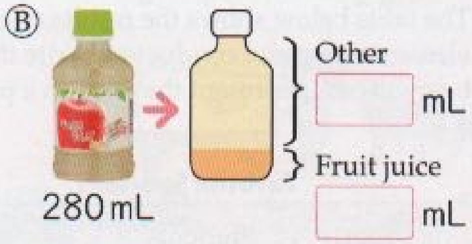
④ $\frac{1}{7}\times\square=0.25\times 4$ ⑤ $\frac{3}{4}\times\square=192\div 32$ ⑥ $\frac{26}{5}\div\square=\frac{2}{3}+\frac{1}{5}$

In what proportions should we mix?

How many mL of fruit juice is in the beverage?

Beverages that are 20% fruit juice

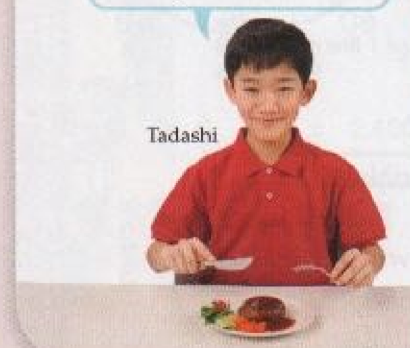




Both Ⓐ and Ⓑ are 20% fruits juice, but their total amounts are different, so...

Tadashi made steak sauce.

This is good! I'm going to share this recipe with my friends.

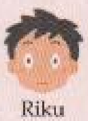


I want to make the steak sauce that tastes the same as Tadashi's.

How can we make steak sauce that tastes the same as Tadashi's in large quantity?

How much Worcestershire sauce and ketchup should be mixed? How can we make 2 or 3 servings?

If we make 30 servings...



I want to explain the amounts of Worcestershire sauce and ketchup in the same simple way regardless of how much steak sauce we make.

Ratios

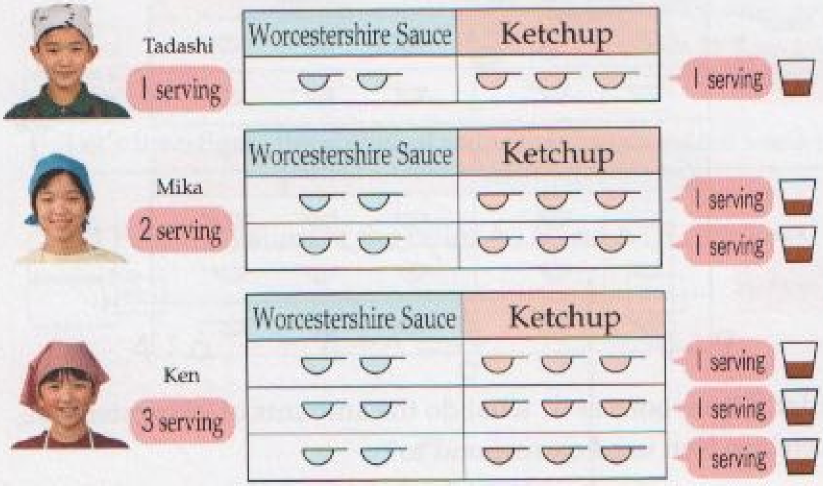
5 Let's Investigate How to Express Rates

Mika and Ken made steak sauce that tasted the same as Tadashi's in the ways as shown on the right.

1 Ratios and Values of Ratios

1 Investigate the relationship between the amounts of Worcestershire sauce and ketchup the three students used.

◡ represents a teaspoon.



Mika made 2 servings and Ken made 3 servings, but they made I serving at a time like Tadashi did. So, the steak sauce the three students made taste the same.

If you consider one teaspoonful amount as I, the amounts of Worcestershire sauce and the ketchup Tadashi used correspond to 2 and 3.

> The proportion of 2 and 3 is sometimes expressed using the symbol, " : " as 2 : 3.
> "2 : 3" is read as "2 to 3."
> The proportion expressed in this way is called **a ratio**.
> 2 : 3 is sometimes referred to as "the ratio of 2 to 3."

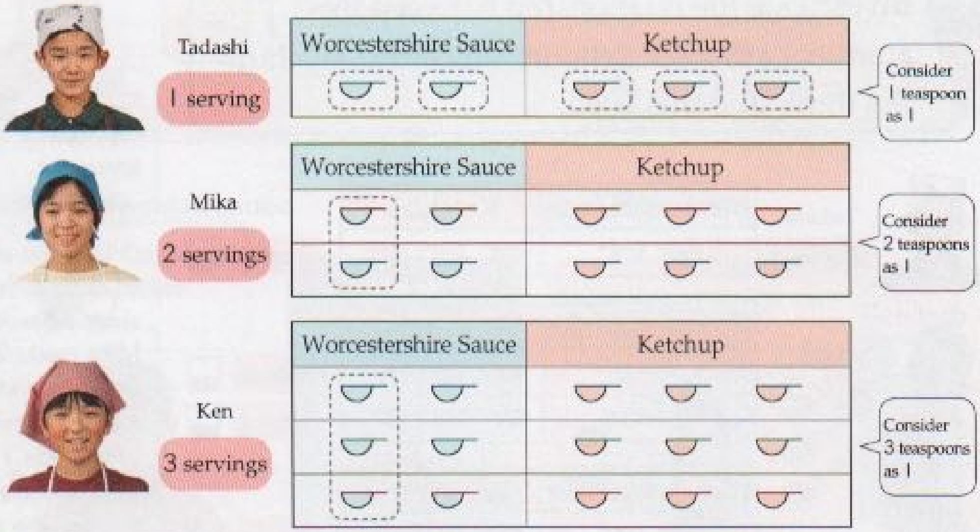
① By considering a teaspoon as 1, express the proportions of the amounts of Worcestershire sauce and ketchup Mika and Ken used on the previous page as ratios.

Riku

The steak sauce each of the three students made tastes the same. I wonder if the proportions of the amounts of Worcestershire sauce and ketchup each of the students used are the same, too.

Let's investigate the proportions of Worcestershire sauce and ketchup the three students used.

② If you consider 2 teaspoons as 1, what do the amounts of Worcestershire sauce and ketchup Mika used correspond to?



③ If you consider 3 teaspoons as 1, what do the amounts of Worcestershire sauce and ketchup Ken used correspond to?

The amounts of Worcestershire sauce and ketchup the three students used are **all in proportions of 2 and 3.** When we examined the proportion for each student, we changed the number of teaspoons we considered as 1.

Misaki

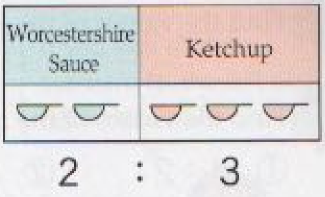
Riku

2 : 3, 4 : 6, and 6 : 9 represent the same proportion, although different numbers are used.

Ami

I want to find out more easily whether different ratios represent the same proportion.

We are going to examine in detail the proportion of the amount of Worcestershire sauce to the amount of ketchup each of the three students used.



2 Find the rate of the amount of Worcestershire sauce Tadashi used based on the amount of ketchup he used.

Tadashi

Quantity being compared

Base quantity

Ratio $2:3 \longrightarrow 2 \div 3 = \frac{2}{3}$ Rate

This is the rate we studied in 5th grade, isn't it?

Rates
Page 273 ⑭

When the ratio of $a:b$ is given, the value representing the rate of a compared to b is called the **value of ratio** of $a:b$.

The value of ratio of $a:b$ is the quotient of a divided by b.

The value of the ratio, $a:b$, expresses what a corresponds to if we consider b as 1.

Let's investigate the values of ratios that represent the same rate.

① Find the values of the ratios $4:6$ and $6:9$.

Mika

$4:6 \longrightarrow 4 \div 6 = \frac{4}{6}$

$= \square$

Ken

$6:9 \longrightarrow 6 \div 9 = \frac{6}{9}$

$= \square$

② Compare the values of the ratios $2:3$, $4:6$ and $6:9$.

When the values of the ratios are equal, we say that "the ratios are equivalent," and express the relationship as

$$2:3 = 4:6$$

Since ratios $2:3$, $4:6$, and $6:9$ are equal in the value of the ratio, these ratios are equivalent.

Haruto

1 Find the values of the ratios 4 : 5 and 5 : 4.

2 Find the values of the ratios and identify the equivalent ratios.

① 1 : 2 ② 6 : 8 ③ 21 : 28

④ 10 : 5 ⑤ 20 : 15 ⑥ 25 : 50

Additional Problems
→Page250 R

Mathlin Newsletter

Ratios and Rates

We studied rates in 5th grade already.

Records of basketball shooting practice

Baskets made	Total attempts
7	10

We expressed the rate of baskets made as $\frac{7}{10}$ or 0.7.

We also studied notations such as 70% and 7 wari.

We can use a ratio to express the rate of the number of baskets made compared to the number of total attempts as shown on the right.

Baskets made : Total attempts = 7:10

The value of the ratio is $\boxed{}$.

The rate of the number of baskets made compared to the number of total attempts is as shown on the right.

(Quantity being compared) (Base quantity)

Baskets made ÷ Total attempts = 7÷10

$= \frac{7}{10}$ (0.7) < (Rates)

This also represents the value of ratio of 7:10.

A ratio is a way to express a proportion using 2 numbers, while the rate we studied in 5th grade expresses the proportion using 1 number.

I wonder what the properties of equivalent ratios could be.

2 Properties of Equivalent Ratios

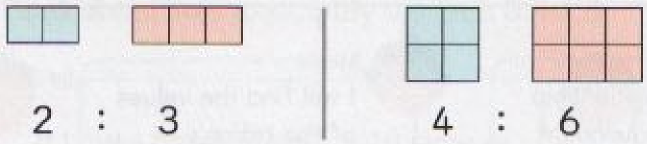
1 The ratios, 2 : 3, 4 : 6, and 6 : 9 are equivalent.
Compare equivalent ratios.


Riku

If you look at the numbers on the left and right sides of " : "...

Let's investigate the relationships among equivalent ratios.

① Investigate the relationship between the ratios, 2 : 3 and 4 : 6.



② Explain the two students' ideas above.


Haruto

$$\underset{\times 2}{\overset{\times 2}{2:3=4:6}}$$


Ami

$$\underset{\div 2}{\overset{\div 2}{4:6=2:3}}$$

③ Investigate to see if the relationships also hold for the ratios, 2 : 3 and 6 : 9.

Summary

Equivalent ratios have the following relationship:

(1) $\underset{\times \boxed{2}}{\overset{\times \boxed{2}}{2:3=4:6}}$ (2) $\underset{\div ③}{\overset{\div ③}{6:9=2:3}}$

Numbers in □ and ○ are the same, aren't they?

If we use this relationship, we can make equivalent ratios.


Shiho

④ What number that goes into each □?

$$\underset{\times \boxed{}}{\overset{\times \boxed{}}{4:6=6:9}}$$

1 There is an error in Kota's idea on the right.
Explain the reason why.


Kota

I made a ratio that was equivalent to 2:3.

$$\underset{\times \boxed{}}{\overset{\times \boxed{}}{2:3=4:9}}$$

2 Make 3 ratios that are equivalent to 6:8.


Misaki

Among equivalent ratios, the one with smaller numbers helps you understand the rate of one number compared to the other more easily.

We are going to think about ratio 49 : 63.


Shiho

The numbers in this ratio are so big that it's hard to understand the rate of one number compared to the other.

2 Express ratio 49 : 63 much more simply.

If we use what we learned about equivalent ratios...


Haruto

Let's think about how to express a ratio with the equivalent ratio that uses the smallest whole numbers.


Kota

Use the relationship between equivalent ratios...

$$\begin{tikzcd}49 \arrow[rrrr, bend left, "\div 7"] & : & 63 \arrow[rrrr, bend right, "\div 7"'] & = & 7 & : & 9\end{tikzcd}$$


Misaki

I will find the values of the ratios...

$$49 : 63 \rightarrow \frac{\overset{7}{\cancel{49}}}{\underset{9}{\cancel{63}}}$$

$$49 : 63 = 7 : 9$$

① Discuss these 2 students' ideas.


Riku

Kouta and Misaki's ideas are similar, aren't they?

When we express a ratio with the equivalent ratio that uses the smallest whole numbers we say we are "simplifying the ratio."

$\frac{\overset{7}{\cancel{49}}}{\underset{9}{\cancel{63}}}$ It's the same as simplifying fractions, isn't it?

Simplifying Fractions
Page 273 ⑧

We can simplify a ratio **if you divide the two numbers in the ratio by their common factor.**


Ami

3 Simplify the following ratios.

① 12 : 9 ② 16 : 24 ③ 18 : 42 ④ 14 : 49

Additional Problems
→Page 250 S

4 Which of the following ratios can be simplified?

Ⓐ 4 : 9 Ⓑ 15 : 51 Ⓒ 32 : 25


Riku

I wonder if some ratios are expressed by using decimal numbers or fractions.

3 Simplify ratios 0.9:1.5 and $\frac{2}{3} : \frac{4}{5}$.

We can simplify ratios with whole numbers, so...


Misaki

Let's think about how to simplify ratios involving fractions and decimal numbers.

① Think about how to simplify the ratio 0.9:1.5.


Riku

If I make both 0.9 and 1.5 10 times as much...

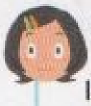
$$0.9 : 1.5 = (0.9 \times 10) : (1.5 \times 10)$$
$$= 9 : \boxed{}$$
$$= \boxed{} : \boxed{}$$


Shiho

If I use 0.1 as the unit...

$$0.9 : 1.5 = 9 : \boxed{}$$
$$= \boxed{} : \boxed{}$$

② Think about how to simplify the ratio $\frac{2}{3} : \frac{4}{5}$.


Ami

If we multiply the numbers in the ratio by a common multiple of their denominators...

$$\frac{2}{3} : \frac{4}{5} = \left(\frac{2}{3} \times 15\right) : \left(\frac{4}{5} \times 15\right)$$
$$= 10 : 12$$
$$= \boxed{} : \boxed{}$$


Haruto

If I find a common denominator...

$$\frac{2}{3} : \frac{4}{5} = \frac{10}{15} : \frac{12}{15}$$
$$= 10 : \boxed{}$$
$$= \boxed{} : \boxed{}$$

Use $\frac{1}{15}$ as the unit.

In all these cases, we should **change the ratios into those made of whole numbers based on the properties of equivalent ratios and ways to look at numbers**, and then simplify the ratios.


Kota

Finding a Common Denominator
Page 273 ⑩

5 Simplify the following ratios.

① 0.5 : 0.6　② 2.5 : 3　③ $\frac{5}{6} : \frac{2}{9}$　④ $\frac{12}{5} : 6$

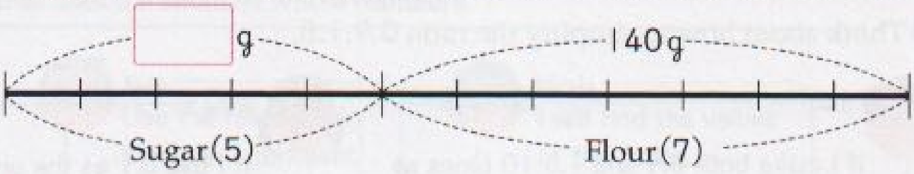
Additional Problems
→Page251 T


Shiho

Since we have studied ratios, I want to use them in my daily life.

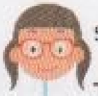
2 Applications of Ratios

1 To make a cake, we mix sugar and flour according to their weight in a ratio of 5 : 7. If we use 140 g of flour, how much sugar do we need?



Let's think about how to find the quantity for one of the numbers of a ratio.

① Explain the following two students' ideas.



Shiho

The ratio of the weight of sugar and flour is 5:7. If I think of the weight of flour as 1, the weight of sugar will be $\frac{5}{7}$.

$$140 \times \frac{5}{7} = \boxed{}$$

Answer $\boxed{}$ g

Kota

I consider the weight of sugar to be x g.

$$\overset{\times 20}{}$$
$$5 : 7 = x : 140$$
$$\underset{\times 20}{}$$

$$x = 5 \times 20$$
$$= \boxed{}$$

Answer $\boxed{}$ g

Summary

You can find the answer if you **consider one of the quantities as 1 or make an equivalent ratio**.

1 We are making a rectangular flag in which the length and the width are in the ratio of 5:8.
If the length is 75 cm, what is the width?

2 Find the value of x in each of the following.

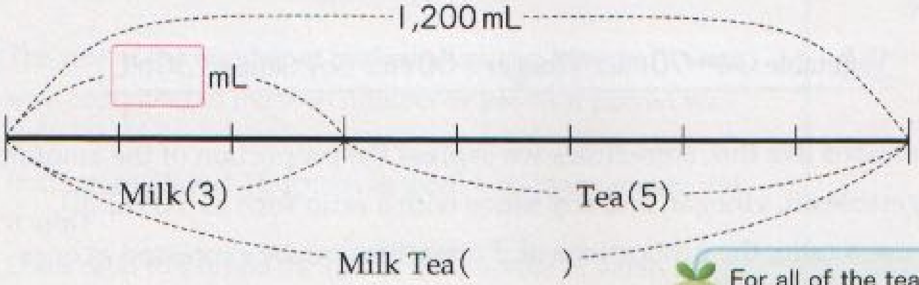
① $15 : 10 = x : 2$ ② $7.5 : 5 = 3 : x$

Additional Problems
→ Page 251 U

Haruto: We were able to solve the problem by focusing on the relationship between the two quantities and using the ratio.

2

We are going to make 1,200 mL of milk tea. If we mix tea and milk in the ratio of 3:5, how much milk do we need?



For all of the tea with milk, add 3 and 5...

Let's think about how to split an entire amount into parts given their ratio.

① Explain the following 2 students' ideas.

Riku

The amount of milk is $\frac{3}{8}$ of the entire milk tea.

$$1{,}200 \times \frac{3}{8} = \square$$

Answer □ mL

Misaki

I considered the amount of milk as x mL.

$$\overset{\times 150}{} \quad 3 : 8 = x : 1{,}200 \quad \underset{\times 150}{}$$

$$x = 3 \times 150$$
$$= \square$$

Answer □ mL

Summary

You can find the answer if you **consider the entire amount as 1 or make a ratio equivalent to the ratio of a part to the entire amount.**

3 Sayuri and Makoto are going to share 250 pieces of colored paper in the ratio of 3 : 2. How many pieces of colored paper will Sayuri and Makoto get?

Mathlin Newsletter

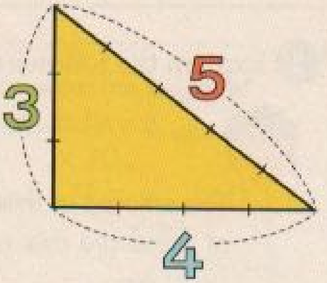
Ratio of 3 Numbers

To make a certain Japanese style salad dressing, vegetable oil, vinegar, and soy sauce are mixed in the proportions shown below.

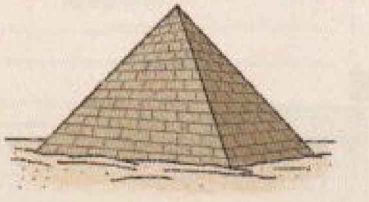
Vegetable Oil···70 mL, Vinegar···50 mL, Soy Sauce···30 mL

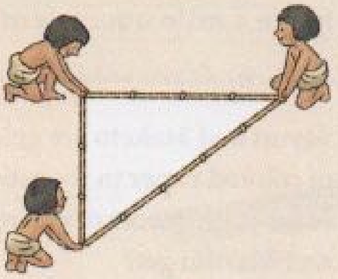
In situations like this, sometimes we express the proportion of the amounts of vegetable oil, vinegar, and soy sauce with a ratio such as 70:50:30.
If we use a ratio, the proportions of 3 quantities can be expressed at once and it is easy to understand.

Express the proportions of the 3 sides of the triangle on the right using a ratio.



When the ratio of the 3 sides of a triangle is 3:4:5, the triangle is a right triangle.
The ancient Egyptians used a square as the base of a pyramid. How did they make a right angle when they didn't have tools like set squares?
As shown on the right, they tied knots to divide a rope equally into 12 segments. They then formed a triangle with sides in the ratio of 3:4:5 to make a right triangle.





Wrapping Up Your Learning: Ratios

Check Your Understanding

☐ **1** Find the values of the ratios.

① 3 : 5 ② 25 : 45 ③ 1.2 : 0.9 ④ 0.8 : 2

Can you find the values of ratios?

Page 79 2

☐ **2** The rate of the number of basketball games Mayumi's team won compared to the total number of games it played was 0.6.

If the team played 15 games in total, how many games did it win?

Use a ratio to express the rate of the number of wins compared to the total number of games.

Can you express the rates you studied in 5th grade by using ratios?

Page 79 2

☐ **3** Simplify the ratios.

① 40 : 120 ② 12 : 21 ③ 5.6 : 2.1 ④ $\frac{3}{5}:\frac{1}{3}$

Can you simplify ratios?

①② Page 82 2
③④ Page 83 3

☐ **4** Find the number that is represented by x.

① $10 : 4 = 5 : x$ ② $2 : 0.5 = x : 2$

Can you find one of the numbers in a ratio?

Page 84 1

☐ **5** We are going to cut a piece of paper into a rectangle with the ratio of its width and length being 5:3.

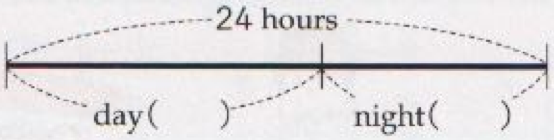
If the width is 45 cm, how many cm is the length?

Can you solve problems using ratios?

Page 84 1

☐ **6** The ratio of daytime to nighttime on a certain day was 7:5.

How long was the day time?



Can you solve problems using ratios?

Page 85 2

Wrapping Up Your Learning: Ratios

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Relationship between Quantities and Think about How to Find the Quantity You Want to Know

The amounts of Worcestershire sauce and ketchup Tadashi used to make steak sauce are as shown on the right.

Worcestershire Sauce	Ketchup
◡ ◡	◡ ◡ ◡

(◡ represents a teaspoon)

① By considering a teaspoon as 1, express the proportions of the amounts of Worcestershire sauce and ketchup as a ratio.
Also, find the value of the ratio.

Ratio ☐ : ☐

Value of the Ratio $\square \div \square = \frac{\square}{\square}$

② We are going to make steak sauce that will taste the same as Tadashi's above.
Riku found the amount of Worcestershire sauce to be mixed with 30 mL of ketchup in the following way.
Based on Ami's idea, explain Riku's way of finding the amount.


Riku

$30 \times \frac{2}{3} = 20$, so we need 20 mL.

Riku used the fact that if 30 mL of ketchup was considered as 1, the amount of Worcestershire sauce corresponded to ☐ ...


Ami

Look back on what you have learned in "Let's Investigate How to Express Rates" and discuss.

Now I know how to express rates by using ratios. I also understand well how ratios are related to the way of expressing rates we studied in 5th grade.

I discovered that simplifying a ratio is the same thing as simplifying a fraction. I want to keep studying new things while paying attention to how they are related to what I studied before.


Kota


Shiho

Challenge Yourself
→Page261

Junior High School In junior high school, you will study how to easily find the value of x in math sentences involving equivalent ratios.

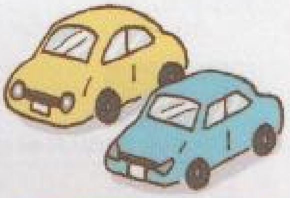
Do You Remember?

Answers → Page 268

1 Express each elapsed times as a fraction using the unit indicated in the ( ).
① 45 min (hr) ② 25 sec (min) ③ 80 min (hr)

2 When a $\frac{2}{3}$ m iron pipe was weighed, it was $\frac{26}{9}$ kg.
① What is the weight of 1 m of this iron pipe?
② How many m is 1 kg of this iron pipe?

3 A car travels 300 km on 25 L of gasoline. Another car travels 250 km on 20 L of gasoline.
Which car travels farther on 1 L of gasoline?



Per unit quantity
Page 273 ⑪

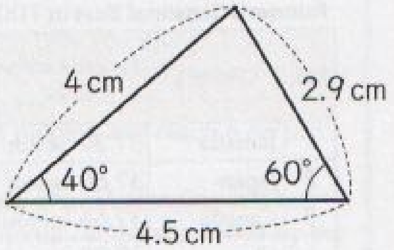
4 Find the speed per minute and per hour of a car that travels 120 km in 2 hours.

5 Answer the following questions.
① Find the circumference of a circle with the diameter of 9 cm.
② What is the radius of a circle with a circumference of 12.56 m?

Length Around Circles
Page 275 ㉒

Warm-up

6 Draw a triangle that is congruent to this triangle.



There are different ways to draw it.

How to Draw Congruent Triangles
Page 274 ⑲

Playing with Numbers and Calculations

Strange Calculations

Calculate math sentences ⓐ and ⓑ, and compare their answers.

① ⓐ $\frac{1}{2}-\frac{1}{3}$ ⓑ $\frac{1}{2}\times\frac{1}{3}$

② ⓐ $\frac{2}{3}-\frac{2}{5}$ ⓑ $\frac{2}{3}\times\frac{2}{5}$

③ ⓐ $\frac{3}{5}-\frac{3}{8}$ ⓑ $\frac{3}{5}\times\frac{3}{8}$

The difference in denominators is [...] the numerator.

Let's Think about Track and Field Records

Previous Olympics and Paralympics saw a wide variety of records.

1

Investigating track and field records, Shiho and her friends found the following data 1, 2, and 3 below.

Data 1

Olympic Records as of 2017

Event	Men's	Women's
100 meters	9.63 (seconds)	10.62 (seconds)
4 × 100 m relay	36.84 (seconds)	40.82 (seconds)
Marathon	2:06:32	2:23:07

〔Developed by Tokyo Shoseki based on World Athletics' and other websites〕

Data 2

Winning Records at 1964 Tokyo Olympics

Event	Men's	Women's
100 meters	10.06 (seconds)	11.49 (seconds)
4 × 100 m relay	39.06 (seconds)	43.69 (seconds)
Marathon	2:12:11	Not held

NOTE: The records of 100 meters and the 4 × 100 m relay were measured with instruments measuring time in $\frac{1}{100}$ seconds.

〔Developed by Tokyo Shoseki based on World Athletics' and other websites〕

Data 3

Records of Top 6 Finalists in 4 × 100 m Relay at Rio de Janeiro Olympics and Each Runner's Seasonal Best in 100 m Race

Rank	Country	Records at the final	The first runner's best record in the 100 m race (seconds)	The second runner's best record in the 100 m race (seconds)	The third runner's best record in the 100 m race (seconds)	The fourth runner's best record in the 100 m race (seconds)
1	Jamaica	37.27 (seconds)	9.92	9.93	9.94	9.81
2	Japan	37.60 (seconds)	10.05	10.36	10.01	10.10
3	Canada	37.64 (seconds)	10.16	9.96	10.34	9.91
4	China	37.90 (seconds)	10.30	10.08	10.08	10.24
5	United Kingdom	37.98 (seconds)	10.01	10.08	10.04	10.19
6	Brazil	38.41 (seconds)	10.21	10.11	10.28	10.26

〔Developed by Tokyo Shoseki based on an article written by the Japan Association of Athletics Federations〕

① In the men's 100 m race, compare the winning record at the 1964 Tokyo Olympics with the Olympic record as of 2017, and discuss what you notice with your classmates.

Misaki

If you compare the records in time, the difference is only ☐ seconds, but...

② In the men's 100 m race, what are the approximate speeds per second of the winning record at the 1964 Tokyo Olympics and the Olympic record as of 2017? Round the answers to the hundredths place.

③ In ②, suppose the winner at the 1964 Tokyo Olympics and the Olympic record holder as of 2017 run a 100 m race at the same speed as they did when they won.
When the faster runner crosses the finish line, about how many m apart will the runners be? Round the answer to the nearest whole number.


Kota

When the Olympic record holder as of 2017 crosses the finish line, the winner at the 1964 Tokyo Olympics will have run...

2

In the final of the men's 4 × 100 m relay at Rio de Janeiro Olympics, Ami noticed that each team's relay record was shorter than the sum of each runner's seasonal best in the 100 m race.


Which data tells you that?


In a relay, the second and later runners start running and reach a certain speed before receiving the baton.
This means, when they receive the baton, they are not standing still but running at a certain speed. So, relay time records can be shortened by the time taken to run the 20 m takeover zones.

① In the final at Rio de Janeiro Olympics, about how many seconds on average did Japan shorten the record per baton pass?
Round the answers to the hundredths place.


Is it safe to say that Japan was relatively good at passing the baton among the top 6 finalists?

Are they the same or different?

The same TV show is on three TV sets.

Compare the three images above, and discuss how they are the same and different.

Misaki

The same mountain is on the screens.

Haruto

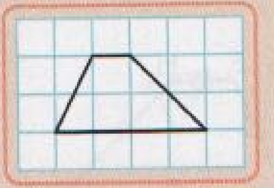
The mountains are different in size, but their shapes are...

6 Enlarged and Reduced Drawings

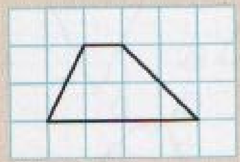
Let's Investigate Geometric Figures that Have the Same Shape but Different Size

We have drawn many trapezoids using a computer.

ⓐ



ⓑ



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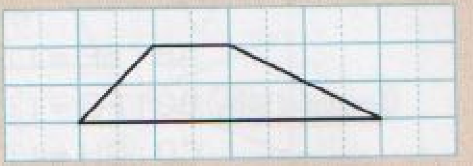

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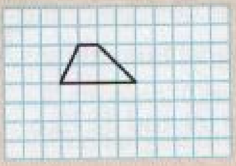

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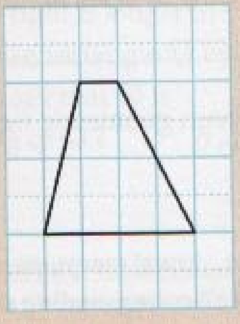
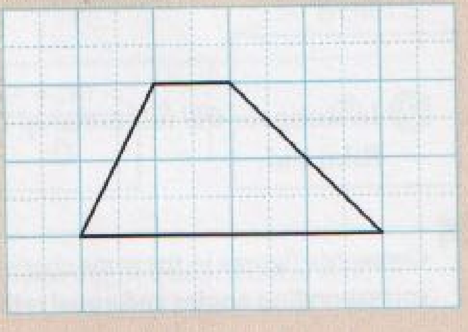


ⓕ



1 Enlarged and Reduced Drawings

1 Which of ⓑ, ⓒ, ⓓ, ⓔ, and ⓕ show the same shape as ⓐ?

Since ⓑ is congruent to ⓐ, they are the same shape.

Congruent Figures Page 274 ⑱

Since the horizontal length is doubled in ⓒ, its shape is...

ⓓ and ⓕ are different in size, but their shapes are...

Let's think about the reason why the geometric figures that are different in size look the same in shape.

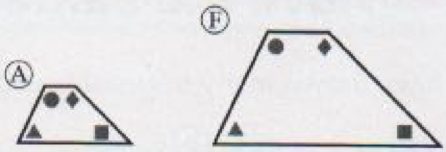
① Compare geometric figure ⓐ with ⓕ, and share what you notice.

When we investigated the relationship between geometric figures, we used to focus on the lengths of sides and the measures of angles...

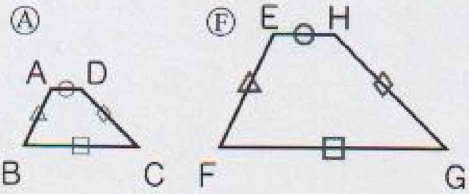
Riku investigated the relationship between geometric figures Ⓐ and Ⓕ as follows.

Riku

Measures of Angles The measures of corresponding angles are equal.



Lengths of Sides
AB : EF = 1 : 2
BC : FG = 1 : 2
CD : GH = 1 : 2
DA : HE = 1 : 2



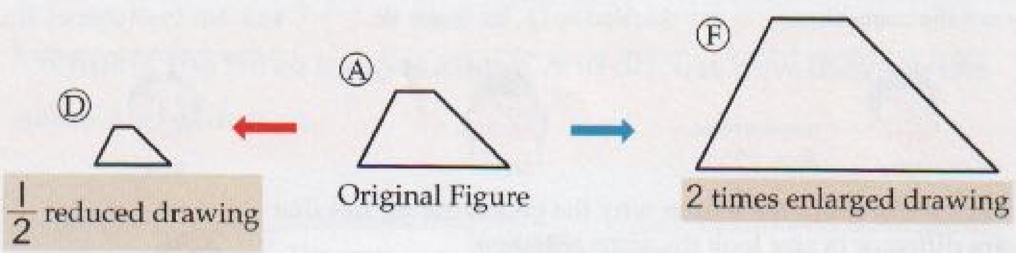
The ratios of the lengths of sides are all equal, 1:2.

② Investigate the relationship between geometric figures Ⓐ and Ⓓ like Riku did.

Geometric figures in the same shape have **equal measures of all corresponding angles and equal ratios of all corresponding sides**.

Ami

When a figure is made larger while the measures of the corresponding angles and the ratios of the lengths of corresponding sides are kept equal, the picture is called an **enlarged drawing**. When a figure is made smaller in the same way, it is called a **reduced drawing**.



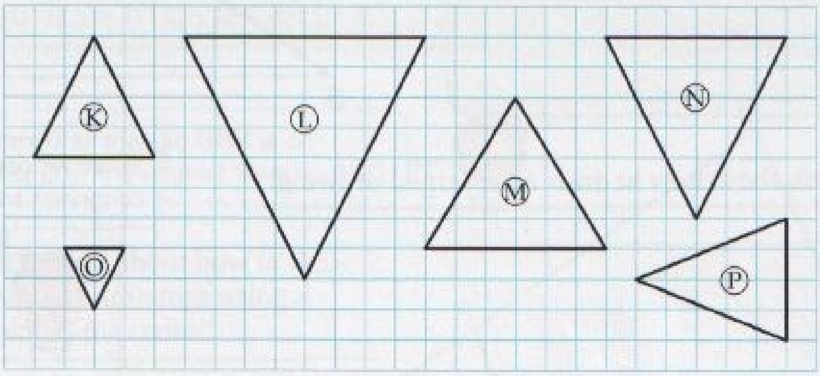
③ Can you say that Ⓒ and Ⓔ are enlarged drawings of Ⓐ ?

Explain the reason why.

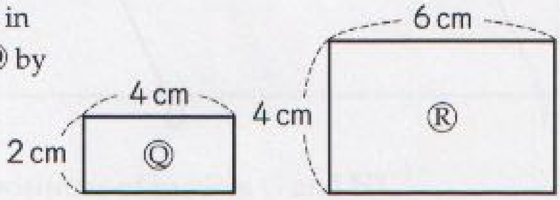
Kota

When we focused on the measures of angles and the lengths of sides just like we did with congruent figures, I understood well the reason why they look like the same shape.

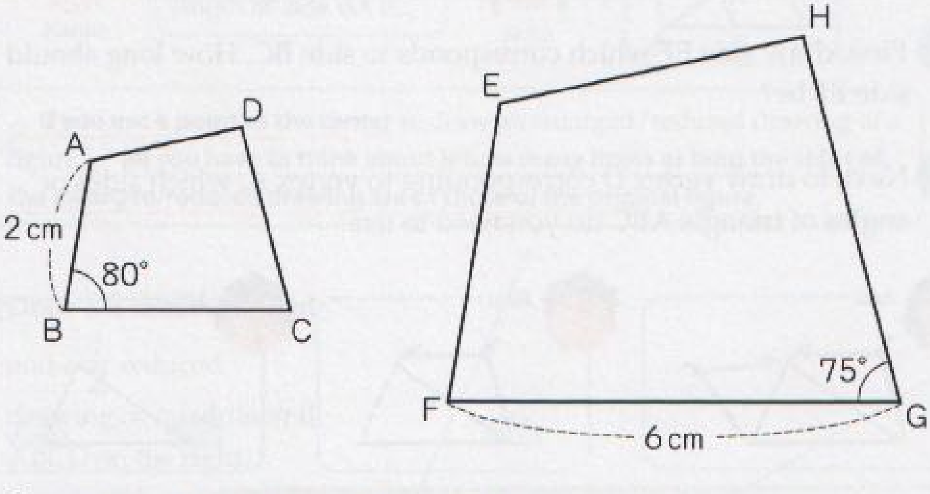
1 Which triangles below are enlarged or reduced drawings of triangle Ⓚ? By how much are they enlarged or reduced?



2 Rectangle Ⓡ on the right is longer in width and length than rectangle Ⓠ by 2 cm each. Can you say that Ⓡ is an enlarged drawing of Ⓠ?



3 Quadrilateral EFGH is a 2 times enlarged drawing of quadrilateral ABCD.

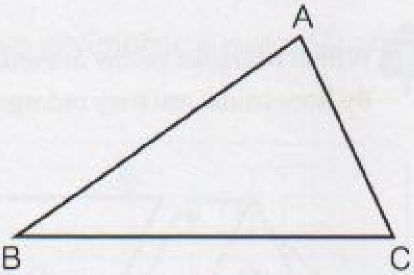


① Which side corresponds to side AB, and how long is it?
② Which angle corresponds to angle B, and what is its measure?
③ How many times a reduced drawing of quadrilateral EFGH is quadrilateral ABCD?
④ Which side corresponds to side FG, and how long is it?
⑤ Which angle corresponds to angle G, and what is its measure?

Shiho

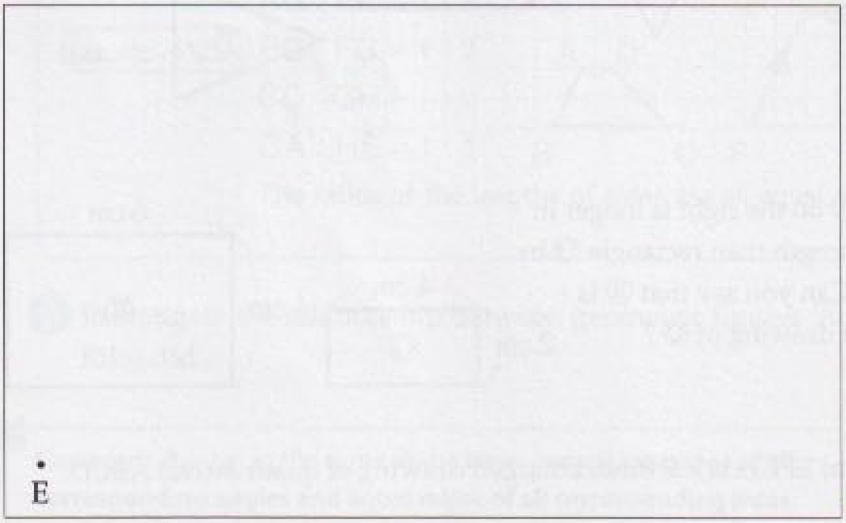
Enlarged and reduced drawings are done by making the sides of the original geometric figure longer or shorter at the same rate, not by the same amount.

2 Draw triangle DEF as a 2 times enlarged drawing of triangle ABC.



Let's think about how to draw an enlarged drawing.

When we drew a congruent triangle...



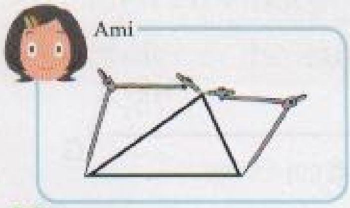
〈Watch〉

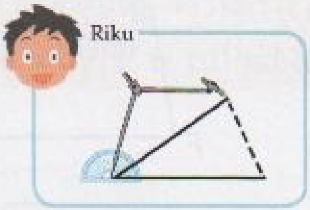
How to Draw Congruent Triangles / How to Draw an Angle

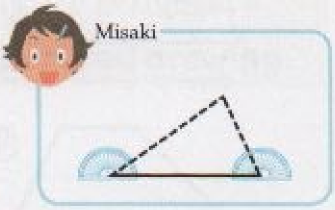
Page 274 ⑲
Page 275 ⑳

① First, draw side EF which corresponds to side BC. How long should side EF be?

② Next, to draw vertex D corresponding to vertex A, which sides or angles of triangle ABC do you need to use?



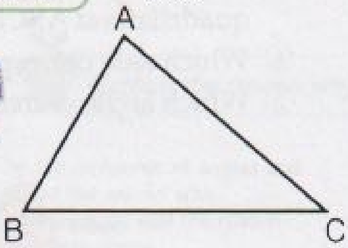




You can make an enlarged drawing without using the lengths of all the sides and the measures of all the angles, just like you can draw a congruent figure without using them all.

4 Draw a $\frac{1}{2}$ reduced drawing of triangle ABC on the right.

Additional Problems
→ Page 251 V



The way of drawing an enlarged or reduced drawing of a triangle is similar to the way of drawing a congruent triangle.

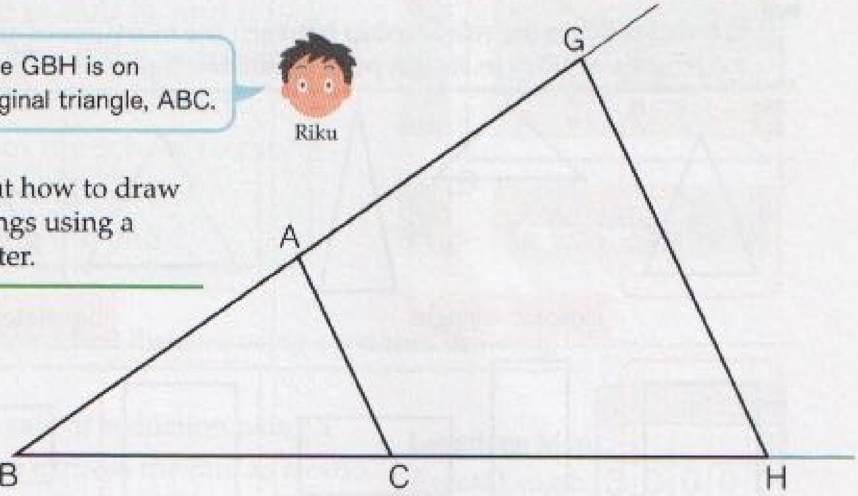
In 2, Kota made an enlarged drawing on top of the original triangle.

3 Triangle GBH below is a 2 times enlarged drawing of triangle ABC from problem 2 on the previous page. Think about how to draw triangle GBH.

Vertex B of triangle GBH is on vertex B of the original triangle, ABC.

Riku

Let's think about how to draw enlarged drawings using a point as the center.

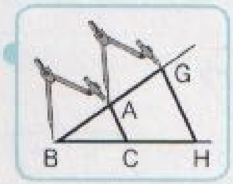


1 How should we decide the positions of vertices G and H?

Haruto

The length of side BG as compared to the length of side BA is...

Shiho

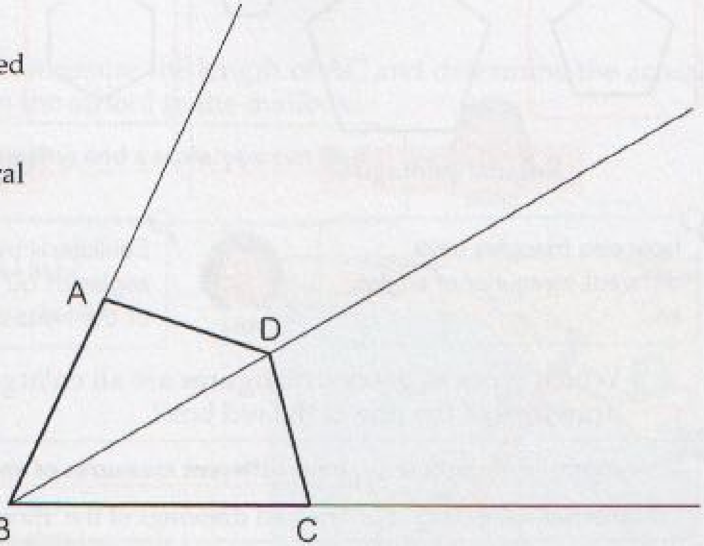


If you use a point as the center to draw an enlarged/reduced drawing of a figure, **all you have to think about is how many times as long the sides of the enlarged/reduced drawing are as those of the original figure.**

Misaki

5 Draw a 2 times enlarged and a $\frac{1}{2}$ reduced drawing of quadrilateral ABCD on the right.

Additional Problems
→ Page 251 W

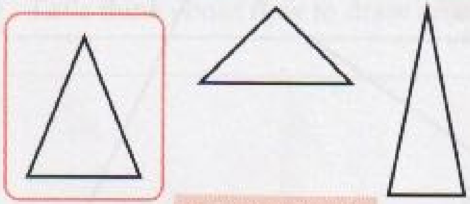


Ami

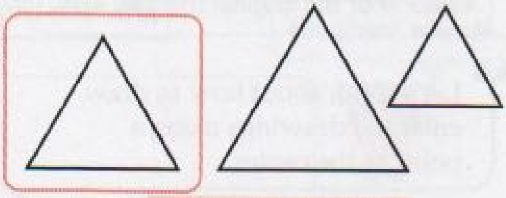
In the 2 times enlarged drawing in 5, the lengths of the diagonals are doubled, too.

4 Of the following types of geometric figure we have studied so far, check to see if all the geometric figures of a given type are enlarged or reduced drawings of the one in the red box.

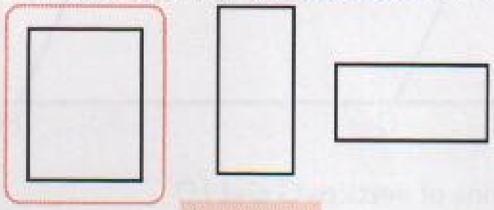
Let's investigate the relationship between the measures of angles and the lengths of sides in each type of geometric figure.



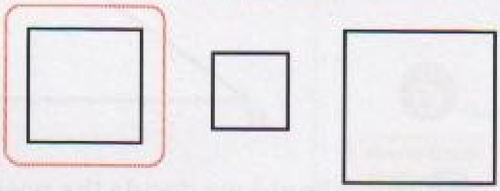
Isosceles triangle



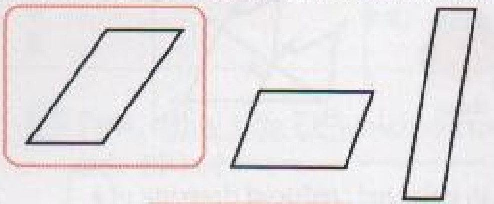
Equilateral triangle



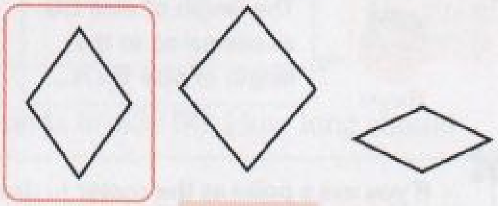
Rectangle



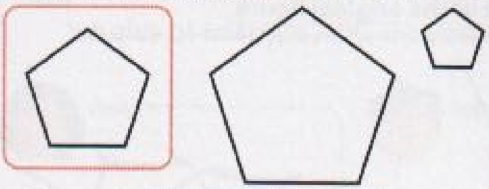
Square



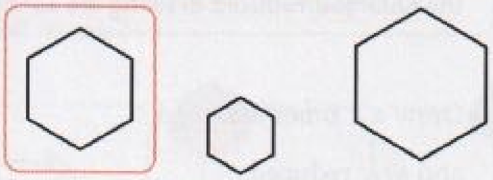
Parallelogram



Rhombus



Regular pentagon



Regular hexagon

Isosceles triangles have different measures of angles, so...

Ami

Equilateral triangles of any size have angles of 60° each, and the lengths of the sides are...

Haruto

① Which types of geometric figures are all enlarged or reduced drawings of the one in the red box?

For example, rhombuses **have different measures of angles**. So, not all rhombuses are enlarged or reduced drawings of the rhombus in the red box.

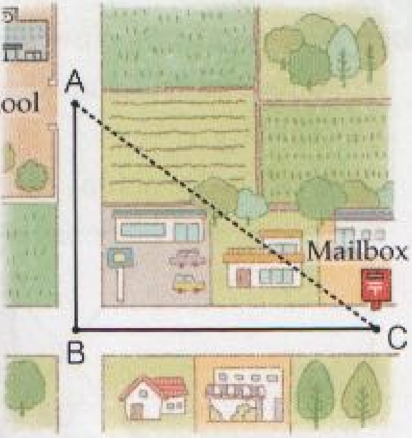
Riku

Misaki

When we took another look at the types of geometric figures we have studied so far by focusing on enlarged or reduced drawings, we made a new discovery.

1 Applications of Reduced Drawings

1 The figure on the right is a reduced drawing of the area around a school. The actual length of AB is 300 m, and it is shown as 3 cm on the reduced drawing. What is the direct distance from the school to the mailbox? How about the distance along the path?



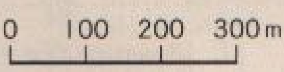
Let's figure out the actual distance using a reduced drawing.

① Express the rate of reduction using a fraction. Also express the rate as a ratio.

			m		cm
Length on Map					3
Actual Length	3	0	0	0	0

The rate in which an actual length is reduced is called a **scale**. A scale may be expressed in the following ways.

Ⓐ $\frac{1}{10,000}$ Ⓑ 1 : 10,000 Ⓒ



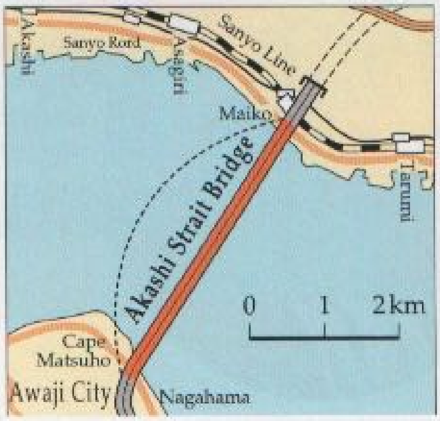
② In the figure above, measure the length of BC and determine the actual distance of BC.
Also, What is the actual distance of the path from the school to the mailbox?

③ In the figure above, measure the length of AC and determine the actual direct distance from the school to the mailbox.

If you use a reduced drawing and a scale, you can find actual distances.


Shiho

1 Find the length of the Akashi Strait Bridge on the map on the right.

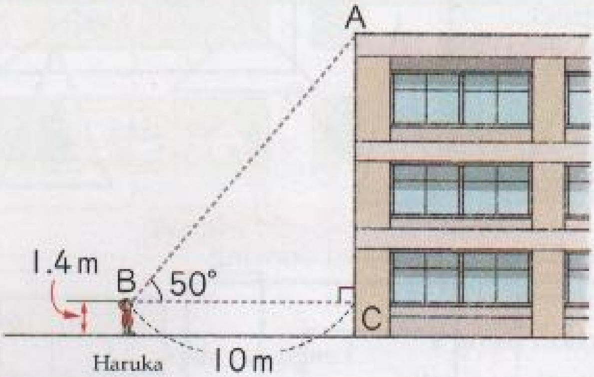


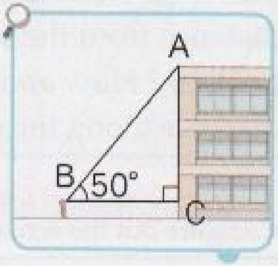

Kota

With a reduced drawing, now I know how to find a distance that is difficult to measure.

2

The diagram below shows Haruka standing 10 m from the school building and looking up at corner A at the top of the building.
What is the actual height of the school building?





Let's think about how to use a reduced drawing to figure out distances that cannot be measured directly.

① Using the method below, draw a $\frac{1}{200}$ reduced drawing of triangle ABC.

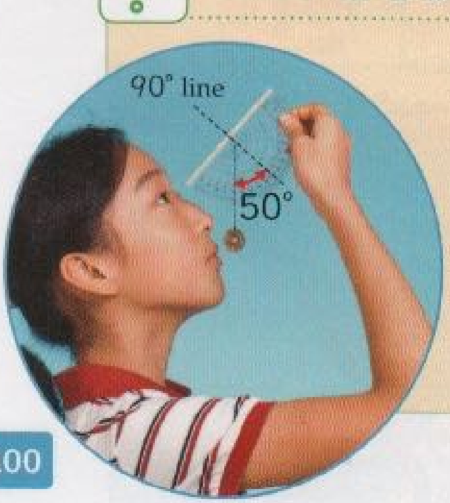
(1) Measure the actual distance of BC. Find the length that is $\frac{1}{200}$ of the distance and draw a corresponding segment.

In the diagram above, it is 10 m

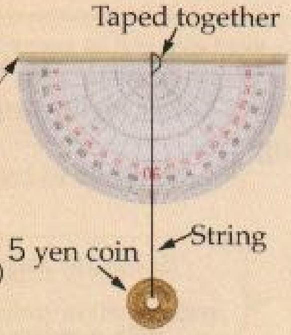
(2) Find the actual measure of angle B using the method below.

In the diagram above, it is 50°

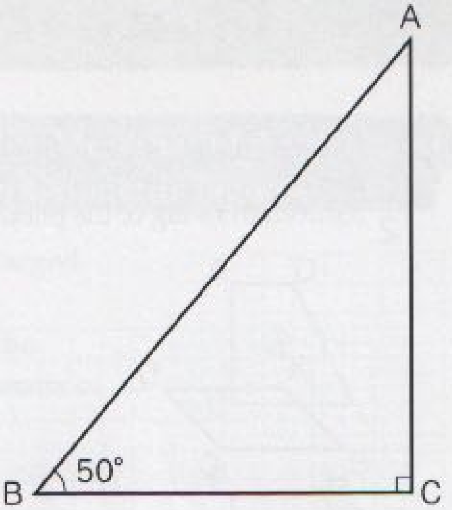
How to measure angle B



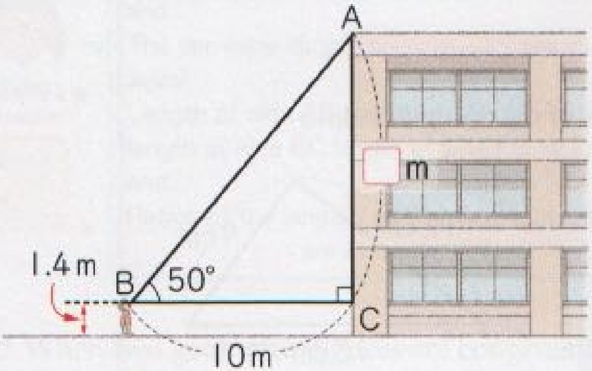
When using a tool like the one shown on the right to look at a target, the measure shown with the ↔ is the angle that you are looking up at.

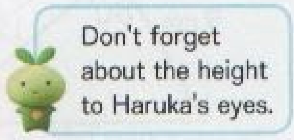


② The figure on the right is a $\frac{1}{200}$ reduced drawing of triangle ABC. Measure the length of side AC. Then, find the actual length of side AC by calculation.



③ What is the actual height of the school building?

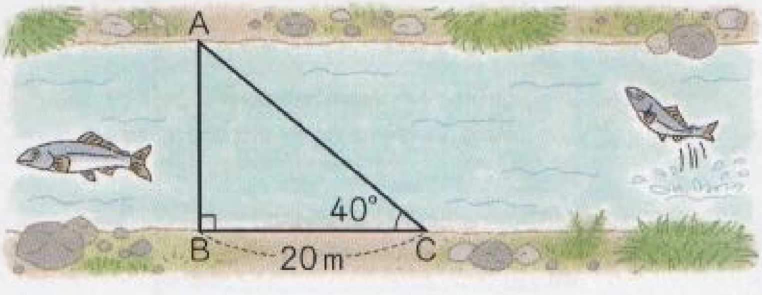




If we know the length BC and angle B of a right triangle ABC, **we can use a reduced drawing to find the actual length of side AC.**

2 In the drawing below, what is the actual width of the river, AB?

Draw a $\frac{1}{500}$ reduced drawing to find the distance.



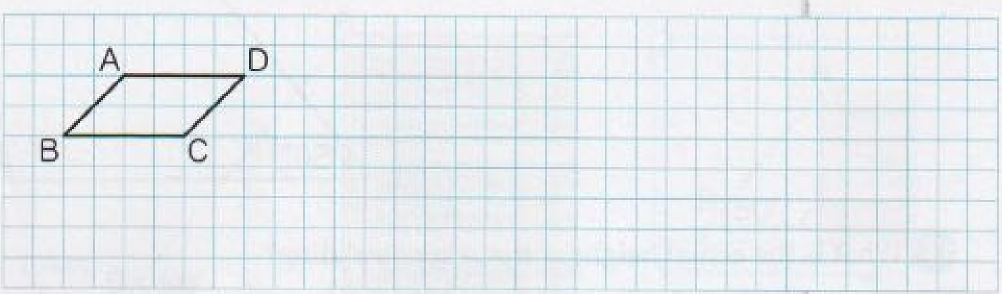
Wrapping Up Your Learning: Enlarged and Reduced Drawings

Check Your Understanding

1 On the grid below, draw a 2 times enlarged drawing and a $\frac{1}{2}$ reduced drawing of the parallelogram ABCD below.

Can you draw enlarged and reduced drawings using a grid?

Page 93 **1**



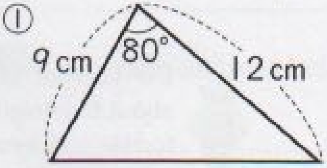
2 Draw $\frac{1}{3}$ reduced drawings of the triangles below.

Can you draw a reduced drawing?

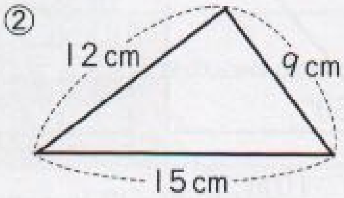
Page 96 **2**

Page 97 **3**

①



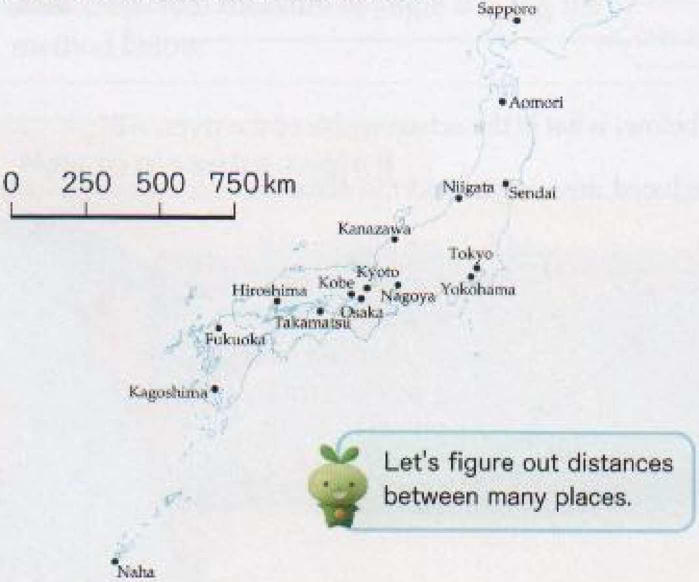
②



3 Using the map on the right, figure out the actual distance from Tokyo to Fukuoka.

Can you figure out the actual distance using a reduced drawing?

Page 99 **1**



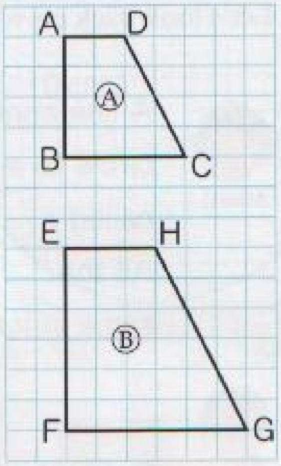
Let's figure out distances between many places.

Wrapping Up Your Learning: Enlarged and Reduced Drawings

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Lengths of Sides and the Measures of Angles and Think about Relationships between Geometric Figures

① Quadrilateral Ⓑ on the right is an enlarged drawing of quadrilateral Ⓐ.
Shiho is explaining the properties of the quadrilaterals on the right. What elements of the figures is she focusing on?




Shiho

Angle A = angle E, angle B = angle F, and...
The corresponding ☐ are all equal.
Length of side AB:length of side EF = 2:3,
length of side BC:length of side FG = 2:3,
and...
Ratios of the lengths of the corresponding ☐ are all equal, 2:3.

② When two geometric figures are congruent, how are the measures of the corresponding angles and the ratios of the lengths of the corresponding sides related?

Properties of the relationship between geometric figures are made clear when you find the lengths of the corresponding sides and the measures of the corresponding angles.

Look back on what you have learned in "Let's Investigate Geometric Figures that Have the Same Shape but Different Size" and discuss.


Haruto

Now I know how to explain that two geometric figures are the same in shape by focusing on the measures of their angles and the lengths of their sides.


Ami

A reduced drawing can be handy. It helps you find a distance you can't actually measure.

Junior High School In junior high school, you will learn about congruent shapes and enlarged and reduced drawings in more detail.

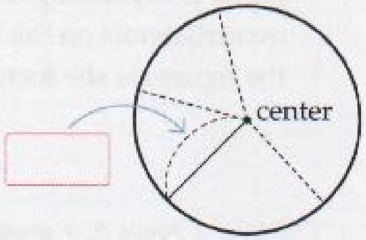
Challenge Yourself
→Page262

What did you learn about circles?

Let's look back on what you learned about circles.

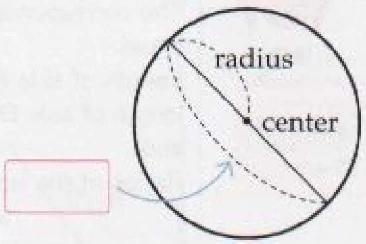
Misaki

A line connecting the center of a circle to a point on the circle is called a ☐ .
All the ☐ of a circle are the same in length.



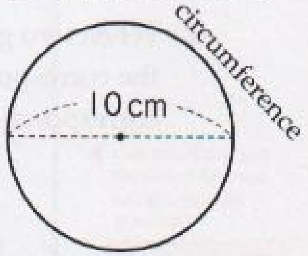
Haruto

The ☐ is twice as long as the radius.



Shiho

The length around a circle is called the circumference.
The number that expresses how many times as long the circumference is as the diameter is called ☐ .
The circumference of the circle on the right is 10 × ☐ = ☐ , or ☐ cm long.



Kota

About how to find the area of a circle, we haven't...

Length Around Circles
Page 275 ㉒

How can we find the area of a circle?

The size of a circle is determined by the length of its radius.

Riku

Ami

I think there may be a relationship between the area and the radius.

Area of Circles

7 Let's Think about How to Calculate the Area of Circles

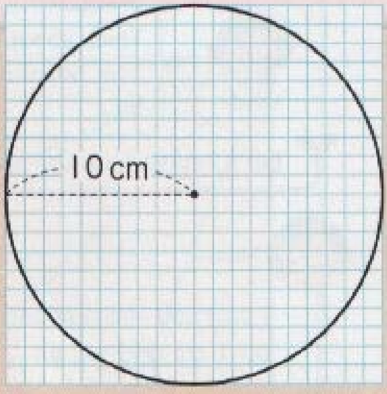
Two students are thinking about how to find the area of the circle on the right.


Shiho

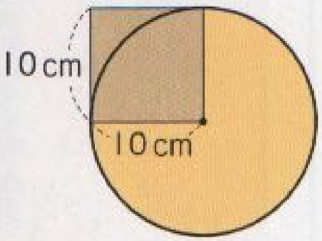
I wonder if we can use the ways of finding the area of geometric figures we have studied.

When we thought about the length of a circumference, we [...] regular polygons in and outside the circle.


Riku



1 We are first going to figure out the area of a circle with a radius of 10 cm. Think about how many times larger the area of the circle is as a square with 10 cm sides.

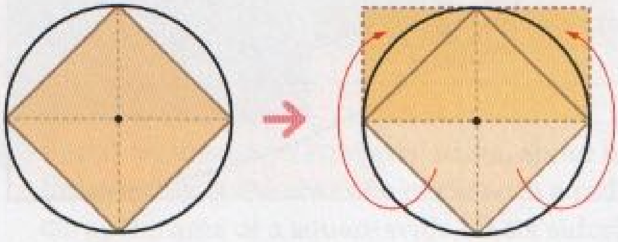


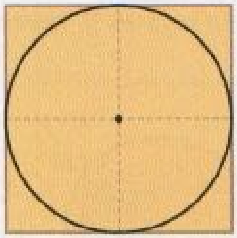
Let's find an approximate area of a circle.

Since the radius of the circle is as long as each side of the square...


Haruto

① Use the figures below to estimate.





As you can see, the area of the circle is **greater than ☐ times and less than ☐ times of the area of a square whose sides are of the same length as the radius of the circle.**


Ami


Misaki

I want to investigate the area of a circle in more details.

2 Think about how to estimate the area of a circle more closely.


Misaki

Think about how many 1 cm^2 squares there will be...

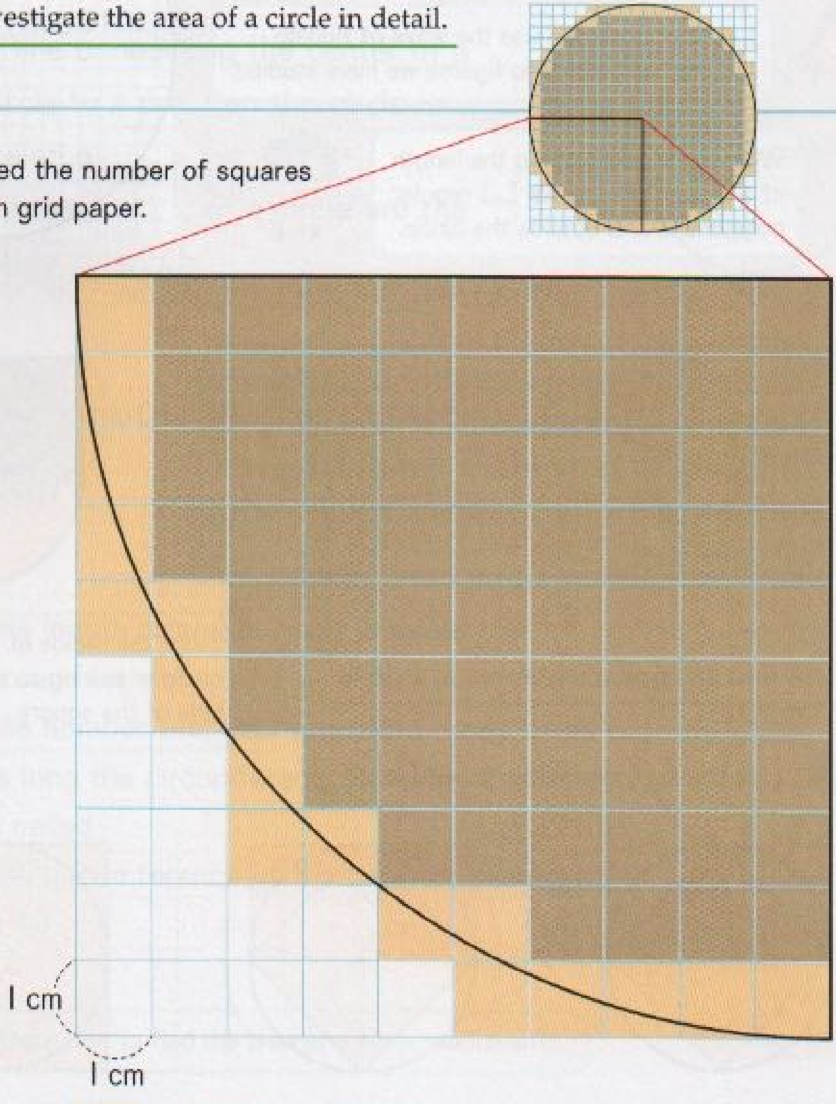
If we draw regular polygons that fit perfectly inside the circle...


Haruto

Let's investigate the area of a circle in detail.


Misaki

I counted the number of squares on 1 cm grid paper.



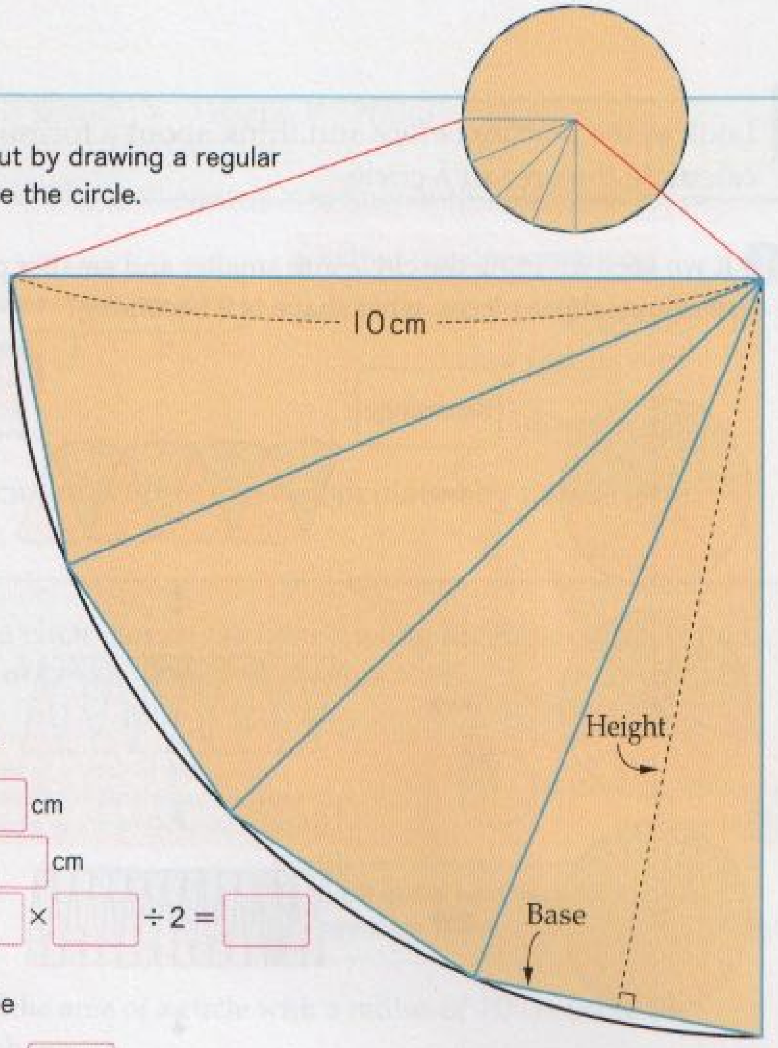
The number of 🟫 is □ …… □ cm^2

The number of 🟧 is □. We count each of these as a half of a square … □ cm^2

} □ cm^2

The area of the circle is □ × 4 = □ Answer about □ cm^2

I figured it out by drawing a regular 16-gon inside the circle.



- One triangle

Base ··· ☐ cm

Height ··· ☐ cm

Area ··· ☐ × ☐ ÷ 2 = ☐

- Area of circle

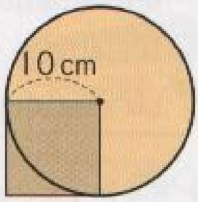
☐ × 16 = ☐

Answer about ☐ cm²

Formula for Calculating the Area of a Triangle
Page 275 ㉓

What will happen if we increase the number of vertices of a regular polygon?

① Based on these two students' ideas, about how many times as big is the area of a circle with a radius of 10 cm as the area of a square with 10 cm sides?



The results these two students obtained are...

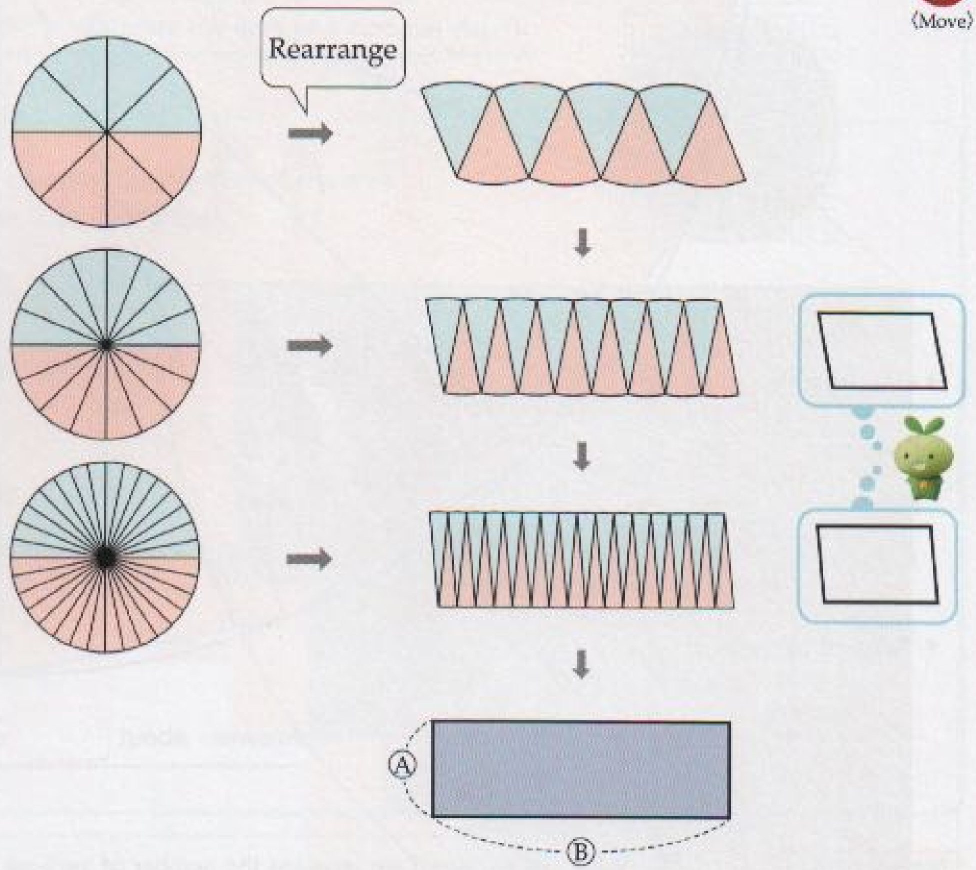
The area of a circle with a radius of 10 cm is about 3.1 times as large as the area of a square with 10 cm sides.

It seems like the area of a circle is also related to pi...

3 Look at the picture below and think about a formula to calculate the area of a circle.

① If we keep dividing the circle into smaller and smaller pieces and rearrange those pieces, what shape is it becoming?

〈Move〉



You may think that if you keep dividing the circle into smaller and smaller pieces and rearrange them, the shape is becoming a rectangle.

Let's come up with a formula to find the area of a circle.

② In the diagram above, are Ⓐ and Ⓑ equal to the lengths of which parts of the circle?

③ Consider the shape made of rearranged pieces of the circle as a rectangle, and make a formula to calculate the area of a circle.

$$\begin{array}{ccccc} \text{Area of Rectangle} & = & \text{Length} & \times & \text{Width} \\ \vdots & & \vdots & & \vdots \\ \text{Area of Circle} & = & \text{radius} & \times & \text{a half of the circumference} \end{array}$$

Circumference=diameter × pi


Haruto

a half of the circumference

$$= \text{diameter} \times \text{pi} \div 2$$

$$= \underbrace{\text{radius}}_{\text{diameter} \div 2} \times \text{pi}$$

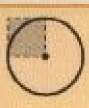
A half of the circumference can be calculated by radius × pi.

Summary

The area of a circle can be calculated using the following formula:

Area of Circle = Radius × Radius × Pi

The area of a circle is about 3.14 times as big as the area of a square that has the radius of the circle as its side, isn't it?


is about 3.14 times as large as

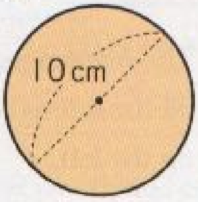


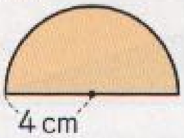
Look at the next page for how to rearrange pieces of a circle into a triangle.

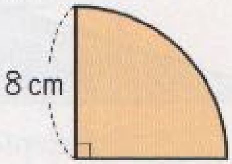
④ Calculate the area of a circle with a radius of 10 cm using the formula above.

1 Calculate the area of the following circles.

①

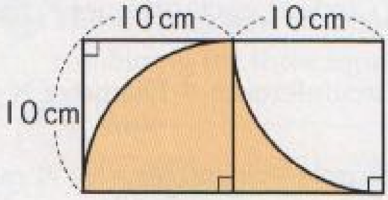

②


③


④


Additional Problems →Page252 X

2 Find the area of the shaded part in the diagram below.




Riku


Turn it...


Ami

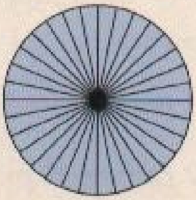

Shiho

We were able to find the areas of many different sized circles and the areas of a half or a quarter of a circle.

Mathlin Newsletter

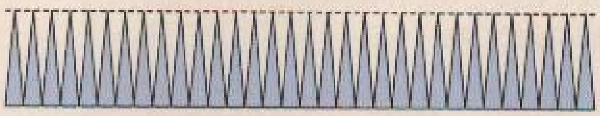
Another Way to Make a Formula to Calculate the Area of a Circle

You can also make a formula to calculate the area of a circle based on the following idea.



① Divide the circle into small pieces.

That's the same as the idea on page 108 so far.

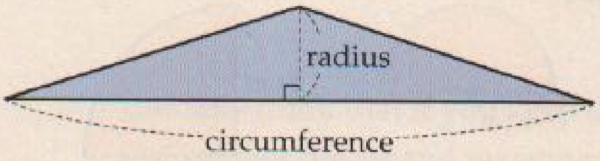


② Open the circle wide.

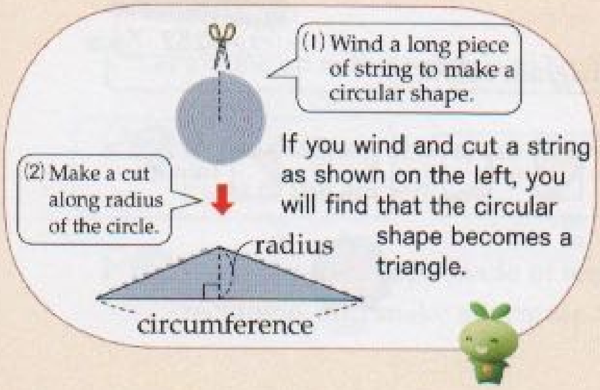
Consider the small pieces of the circle ▲ as triangles.

③ Gather the vertices of ▲ in one point.

Remember that triangles of any shape have the same area as long as their bases are of equal length and their heights are of equal length.



The rearranged pieces of the circle can be considered as a triangle as shown on the left.



(1) Wind a long piece of string to make a circular shape.

(2) Make a cut along radius of the circle.

If you wind and cut a string as shown on the left, you will find that the circular shape becomes a triangle.

Area of triangle = Base × Height ÷ 2

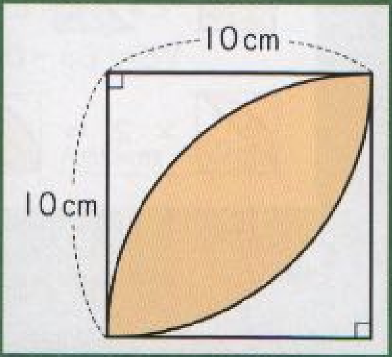
Area of Circle = circumference × radius ÷ 2

Since Circumference = Diameter × pi,

Area of Circle = diameter × pi × radius ÷ 2

$$= \underbrace{\text{radius}}_{\text{diameter} \div 2} \times \text{radius} \times \text{pi}$$

4 Think about how to find the area of the shaded part in the diagram below.



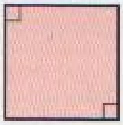
① Plan how to find the answer.

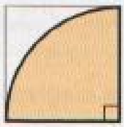
Which part of the figure can you calculate the area of easily?


Ami

② Look at Kota's idea and find the areas of the three geometric figures below.


Kota

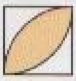
A

10×10
= ☐(cm²)

B

10×10×3.14÷4
= ☐(cm²)

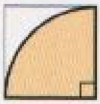
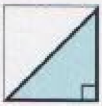
C

10×10÷2
= ☐(cm²)

Let's think about how you should consider the shape of  to find its area.

③ Think about how to calculate the area of the  by combining the three figures above.


Haruto

 −  =

(Move)

Grasp the problem.

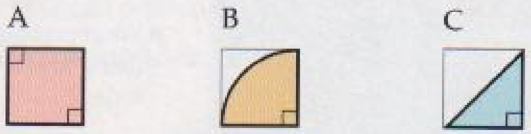
- What problem are we going to work on today?
- What idea may be useful to solve the problem?
- Is there anything you have learned before that you can use to solve this problem?

Write down your ideas.

- Is your idea clear to others?
- If you figured it out one way, try to think of another way.

Shiho and some other students are explaining their classmates' ideas.

Geometric figures we know how to find the area of

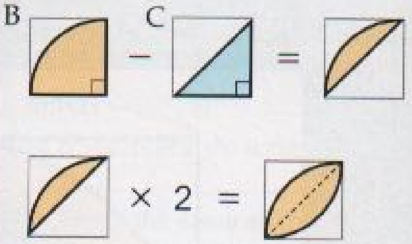


10×10
$=100(\text{cm}^2)$

$10\times10\times3.14\div4$
$=78.5(\text{cm}^2)$

$10\times10\div2$
$=50(\text{cm}^2)$

Haruto



Learn with your classmates.

- Can you understand your classmates' ideas based on their diagrams and math sentences?
- What is common and what is different about your own idea and your classmates' ideas?
- What are the good points in your classmates' ideas?

④ Look at Haruto's idea, and then explain his idea by writing math sentences.

⑤ Look at the math sentences Ami wrote on the next page. Explain how she found the area.

The area of what part is the answer to $100-78.5$?
Draw a diagram like Haruto did.

⑥ Look at Riku's idea on the next page, and explain how he figured out the area.

⑦ What is common among the three students' ideas?

Ami

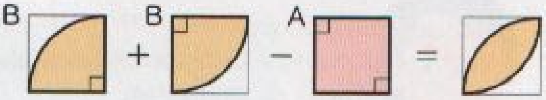
$100 - 78.5 = 21.5$

$21.5 \times 2 = 43$

$100 - 43 = 57$

Answer 57cm²

Riku



$78.5 + 78.5 - 100 = 57$

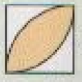
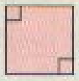
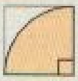
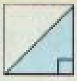
Answer 57cm²

Kota

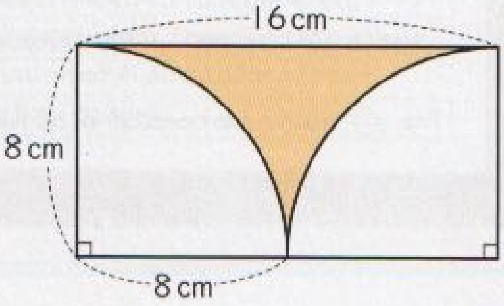
Misaki

⑧ Look back and summarize today's lesson.

Summary

Even the areas of shapes like the  can be figured out **if we think about how to combine geometric figures whose area we know how to find**, such as , , and .

3 Calculate the area of the shaded region.



Additional Problems → Page 252 Y

Look back and summarize today's lesson.

- What did you learn from today's investigation?
- Which way of thinking was useful?

Put it into use.

- Can you what you learned in a new problem?

Use Your Math Notebook for Better Learning

Let's look back at the ideas you used to solve problems.

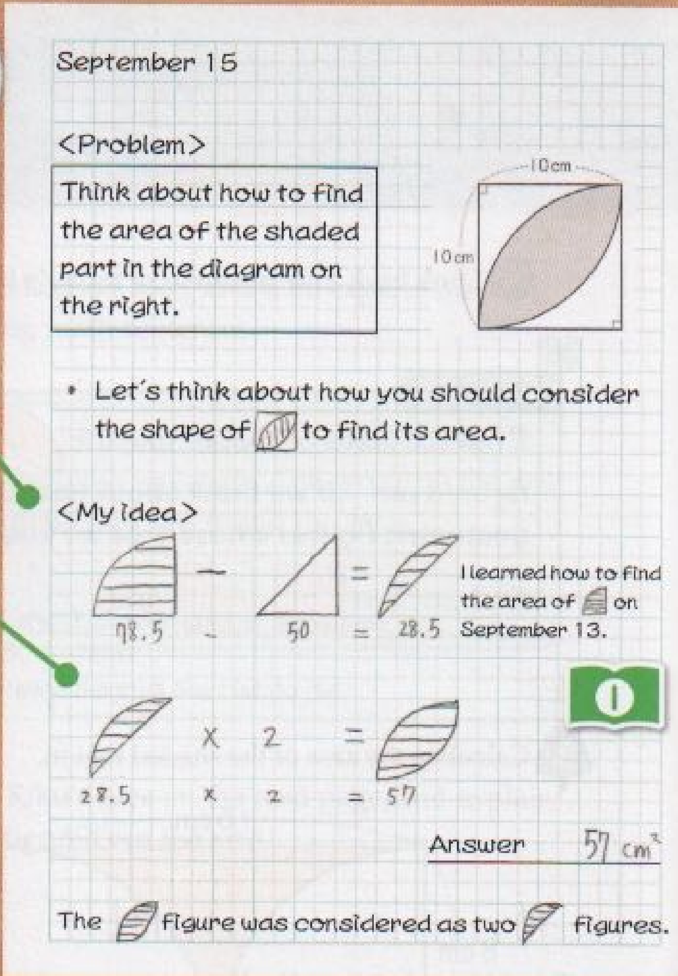
Haruto

He focused on the shapes he knew how to find the area of.

He used diagrams to show the combinations clearly.

When you work on problems, not only write math sentences and answers but also consider using:

- Diagrams
- Tables
- Graphs



September 15

<Problem>

Think about how to find the area of the shaded part in the diagram on the right.

- Let's think about how you should consider the shape of to find its area.

<My idea>

78.5 − 50 = 28.5

I learned how to find the area of on September 13.

28.5 × 2 = 57

Answer 57 cm²

The figure was considered as two figures.

Classmates' reflection

Ami

Although today's shape was different from the shape we studied on September 13, the idea of focusing on the shapes we already know how to find the area for could be applied today, too.

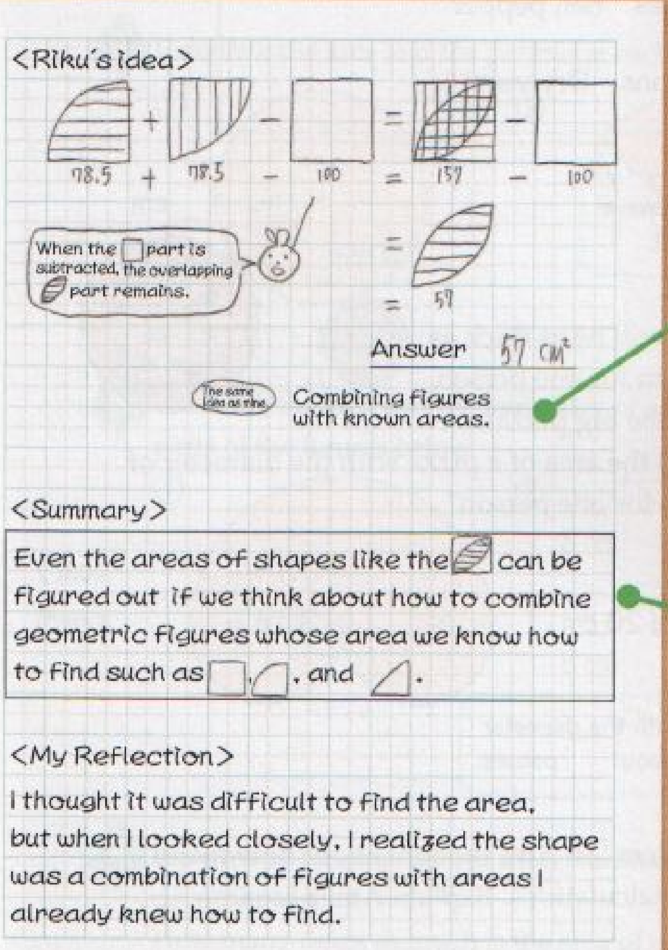
She wrote how her learning from before was useful.

Note Taking Tip ❶

Today's lesson was based on what he had learned. So, he added the date of the lesson so that he can go back to the page of his notebook.

Note Taking Tip ❷

He combines diagrams with math sentences to show his idea clearly.



<Riku's idea>

78.5 + 78.5 − 100 = 157 − 100

= 57

When the □ part is subtracted, the overlapping ⬯ part remains.

Answer 57 cm²

The same idea as mine. Combining figures with known areas.

<Summary>

Even the areas of shapes like the ⬯ can be figured out if we think about how to combine geometric figures whose area we know how to find such as □, ◠, and ◺.

<My Reflection>

I thought it was difficult to find the area, but when I looked closely, I realized the shape was a combination of figures with areas I already knew how to find.

He found out what their ideas had in common.

He summarized the usefulness of focusing on the combination of geometric figures.

Riku

Ami's idea was to pay attention to the unshaded part ◸. I want to try to use Ami's idea in other problems.

He wrote about further ideas he would like to investigate.

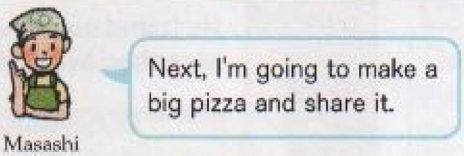
Wrapping Up Your Learning: Area of Circles

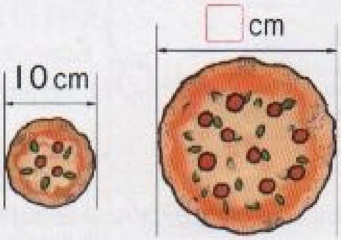
Use What You Have Learned

Masashi made a pizza for one person. Listed below are the ingredients necessary to make the pizza for one person.

Pizza for one person (10 cm in diameter)

- 25 g Flour
- 10 g Pizza sauce
- 20 g Cheese
- 5 slices Salami
- 5 slices Bell pepper
- $\frac{1}{2}$ teaspoons Dry yeast





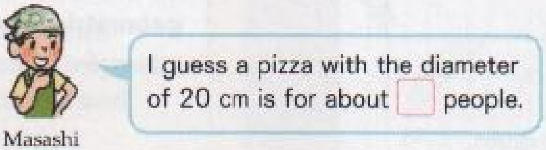
He is going to make a pizza that will be as thick as but twice the diameter as the pizza for one person.

① What will be the diameter of the big pizza?

② How many times as large will the area of a pizza with the diameter of 20 cm as the area of the pizza for one person?

Area of pizza for one person ☐ × ☐ × 3.14 = ☐ (cm^2)

Area of pizza with diameter of 20 cm ☐ × ☐ × 3.14 = ☐ (cm^2)



③ When working on math problem ②, Ami thought about how many times as large without doing calculations. Explain Ami's idea.

She focused on the double radius...

$$\begin{array}{lccccc} \text{Area of pizza for one person} & 5 & \times & 5 & \times & 3.14 \\ & \downarrow\boxed{\times 2} & & \downarrow\boxed{\times 2} & & \\ \text{Area of pizza with diameter of 20 cm} & 10 & \times & 10 & \times & 3.14 \end{array}$$

④ Find the amount of flour necessary to make a pizza with the diameter of 20 cm.

Wrapping Up Your Learning: Area of Circles

Check Your Understanding

1 There is a circle with the radius of 10 cm. Select the math sentences that will calculate ① and ② from Ⓐ to Ⓒ below. Use 3.14 as the value for pi.

① Length of the circumference ② Area of the circle

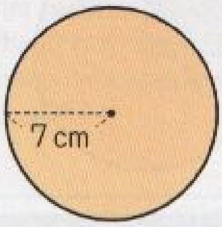
Ⓐ 10×3.14 Ⓑ 10×2×3.14 Ⓒ 10×10×3.14

Do you understand the formula to calculate the area of a circle?

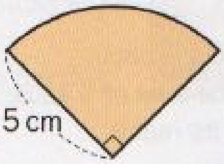
Page 108 3

2 Calculate the area and the perimeter each of the figures below.

①



②

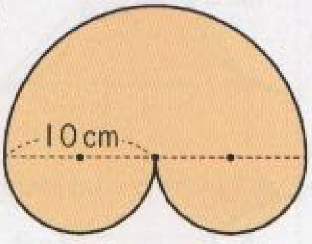


Can you find the area of a circle and the perimeter of a shape with circular parts.

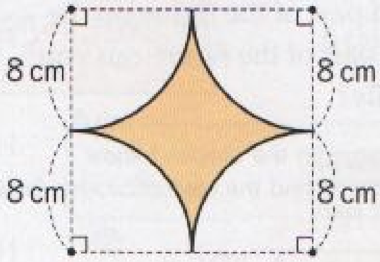
Page 108 3

3 Calculate the area and the perimeter of each of the shaded parts of the figures below.

①



②



Can you figure out the area and the perimeter of figures made up of figures for which you already know how to calculate the area?

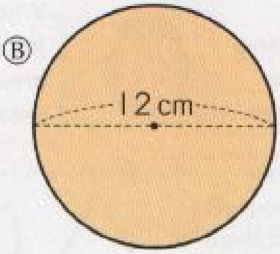
Page 108 3
Page 111 4

4 How many times as long is the circumference of circle Ⓑ as the circumference of circle Ⓐ?

How many times as much is the area of circle Ⓑ as the area of circle Ⓐ?

Ⓐ

Ⓑ



Can you compare the lengths of circumferences and the areas of circles by using formulas?

Page 108 3

Wrapping Up Your Learning: Area of Circles

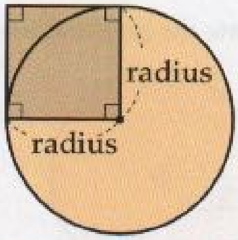
Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

1 Review Formulas and Think about the Relationship between the Area of a Circle and the Area of a Square

Riku is reviewing the formula for finding the area of a circle. Write the numbers that go in the □ and explain Riku's idea.

Area of Circle = Radius × Radius × □

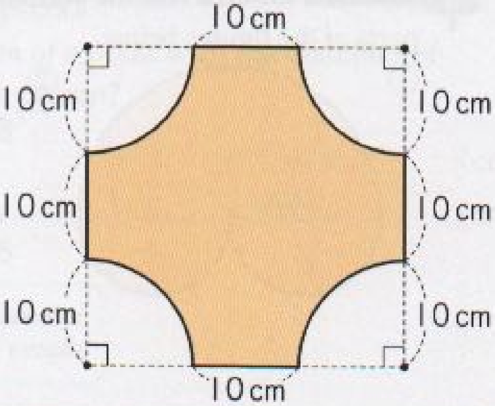
The area of a circle is about □ times as big as the area of a square you can draw on its radius.



2 Focus on the Characteristics of Geometric Figures and Think about How to Calculate Their Areas

Think about how to calculate the area of the shaded part of the figure on the right. Which part of the figure can you calculate easily?

Focus on the shapes I know how to find the area of...



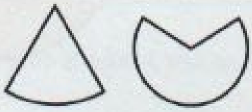
Look back on what you have learned in "Let's Think about How to Calculate the Area of Circles" and discuss.

Now that I've learned about how to find the area of a circle, I know more about circles than before.

When we focused on the geometric figures we knew how to find the area for, we were able to find the area of a shape with curves.

Junior High School

In junior high school, you will think about how to find the area of geometric figures as shown on the right.



Challenge Yourself
→Page263

Do You Remember?

Answers→ Page 269

1 Which ratios in Ⓐ through Ⓒ are equivalent to 4 : 3?

Ⓐ 0.4 : 0.03　Ⓑ 16 : 9　Ⓒ 28 : 21　Ⓓ $\frac{2}{5} : \frac{3}{10}$　Ⓔ $\frac{4}{3} : \frac{3}{5}$

2 Which will have a lower price for one notebook, 8 notebooks for 1,000 yen, or 6 notebooks for 780 yen?

Per unit quantity
Page 273 ⑪

3 The table on the right shows the number of students in Atsushi's class who borrowed books from the library last week.
How many students borrowed books per day on average?

Number of Students Who Borrowed Books

Days of the week	Mon	Tue	Wed	Thu	Fri
Number of Students	12	6	0	14	10

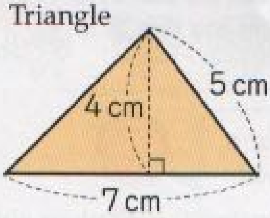
Average
Page 273 ⑫

Warm-up

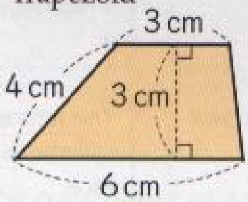
4 Calculate the area of the following figures.

The Formulas for Calculating Area of Triangles and Quadrilaterals
Page 275 ㉓

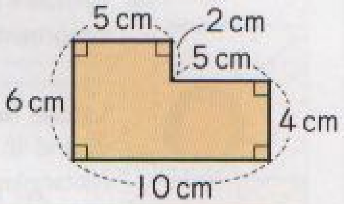
① Triangle



② Trapezoid



③

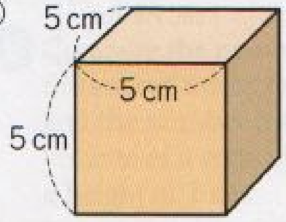


Warm-up

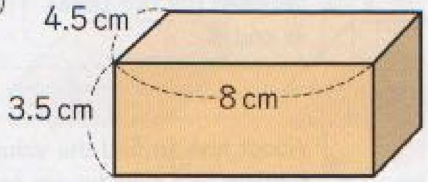
5 Calculate the volume of the cube and cuboid below.

The Formulas for Calculating Volume of Cubes and Cuboids
Page 275 ㉕

①



②



Playing with Numbers and Calculations

Addition Puzzle

Fill in the blanks Ⓐ to Ⓙ with fractions so that the sum of the three numbers in any column, row, or along a diagonal is equal.

①

Ⓐ	0.9	0.6
Ⓑ	1	Ⓒ
Ⓓ	$\frac{11}{10}$	Ⓔ

②

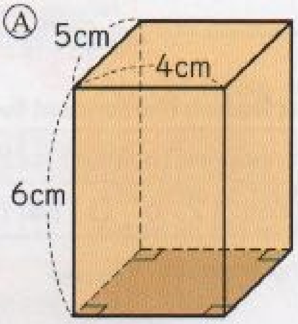
Ⓕ	Ⓖ	0.25
$\frac{1}{12}$	Ⓗ	$\frac{5}{4}$
Ⓘ	Ⓙ	0.5

What did you learn about prisms and cylinders?

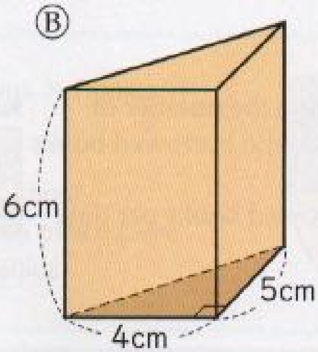
Let's look back on what you learned about prisms and cylinders

Prisms
Page 275 ㉔

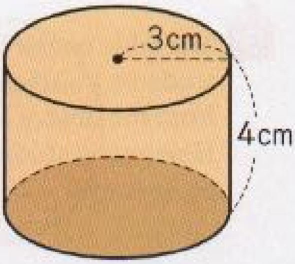
Ⓐ



Ⓑ



Ⓒ



Bases of prisms and cylinders are congruent.

If you carefully look at the shape of the base, you can tell that Ⓐ is a rectangular prism and that Ⓑ is a ☐ .

Lateral faces of prisms Ⓐ and Ⓑ are all rectangles.

The lateral face of a cylinder is a curved face.

The height of a prism or a cylinder is the perpendicular line between the two bases. The height of prism Ⓐ is ☐ cm. As for Ⓑ and Ⓒ ...

About how to find the volume of a prism or a cylinder, we haven't...

How can we find the volume of the prism or the cylinder above?

We know the lengths of the edges, the radius, and the height, but...

I wonder if we can make a formula for calculating the volume of prisms and cylinders.

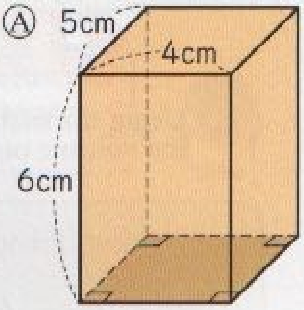
8

Volume of Prisms and Cylinders

Let's Think about How to Calculate the Volume of Prisms and Cylinders

Formula for calculating volume of cuboids
Page 275 ㉕

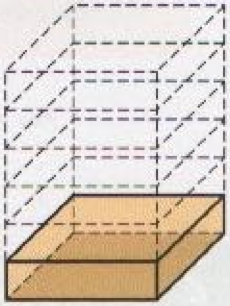
1 Think about how to find the volume of the rectangular prism Ⓐ on page 120.



① Using the formula for finding the volume of a cuboid, find the volume of the rectangular prism on the right.

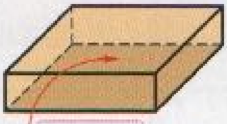
Length × Width × Height = Volume

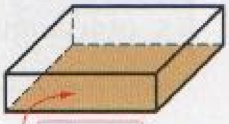
□ × □ × □ = □ (cm³)



This rectangular prism can be considered as a stack of six rectangular prisms with the height of 1 cm, as shown on the right.

② Compare the number that expresses the area of the rectangle at the base and the number that expresses the volume of a rectangular prism with a height of 1 cm.


□ cm³


□ cm²

The area of the base of a prism is called **Area of the Base.**

When the height of a prism is 1 cm, the number expressing the Area of the Base and the number expressing the volume are equal.

Let's think about how to find the volume of a rectangular prism by using the Area of the Base.

③ Using Area of the Base, take another look at the math sentence we used to find the volume in ①.

〔Cuboid〕

$$\underbrace{\overset{\text{Length}}{5} \times \overset{\text{Width}}{4}}_{\text{Area of the Base}} \times \overset{\text{Height}}{\underset{\text{Height}}{6}} = \overset{\text{Volume}}{\underset{\text{Volume}}{120}}$$

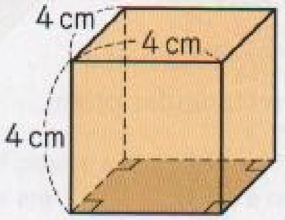
〔Rectangular Prism〕

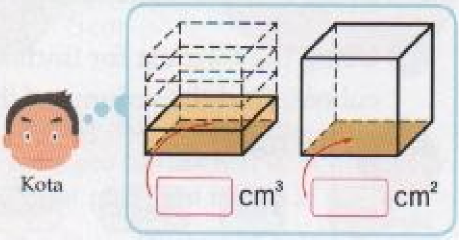
Summary

The volume of a rectangular prism can be calculated by Area of the Base × Height.

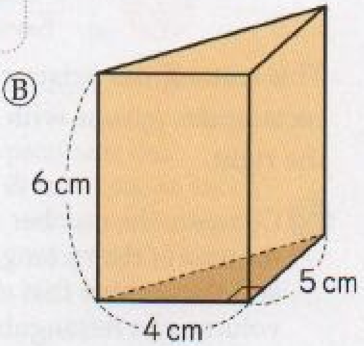
Length × Width in the formula for finding the volume of a cuboid is considered as Area of the Base.

④ Using the math sentence of Area of the Base × Height, can we also find the volume of a cube as shown below?



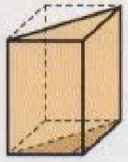


I wonder if we can also calculate the volume of triangular prism Ⓑ on page 120 by the math sentence of Area of the Base × Height.



2 Think about how to find the volume of triangular prism Ⓑ on page 120.

Think about the volume as half the volume of the rectangular prism...



$5 \times 4 \times 6 \div 2 = \square\ (\text{cm}^3)$

Also in the case of a triangular prism, consider if the number expressing the volume of a triangular prism with the height of 1 cm is equal to the number expressing the area of the base, then...

$4 \times 5 \div 2 \times 6 = \square\ (\text{cm}^3)$

① Compare the math sentences the two students wrote on the previous page, and think if the volume of a triangular prism can also be calculated by the math sentence of Area of the Base × Height.

〔Misaki〕 $5\times4\times6\div2=\boxed{}$ (cm³)

〔Haruto〕 $\underbrace{4\times5\div2}_{\text{Area of the Base}}\times\underbrace{6}_{\text{Height}}=\boxed{}$ (cm³)

The math sentence Haruto wrote is the same as that Misaki wrote. So, the volume of a triangular prism can also be calculated by the math sentence of Area of the Base × Height.

Let's summarize the formula for finding the volume of a prism.

Both rectangular and triangular prisms...

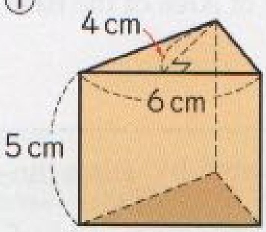
Summary

The volume of prisms can be calculated by using the following formula.
Volume of Prisms = Area of the Base × Height

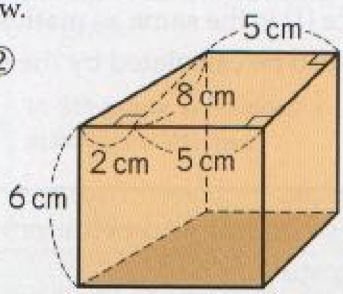
The same formula can be used to find the volume of any prism.

1 Find the volume of the prisms below.

①



②

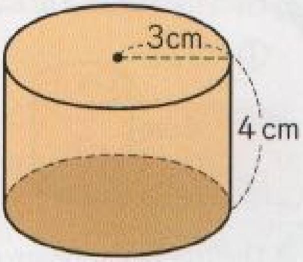


Additional Problems
→Page253 Z

Among the solid figures on page 120, we haven't found the volume of cylinder ©.

3 Think about how to find the volume of cylinder © on page 120.

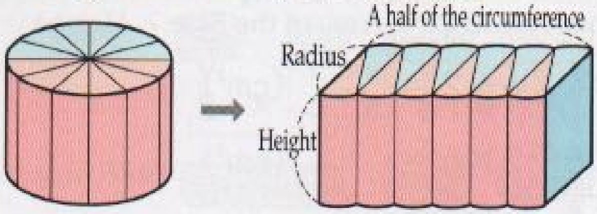
©



The volume of a prism can be calculated by the math sentence of Area of the Base × Height, but...

Let's think if we can also calculate the volume of a cylinder by the math sentence of Area of the Base × Height.

① Look at the diagram below and think about how to find the volume of the cylinder on the previous page.



This is similar to the way of finding the area of a circle on page 108...

$$\underset{\text{Radius}}{3}\times\underbrace{3\times2\times3.14\div2}_{\text{A half of the circumference}}\times\underset{\text{Height}}{4}=\boxed{}\ (\text{cm}^3)\quad\cdots(1)$$

② What math sentence can you write to find the volume of the cylinder on the previous page if you put Area of the Base × Height in the math sentence?

$$\underbrace{3\times3\times3.14}_{\text{Area of the Base}}\times\underset{\text{Height}}{4}=\boxed{}\ (\text{cm}^3)\quad\cdots(2)$$

Is the answer to this math sentence equal to that to math sentence (1)?

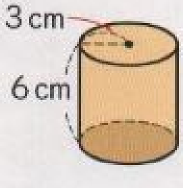
Math sentence (1) is the same as math sentence (2). So, the volume of a cylinder can also be calculated by the math sentence of Area of the Base × Height.

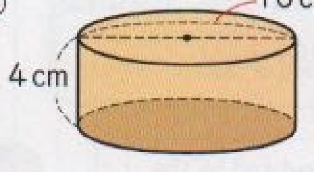
Summary

The volume of prisms and cylinders can be calculated by using the following formula.
Volume of Prisms/Cylinders = Area of the Base × Height

You can find the volume of a prism and a cylinder with the same formula.

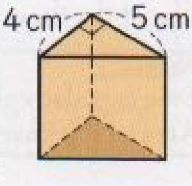
2 Find the volume of each cylinder below.

① 

② 

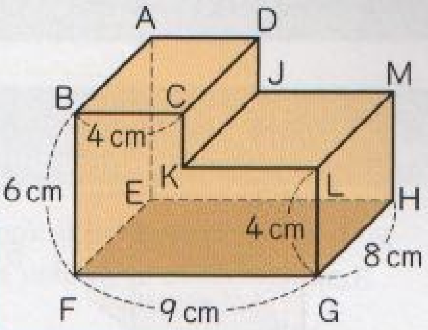
Additional Problems
→Page253 AA

3 The triangular prism on the right has the volume of 50 cm^3. Find the height of this prism.



A single formula can be used to find the volume of various prisms and cylinders.

4 Think about how to find the volume of the solid figure on the right.

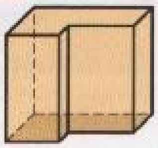


I wonder if we can use the formula for finding the volume of a prism here.

Bases of a prism are the top and bottom faces opposite of each other, so...

Let's think if we can use the math sentence of Area of the Base × Height here.

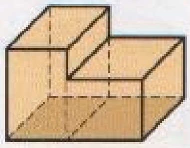
① In order to use the math sentence of Area of the Base × Height, which face should be considered as the base?



② Try to find the volume by using the math sentence of Area of the Base × Height.

③ Is the volume you found in ② the same as the volume you found by using the way you studied in 5th grade?

In 5th grade, we divided a solid figure into two □.



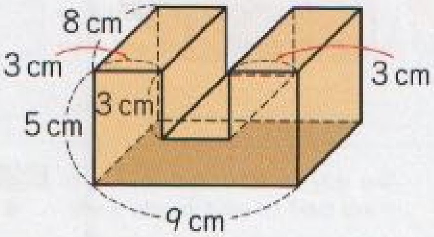
Summary

You can even find the volume of a solid figure like  by calculating Area of the Base × Height if you **consider the figure as a prism**.

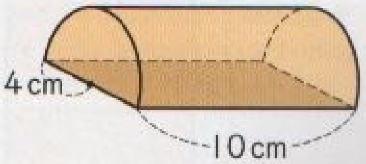
We thought about which face should be considered as the base.

4 Find the volume of each of the solid figures below.

①



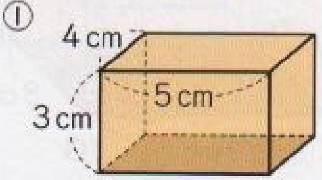
②

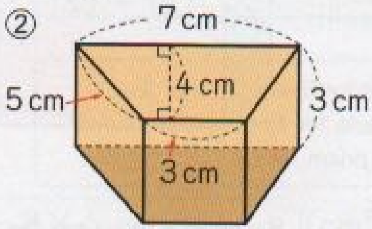


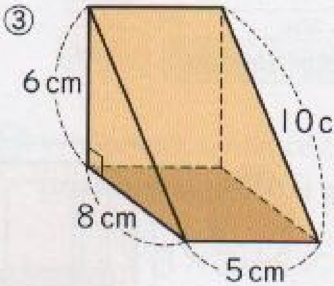
Wrapping Up Your Learning: Volume of Prisms and Cylinders

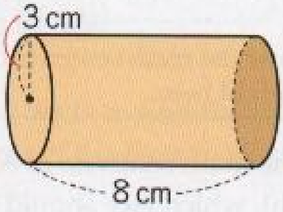
Check Your Understanding

1 Calculate the volume of the following solids.

①


②


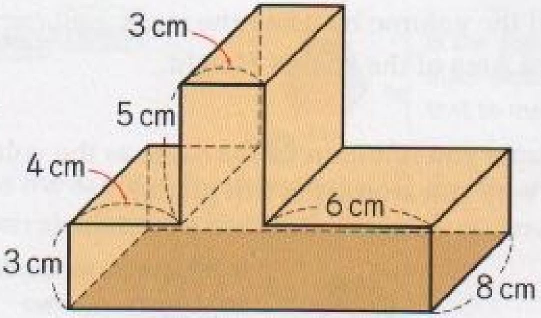
③


④


Can you calculate the volume of prisms and cylinders?

①~③
Page 121 **1**
Page 122 **2**
④ Page 123 **3**

2 Calculate the volume of the solid figure on the right by using the math sentence of Area of the Base × Height.



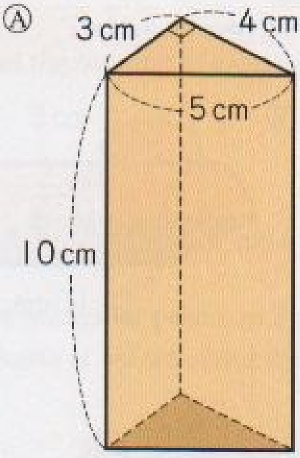
Can you consider the solid figure as a prism and calculate its volume by using the formula?

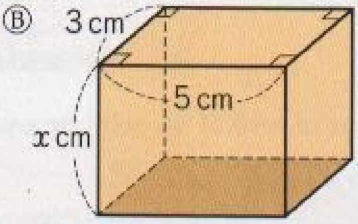
Page 125 **4**

3 Calculate the volume of triangular prism Ⓐ below. We are going to make rectangular prism Ⓑ, which will have the same volume as the triangular prism. How many cm will the height of rectangular prism Ⓑ be?

Can you find the height of a solid figure based on its volume?

Page 121 **1**
Page 122 **2**

Ⓐ


Ⓑ


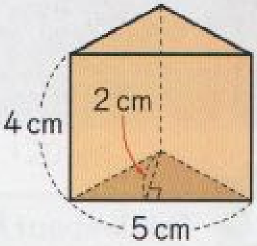
Wrapping Up Your Learning: Volume of Prisms and Cylinders

Grow Your "Eyes for Math" – Key Viewpoints and Ways of Thinking

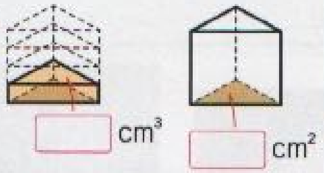
Look back on and take another look at the meaning of formulas, and think about how to find volume

① In **1** on page 121, we took another look at the formula for finding the volume of a cuboid, and found the volume of a rectangular prism.
Which part of the following formula represents the area of the base?
Volume of a Cuboid = Length × Width × Height

② We are going to find the volume of the prism on the right. Based on Kota's idea, explain how to find the volume.

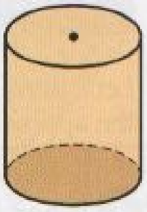


This is a triangular prism with a triangular base. Since the number expressing the volume of a prism with the height of 1 cm is equal to the number expressing the area of the base, you can use the math sentence of ☐ × Height...




Kota

③ We are going to find the volume of the cylinder on the right. Name all the segments you need to know the lengths of.



Look back on what you have learned in "Let's Think about How to Calculate the Volume of Prisms and Cylinders" and discuss.


Shiho

We have found that we can find the volume of prisms and cylinders with the same formula.


Haruto

Now, among all the solid figures we have studied so far, the sphere is the only one we don't know how to find the volume of. I wonder if there is also a formula for finding the volume of a sphere.

Junior High School

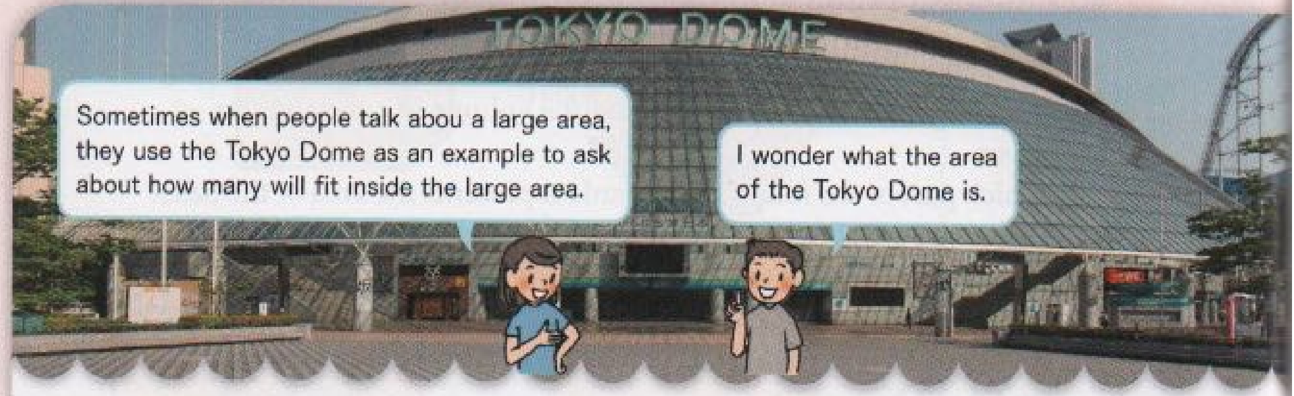
In junior high school, you will think about how to find the volume of solid figures as shown on the right.

Challenge Yourself
→ Page 263

9

Approximate Area and Volume

Let's Determine Approximate Area and Volume



1 Think about how to determine the area of the Tokyo Dome.

The Formulas for Calculating Area of Triangles and Quadrilaterals
Page 275 ㉓

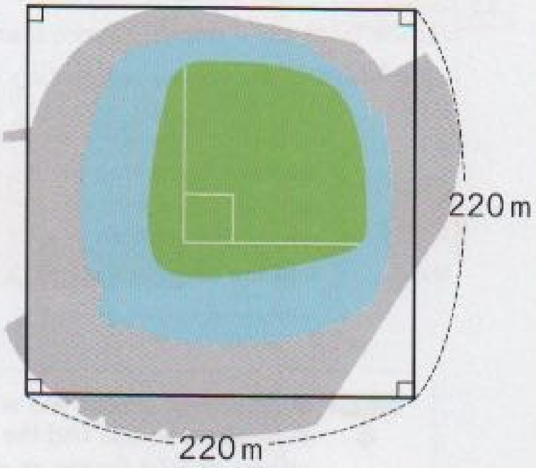
Let's think about how to calculate the approximate area.

① When you look at the Tokyo Dome from directly above, what shape does it look like?

② Determine the approximate area of the Tokyo Dome by assuming it is in the shape of a square.

Math Sentence

Answer about ☐ m^2

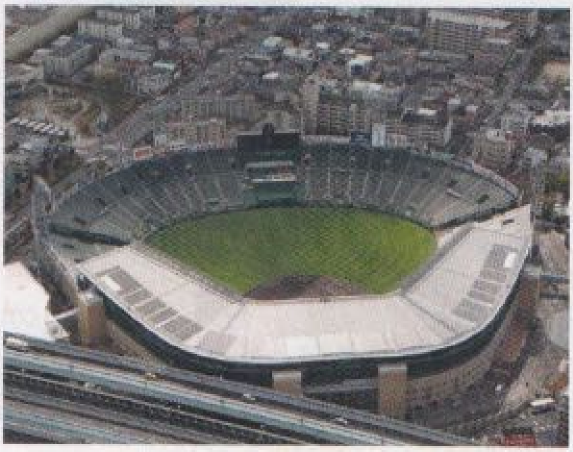


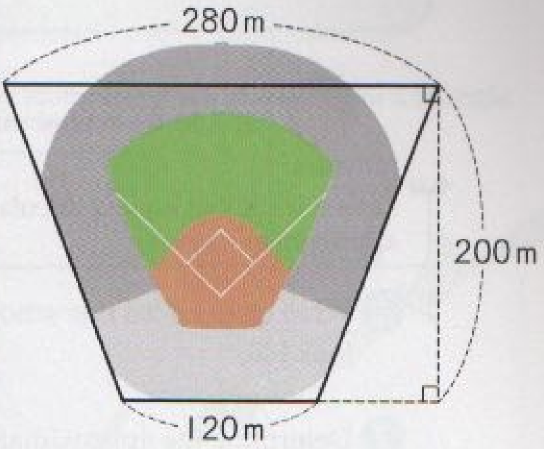
Riku: I guess we may consider this shape as a circle, too.

Shiho: I want to find the area of various shapes.

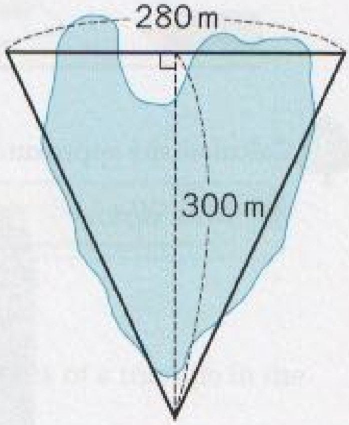
1 Calculate the approximate areas of the following.

Hanshin Koshien Stadium (Nishinomiya City, Hyogo Prefecture)





Lake Toyoni (Erimo-cho, Hokkaido)



Summary

You can also determine the approximate area of things around you **if you consider them as the geometric figures you know how to find the area of**.

2 Calculate the approximate areas of various places where you live.

Kota

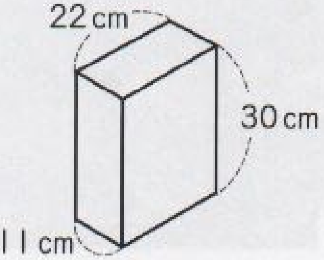
I want to think about the approximate capacity and volume of solid figures in the same way.

2 Determine the approximate capacity of the school backpack on the right.


Riku

Like the approximate area...

Let's think about how to calculate the approximate capacity.



① What shape does this school backpack look like?

② Determine the approximate capacity of the school backpack by assuming that it is a cuboid.

The Formulas for Calculating Volume of Cubes and Cuboids
Page 275 ㉕

Math Sentence

Answer about ☐ cm^3

3 Calculate the approximate capacity or volume of following.

① Milk carton

② Cake

Summary

You can also determine the approximate capacity or volume of things around you **if you consider them as the solid figures you know how to find the volume for**.

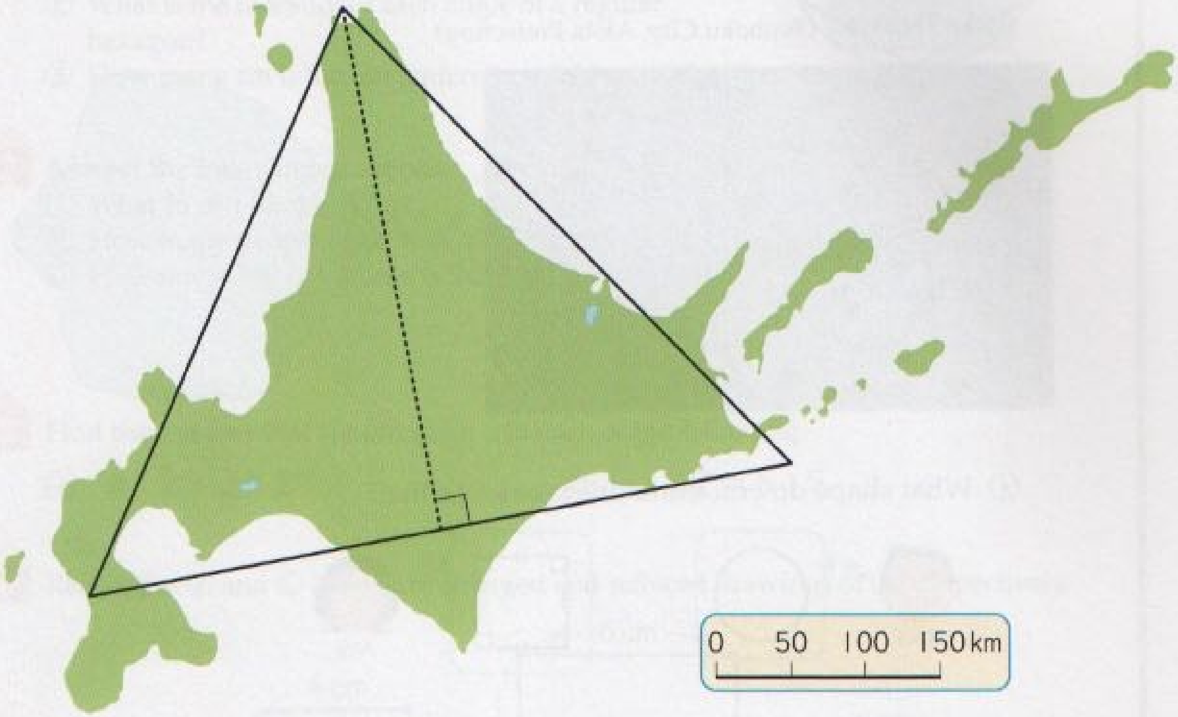
4 Determine the approximate capacity or volume of various things around you.


Shiho

Use What You Have Learned

Using a Map, Calculate the Approximate Area of Various Prefectures, Cities, Towns, and Villages.

(Example) Calculate the area of Hokkaido by assuming it is in the shape of a triangle.



① Measure the lengths you need to calculate the area of a triangle in the map above.

② Using the scale provided, determine the actual length of the measurements you made in ①.

In the map above, 1 cm represents 50 km, doesn't it?

③ Using the actual length obtained in ②, calculate the approximate area.

The actual area of Hokkaido is 83,424 km².

[Source: Website of the Statistics Bureau of Japan]

Shiho: I wonder what the approximate area is of the prefecture I live in.

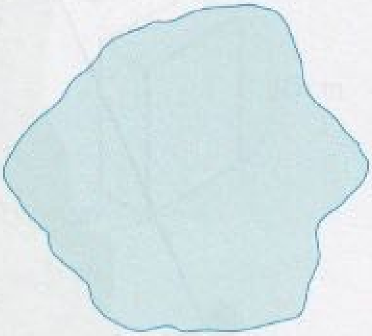
Wrapping Up Your Learning: Approximate Area and Volume

Grow Your "Eyes for Math" – Key Viewpoints and Ways of Thinking

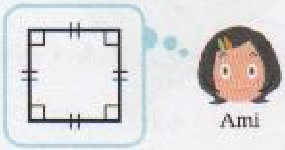
Focus on the Characteristics of a Shape and Think about How to Determine its Area Based on a Similar Geometric Figure

Think about how to determine the approximate area of the lake below, and explain how to determine it.

Lake Tazawa (Senboku City, Akita Prefecture)



① What shape do you think this lake looks like?



② Determine the approximate area of this lake by assuming that it is the circle as shown on the right.

The actual area of Lake Tazawa is 25.8 km².

[Source: Website of the Statistics Bureau of Japan]

Look back on what you have learned in "Let's Determine Approximate Area and Volume" and discuss.

Now I know that we can determine the approximate area or volume of any shape if you consider the shape as a figure you know how to find the area or volume for.

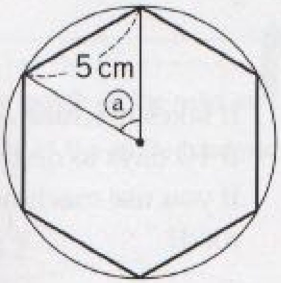
I thought we could consider the same shape differently, for example, as a circle or a square.

Do You Remember?

Answers→ Page 269

1 There is a regular hexagon in a circle as shown on the right.

Sum of the Angles of a Triangle and a Quadrilateral / Length Around Circles
Page 275 ㉑㉒



① What is the measure of angle ⓐ ?
② What is the measure of each angle of a regular hexagon?
③ How many cm is the circumference of this circle?

2 Answer the following questions.

Rates
Page 273 ⑭

① What % of 145 g is 87 g?
② How many people is 48 % of 250 people?
③ How many cm^3 of volume is 70 % of $63cm^3$?

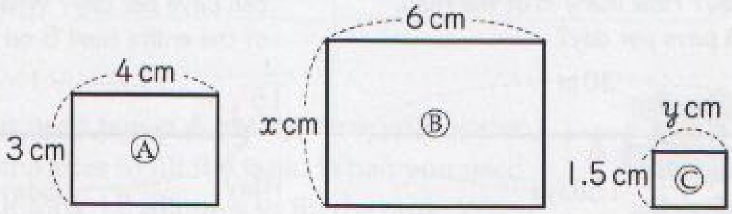
Warm-up

3 Find the number that should go in x in each of the following.

① $4 : 3 = x : 27$　　② $x : 42 = 5 : 7$

Warm-up

4 Rectangles Ⓑ and Ⓒ below are enlarged and reduced drawings of Ⓐ, respectively.



① Find the number for x and y.
② Rectangle Ⓑ is an enlarged drawing of rectangle Ⓐ. Rectangle Ⓒ is a reduced drawing of rectangle Ⓐ. By how much are Ⓑ and Ⓒ enlarged or reduced?

Playing with Numbers and Calculations

Let's Complete the Math Sentences

Fill the □ with +, −, ×, or ÷ to complete each of the following math sentences.

① $\frac{1}{3} \square \frac{1}{3} \square \frac{1}{3} = 1$　　② $\frac{1}{3} \square \frac{1}{3} \square \frac{1}{3} = 3$

③ $\frac{1}{4} \square \frac{1}{4} \square \frac{1}{4} \square \frac{1}{4} = 0$　　④ $\frac{1}{4} \square \frac{1}{4} \square \frac{1}{4} \square \frac{1}{4} = 1$

Some of the math sentences have more than one set of answers.

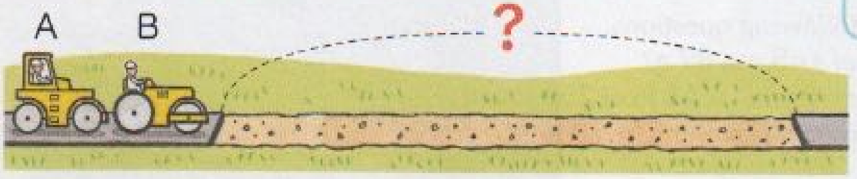
Extending Mathematical Thinking

Fix the Whole Quantity

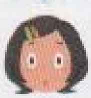
Think Using a Diagram

1

It takes machine A 15 days to pave a certain road, while it takes machine B 10 days to do that.
If you use machines A and B together, how many days will it take to pave the road?



We don't know how long the road is.



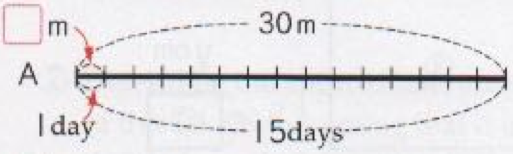
Ami

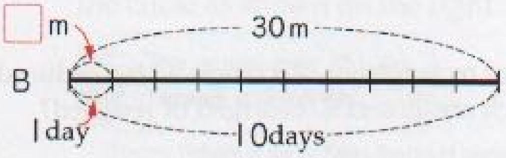
① Find the answer based on the two students' ideas.

Riku

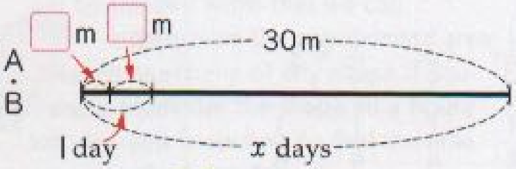
Since the least common multiple of 15 and 10 is 30, assume that the road is 30 m long.

(1) How many m of the road can A pave per day? How many m of the road can B pave per day?





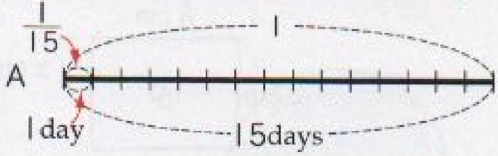
(2) If you use machines A and B together, how many m of the road can they pave per day?
How many days will it take to pave the entire road?

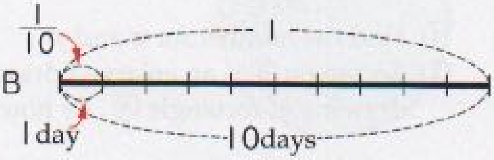


Misaki

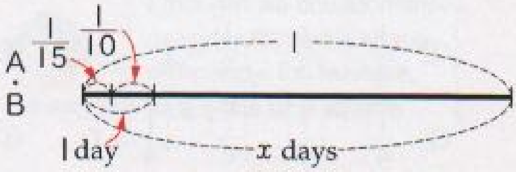
Consider the length of the road as 1.

(1) What is the fraction of the entire road A can pave per day? What is the fraction of the entire road B can pave per day?





(2) If you use machines A and B together, what is the fraction of the entire road A and B can pave per day? How many days will it take to pave the entire road?

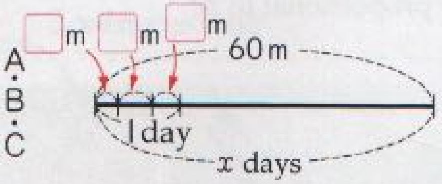


Assume that it takes x days.

② It takes machine C 12 days to pave this road.
If you use machines A, B, and C together, how many days will it take to pave the road?

Riku

The least common multiple of 15, 10, and 12 is 60. Assume that the road is 60 m long, and find the length of the road that can be paved per day.

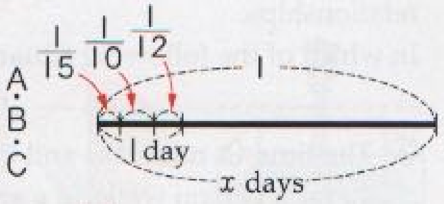


Since ☐ m of the road can be paved per day,

60 ÷ ☐ = ☐ Answer ☐ days

Misaki

Consider the length of the road as 1. C can pave $\frac{1}{12}$ of the entire road per day.



Since ☐ of the entire road can be paved per day,

1 ÷ ☐ = ☐ Answer ☐ days

③ Compare the two students' ideas and discuss what you notice.

Haruto

It's easy to think about if you assume a length, but it's more difficult to change that assumption once it's made.

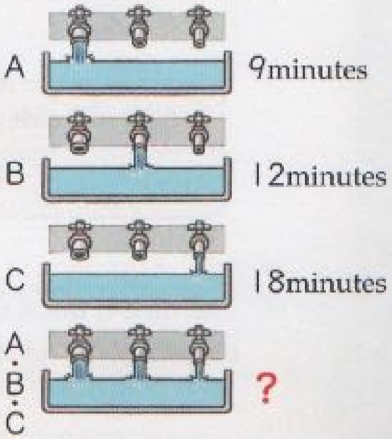
Even if machine C is added, you still consider the entire length as 1.

Shiho

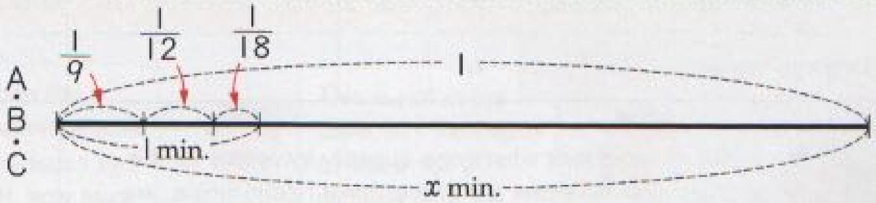
2

When you used faucet A to fill a tank with water, it took 9 minutes to fill the tank. When you used faucet B, it took 12 minutes to fill the tank. When you used faucet C, it took 18 minutes to fill the tank.

If you use faucets A, B, and C together, how many minutes will it take to fill the tank?



Consider the whole time as 1.



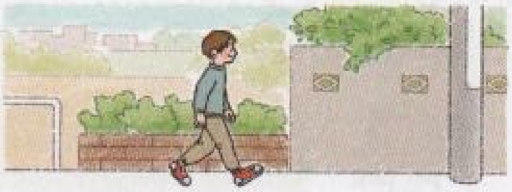
Let's review what you learned about proportional relationships

Proportional Relationships
Page 273 ⑬

We have studied how two quantities changed in relationship to each other. We also studied proportional relationships.
In which of the following situations is y proportional to x?

Add ○ to the tables and find it out.

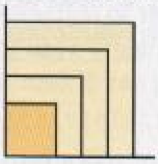
Ⓐ The time (x minutes) and distance (y m) a person walks at a speed of 60 m per minute



Speed per minute × Time = Distance
60 × x = y

Time x(min)	1	2	3	4	5	6
Distance y(m)						

Ⓑ The length of 1 side of a square and its area



Length of 1 side × Length of 1 side = Area
x × x = y

Length of 1 side x(cm)	1	2	3	4	5	6
Area y(cm^2)						

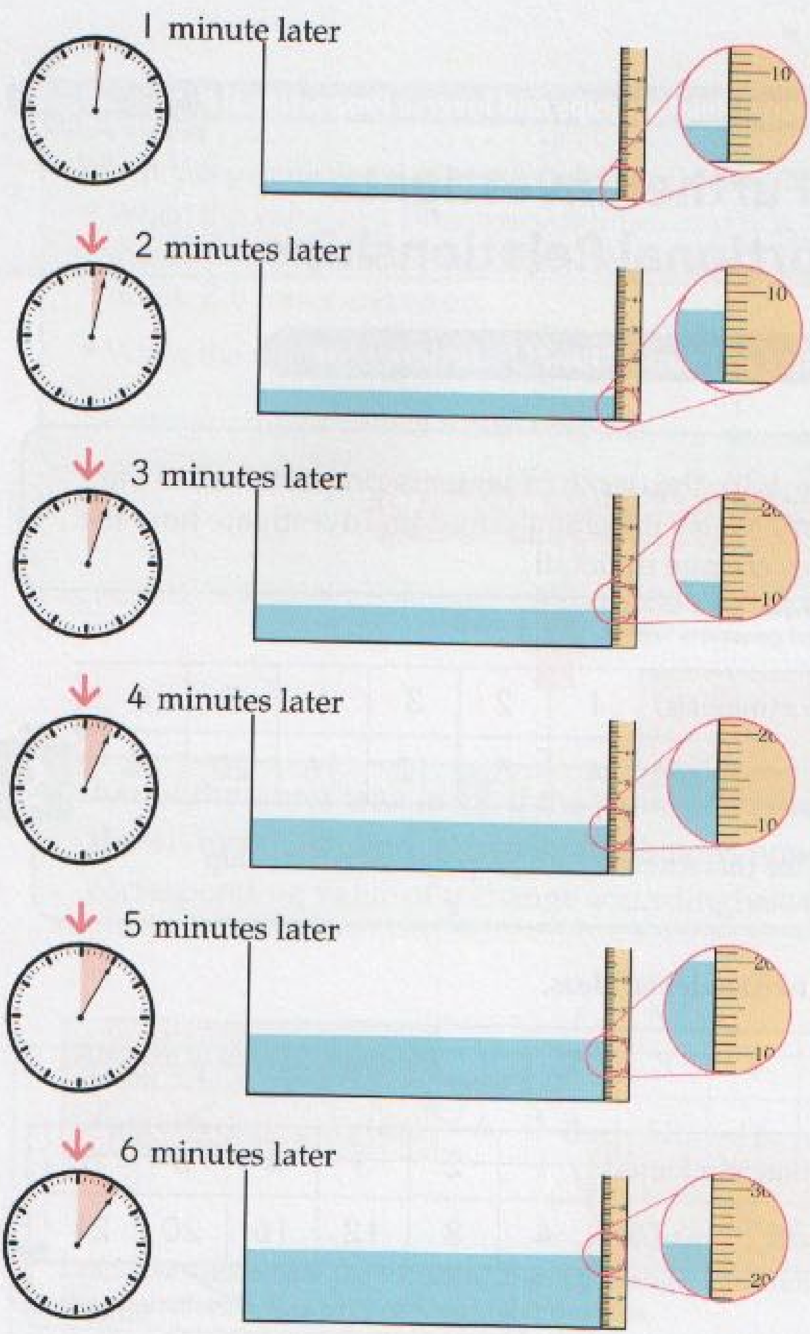
Ⓒ The time it takes to fill a water tank in the shape of a cuboid, and the depth of water

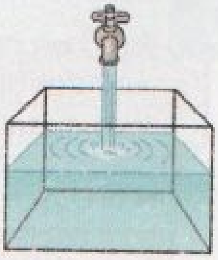
Look at the pictures on the next page to complete the table.

Amount of time x(minutes)	1	2	3	4	5	6
Depth of water y(cm)	4					

Misaki

Not all situations where one quantity increases as the other also increases are proportional relationships, are they?





The depth of water poured in 1 minute is always ☐ cm, isn't it?

In ©, depth of water is proportional to the amount of time water is being poured in.
What else can you tell about the proportional relationship?

Kota: When the amount of time is reduced by half, how much will the depth of water be?

Riku: This is not in the table, but the depth of water for 2 minutes and 30 seconds is also...

Shiho: It gets deeper by 4 cm per minute. I want to show this clearly on a graph.

10 Let's Further Investigate Proportional Relationships

1 Characteristics of Proportional Relationships

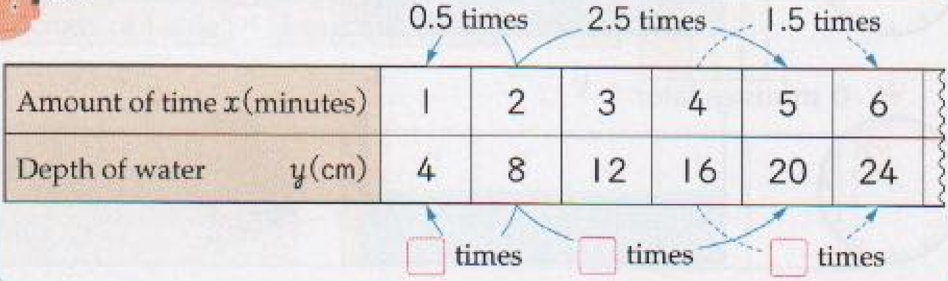
1 In © on page 136, the depth of water is proportional to the amount of time water is being poured in. Investigate how the two quantities change in detail.

Amount of time x (minutes)	1	2	3	4	5	6
Depth of water y (cm)	4	8	12	16	20	24

Let's investigate the characteristics of a proportional relationship between 2 quantities.

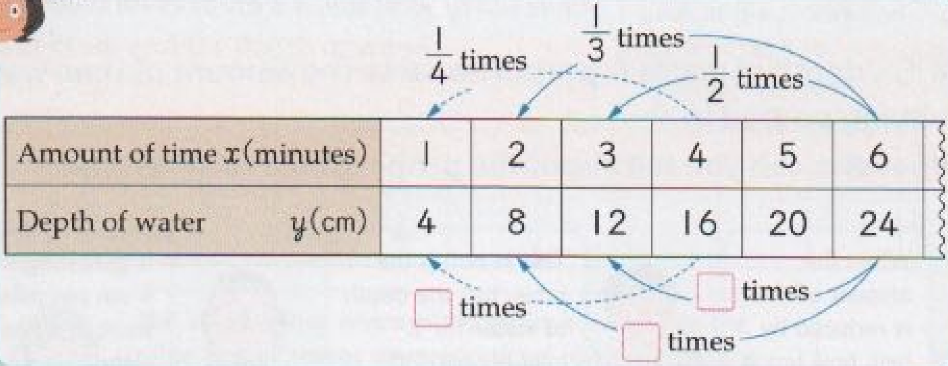
① Explain the two students' ideas.

Kota



Amount of time x (minutes)	1	2	3	4	5	6
Depth of water y (cm)	4	8	12	16	20	24

Ami



Amount of time x (minutes)	1	2	3	4	5	6
Depth of water y (cm)	4	8	12	16	20	24

Summary

When y is proportional to x, the following characteristics are true.

- When the value of x becomes 0.5 times as much, 2.5 times as much, and so forth, the corresponding value of y also becomes 0.5 times, 2.5 times and so on.
- When the value of x becomes $\frac{1}{2}$, $\frac{1}{3}$, ... times as much, the corresponding value of y also becomes $\frac{1}{2}$, $\frac{1}{3}$, ... times as much.

We know that this property is true when the numbers expressing times as much are whole numbers. The same is also true with decimal numbers and fractions.

Shiho: I wonder if this property is true whatever the number expressing times as much may be.

2

About the water tank in **1**, if the value of x changes as shown by ⓐ, ⓑ, and ⓒ in the table below, how does the corresponding value of y change accordingly?

ⓑ ⓐ ⓒ

Amount of time x (minutes)	1	2	3	4	5	6
Depth of water y (cm)	4	8	12	16	20	24

☐ times ☐ times ☐ times

Let's investigate how the values of x and y change, and summarize the characteristics of proportional relationships.

① When the value of x changes from 3 to 5 as shown with ⓐ, how many times as much does the value of x become?
How many times as much does the corresponding value of y become?

ⓐ Changes in the value of x $3 \rightarrow 5$ $5 \div 3 = \frac{5}{3}$ (times)

Changes in the value of y $12 \rightarrow 20$ $20 \div 12 = \frac{20}{12}$

$= \square$ (times)

When the value of x becomes $\frac{5}{3}$ times as much, the corresponding value of y also becomes $\frac{5}{3}$ times as much.

② Also, find how the values of x and y change in the cases of ⓑ and ⓒ.

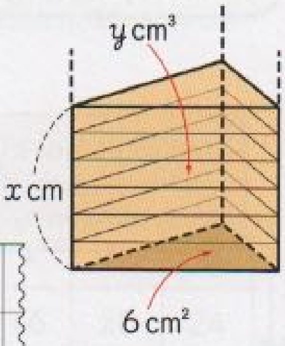
Summary

Suppose y is proportional to x. When the value of x becomes ■ times as much, the corresponding value of y also becomes ■ times as much.

■ represents the same number. Not only whole numbers but also decimal numbers and fractions can fit in the ■.

In other words, when y is proportional to x, these two quantities change at the same rate.

1 The table below shows the height (x cm) and volume (y cm^3) of a triangular prism with the area of the base of 6 cm^2.



$\frac{1}{3}$ times　　Ⓑ times

Height x(cm)	1	2	3	4	5	6
Volume y(cm^3)	6	12	18	24	30	36

Ⓐ times　　Ⓒ times

① Is the volume of the triangular prism proportional to its height? Also, explain the reason why.

② Find the numbers go into Ⓐ, Ⓑ, and Ⓒ.

③ How many times as much is the volume of the prism with the height of 7 cm as the volume of the prism with the height of 3 cm?
If the height is 7 cm, how many cm^3 is the volume?

Additional Problems
→Page253 AB

Misaki: We have looked at tables horizontally to examine changes.

2 Math Sentences of Proportional Relationships

1 Express the proportional relationship in © on page 136 as a math sentence.

Amount of time x (minutes)	1	2	3	4	5	6
Depth of water y (cm)	4	8	12	16	20	24

Let's find a property of the proportional relationship and express the property as a math sentence using x and y.


Riku

If you look at the table vertically...

① Explain the two students' ideas.


Haruto

The value of y is always equal to ☐ times as much as the value of x.

x	1	2	3	4	5	6
y	4	8	12	16	20	24

×☐

$x \times \square = y$


When x is 1, this is equal to the value of y...


Misaki

Dividing the value of y by the value of x always makes ☐.

x	1	2	3	4	5	6
y	4	8	12	16	20	24
$y \div x$						

$y \div x = \square$


This means the depth of water being poured in will be 4 cm for each minute.

Summary

When y is proportional to x, the **quotient of the value of y divided by the corresponding value of x always remains constant**.

$y \div x =$ constant

y can be expressed as the following math sentence with x:

$y =$ constant $\times x$

+1 +1 +1 +1 +1

x	1	2	3	4	5	6
y	4	8	12	16	20	24

+4 +4 +4 +4 +4

When y is proportional to x, you can express this relationship as a multiplication math sentence, like $y = 4 \times x$.

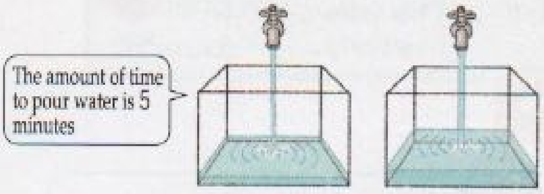

You can consider the constant number, 4, as the quantity by which y increases as the value of x increases by 1.


Kota

Depth of water being poured in each minute × Amount of time = Depth of water in the water tank.
Depth of water being poured in each minute is constant, and it's 4.

Next, we fix **Amount of time** to be 5 minutes.

2 We are going to pour water in a water tank in the shape of a cuboid for 5 minutes. We will vary the amount of water being poured in so that the depths of water for each minute will be different. What is the relationship between the depth of water for each minute and the depth of water in the water tank? Let the depth of water for each minute be x cm, and the depth of water in the water tank be y cm.



Depth of water for each minute × **Amount of time** = **Depth of water**

$$x \times 5 = y$$

Depth of water for each minute x(cm)	1	2	3	4	5	6
Depth of water y(cm)						

Let's investigate the relationship between 2 quantities.

① Is y proportional to x?

② Express y as a math sentence with x.

$$y = \square \times x$$

Look at the table horizontally...

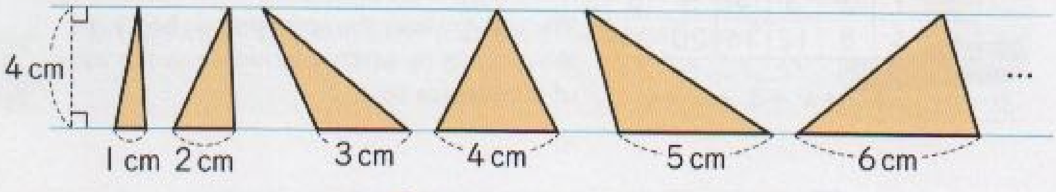
Look at the table vertically...

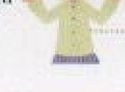
Depth of water for each minute × **Amount of time** = **Depth of water**.

There is a proportional relationship also when Amount of time is fixed.

1 We are increasing the length of the base of a triangle from 1 cm to 2 cm, 3 cm ... without changing the height of the triangle, 4 cm. Is the area of the triangle proportional to the length of the base? Explain the reason why, letting the length of the base be x cm and the area be y cm^2.
Also, express y as a math sentence with x.

The Formulas for Calculating Area of Triangles
Page 275 ㉓





What will happen if we graph proportional relationships?

3 Graphs of Proportional Relationships

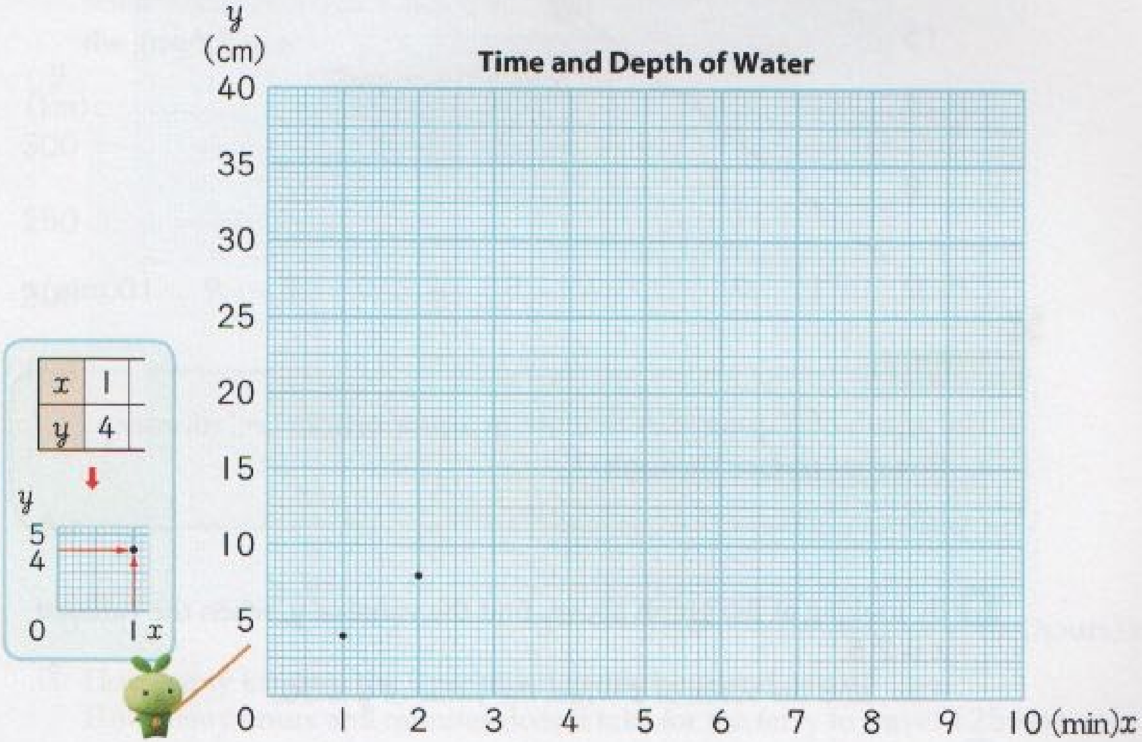
1

Graph the proportional relationship of © on page 136.

Amount of time x(min)	1	2	3	4	5	6
Depth of water y(cm)	4	8	12	16	20	24

Let's investigate the characteristics of graphs of proportional relationships.

① Plot the values of the time, x, and the depth of water, y, in the graph below.

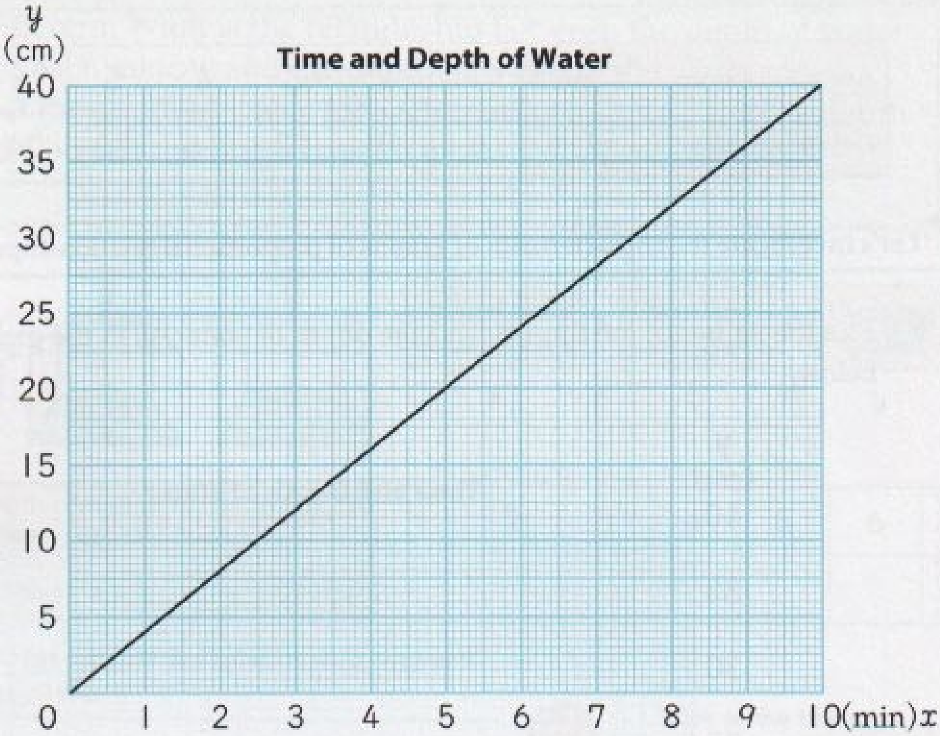


② Calculate the values of y when x is 0.5, 1.5, and 4.5 by using the math sentence, $y = 4 \times x$, and then graph them.

Put various numbers in x and find the corresponding values of y. Then, graph them.

③ How are the points that were graphed arranged?

The graph representing the relationship between the amount of time, x, and the depth of water, y is a straight line as shown below.



Summary

The graph of 2 quantities that are in a proportional relationship is a **line that passes through 0.**

④ By looking at the graph above, find the value of y, when the value of x is 2.5.
Also, find the value of x when the value of y is 30.

⑤ As the value of x increases by 1, by how much does the value of y increase?
Also, compare that number with the value of y when the value of x is 1.

⑥ Find the values of y when the values of x are 8 and 1.2 in the math sentence $y = 4 \times x$.
Then, check to see if those points are on the graph.


Haruto

In a graph, you can see at a glance that there are many pairs of the values of x and y, not just those in the table.

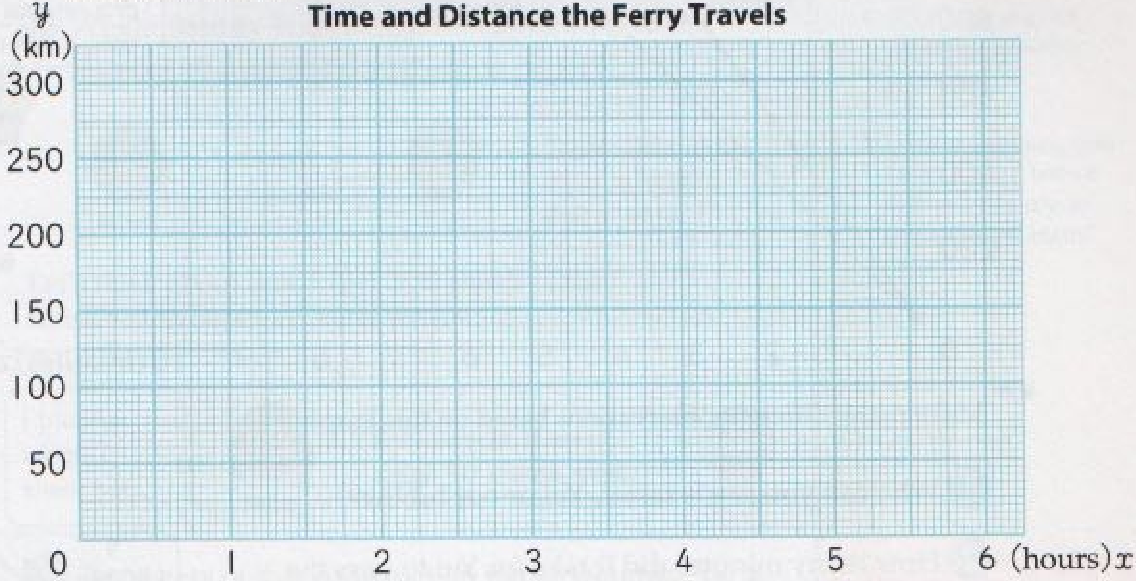
1 The ferry on the right travels 50 km per hour. The table below shows the relationship between the time and distance the ferry travels. The distance, y km, is proportional to the time, x hours. Answer the following questions about the relationship between these two quantities:

Time x(hours)	1	2	3	4	5	6
Distance y(km)	50	100	150	200	250	300

① Express y as a math sentence with x.

② Plot the proportional relationship between x and y in the graph below.

Put various numbers in x and find the corresponding values of y. Then, graph them.


Time and Distance the Ferry Travels

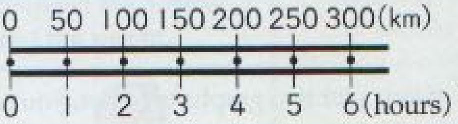
③ How many km does the ferry travel in one hour and a half? How many hours and minutes does it take for the ferry to travel 125 km?

Circle those points on the graph above.


Kota

Also from the math sentence and the table...

Remember that, like the graph above, number line diagrams show a proportional relationship.

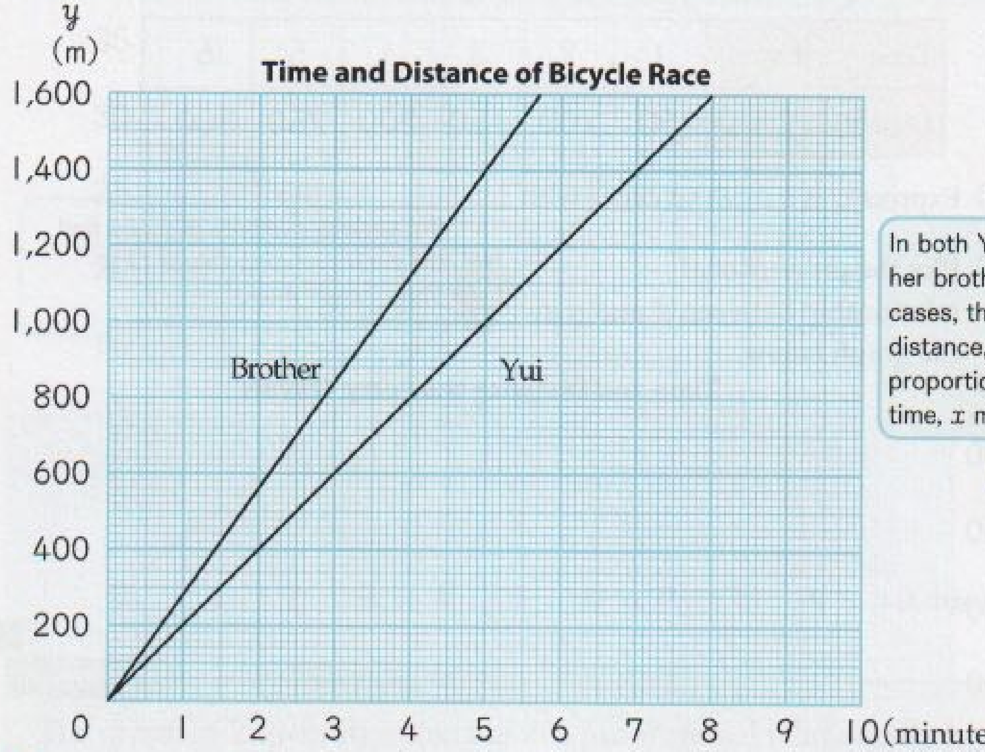



Misaki

I want to solve many questions by using a graph.

2

The graph below shows the time and distance Yui and her brother ran a bicycle race. They started together. What can you tell from this graph?

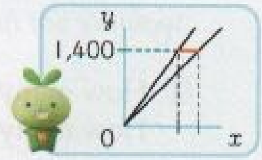


In both Yui' and her brother's cases, the distance, y m, is proportional to the time, x minutes.

Make many different observations based on this graph.

① Who can you say is faster, Yui or her brother?

② How many minutes did it take for Yui to pass the 1,400 m point after her brother passed it?



③ How far apart were Yui and her brother 5 minutes after the start of the race?

④ If they had kept riding at the same speed, how many m apart would they have been 10 minutes after the start of the race?

If you plot two graphs together, you can easily examine the relationships between two quantities in different cases.

To solve a problem, I want to use math sentences and tables, too.

4 Applications of Proportional Relationship

1 Let's Think about how to get 300 sheets of construction paper without counting them all.

① What quantity will change in relationship to the number of sheets of construction paper?

Misaki

Weight changes.

Riku

There seem to be other quantities that change.

Let's think about how to get 300 sheets of the construction paper by using weight.

I tried to weigh 1 sheet but...

Shiho

For example, if we weigh 10 sheets, then...

Haruto

② Ten sheets of this construction paper weighed 92 g. Based on this information, think about how to get 300 sheets of the construction paper. Express your idea using a table and a math sentence.

Number of Sheets and Weight of Construction Paper

Number of sheets x	10	300
Weight y (g)	92	□

The relationship between the number and the weight...

Kota

Grasp the problem.

- What problem are we going to work on today?
- What idea may be useful to solve the problem?
- Is there anything you have learned before that you can use to solve this problem?

Write down your ideas.

- Is your idea clear to others?
- If you figured it out one way, try to think of another way.

Haruto and some other students are explaining their classmates' ideas.

Misaki

Thinking that the weight of sheets is proportional to the number of sheets, find the weight per sheet.

300 times

$\frac{1}{10}$ times

Number of sheets x	1	10	300
Weight y(g)	□	92	□

$\frac{1}{10}$ times

300 times

Kota

I thought the weight of sheets was proportional to the number of sheets. So, I used a characteristic of proportional relationships.

Number of sheets x	10	300
Weight y(g)	92	□

$300 \div 10 = 30$

$92 \times 30 = \square$

Learn with your classmates.

- Can you understand your classmates' ideas based on their tables and math sentences?
- What is common and what is different about your own idea and your classmates' ideas?
- What are the good points in your classmates' ideas?

③ Look at the table Misaki made. Express her idea as a math sentence and explain it.

④ Look at the math sentences Kota wrote. Explain his idea using a table.

□ times

Number of sheets x	10	300
Weight y(g)	92	□

□ times

⑤ Look at the math sentences Shiho wrote on the next page. Explain her idea using a table.

⑥ What do the ideas of the three students have in common? What is different?

Shiho

Assuming that the weight of sheets is proportional to the number of sheets, find the constant.

Number of sheets x	10	×□	300	×□
Weight y(g)	92		□	

$10 \times \square = 92$

$\square = 92 \div 10$

$= 9.2$ Constant

$300 \times 9.2 = 2{,}760$


Riku

7 Look back and summarize today's lesson.

Summary

Assuming that the weight of construction paper is proportional to the number of sheets of construction paper, we can estimate the number of sheets without counting.

Tables and math sentences help you explain.

Thickness also changes in relationship to the number of sheets.

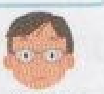

Riku

8 Ten sheets of this construction paper measured 2 mm thick. Based on this information, explain how to get 300 sheets of the construction paper.

Number of Sheets and Thickness of Construction Paper

Number of sheets x	10	300
Thickness y(mm)	2	□

Thickness of sheets of the construction paper is [...] to the number of sheets.


Haruto

Look back and summarize today's lesson.

- What did you learn from today's investigation?
- Which way of thinking was useful?

Dig deeper into your study.

- Whose ideas can you use?

Deepen **Your** Learning

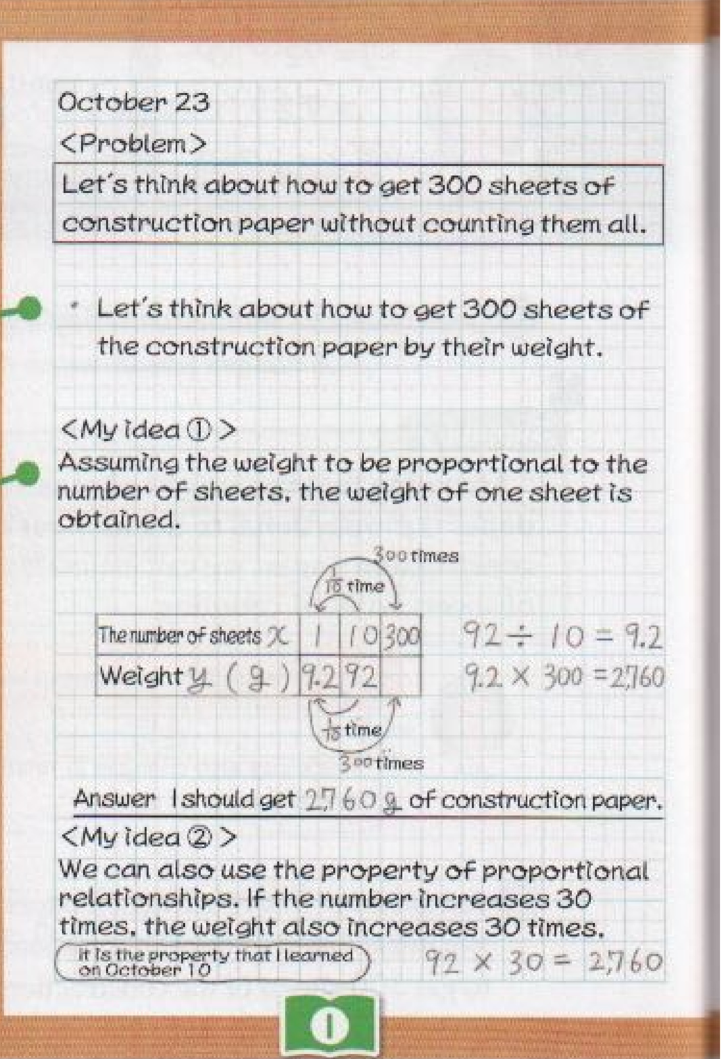
Use Your Math Notebook for Better Learning

Look back at the ideas you used to solve problems.

Misaki

She focused on the weight that changed in proportion to the number of sheets.

She thought the weight was proportional to the number of sheets.



October 23

<Problem>

Let's think about how to get 300 sheets of construction paper without counting them all.

- Let's think about how to get 300 sheets of the construction paper by their weight.

<My idea ①>

Assuming the weight to be proportional to the number of sheets, the weight of one sheet is obtained.

300 times

$\frac{1}{10}$ time

The number of sheets x	1	10	300
Weight y (g)	9.2	92	

$\frac{1}{10}$ time

300 times

$92 \div 10 = 9.2$

$9.2 \times 300 = 2{,}760$

Answer I should get 2,760 g of construction paper.

<My idea ②>

We can also use the property of proportional relationships. If the number increases 30 times, the weight also increases 30 times.

It is the property that I learned on October 10

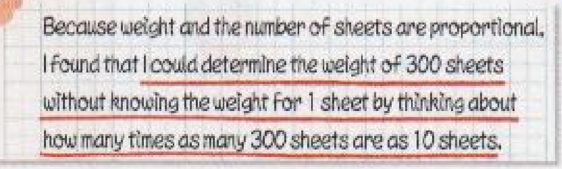
$92 \times 30 = 2{,}760$

When you work on problems, not only write math sentences and answers but also consider using:

- **Diagrams**
- **Tables**
- **Graphs**

Classmates' reflection

Kota



Because weight and the number of sheets are proportional, I found that I could determine the weight of 300 sheets without knowing the weight for 1 sheet by thinking about how many times as many 300 sheets are as 10 sheets.

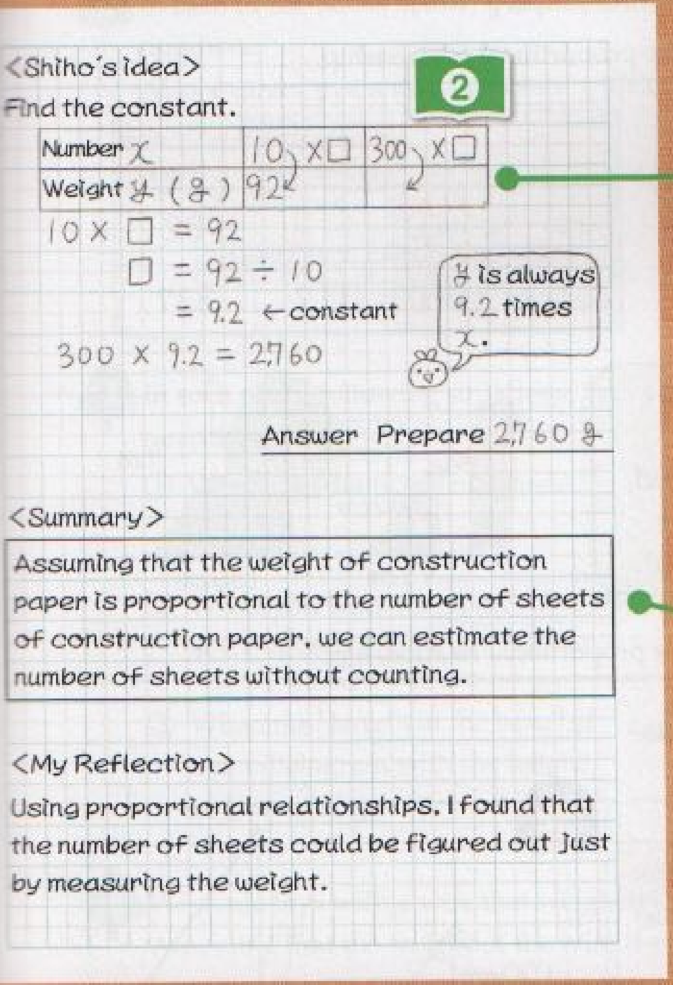
He wrote how his learning from before was useful.

Note Taking Tip ①

Today's lesson was based on what she had learned. So, she added the date of the lesson so that she can go back to the page of her notebook.

Note Taking Tip ②

She drew arrows in the tables to clearly show the changes in quantities and the relationships between quantities.



<Shiho's idea>

Find the constant.

②

Number x	10 ×□	300 ×□
Weight y (g)	92	

$10 \times \square = 92$

$\square = 92 \div 10$

$= 9.2$ ← constant

$300 \times 9.2 = 2{,}760$

y is always 9.2 times x.

Answer Prepare 2,760 g

<Summary>

Assuming that the weight of construction paper is proportional to the number of sheets of construction paper, we can estimate the number of sheets without counting.

<My Reflection>

Using proportional relationships, I found that the number of sheets could be figured out just by measuring the weight.

She used tables to examine quantities. She looked at the tables horizontally and vertically.

She summarized the usefulness of a solution that used the proportional relationship.

Shiho

If I assume a proportional relationship, I can figure out, for example, the number of same paper clips using their weight. I want to check this out.

She wrote about further ideas she would like to investigate.

2

From Shin-Fuji Station along the Tokaido Shinkansen, you can see Mount Fuji very well.
If you use the bullet train, Nozomi, about how many minutes after you leave Shin-Yokohama Station will you pass Shin-Fuji Station?

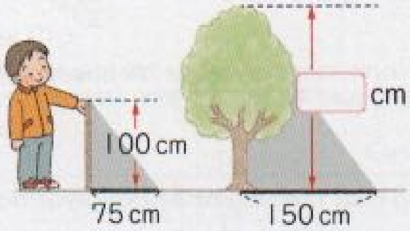
Let's solve the problem by using the proportional relationship.

Based on the time and distance between Shin-Yokohama Station and Nagoya Station, assume that these two quantities are proportional to each other.

	Shin-Yokohama to Shin-Fuji	Shin-Yokohama to Nagoya
Time x(min)	☐	82
Distance y(km)	117	337

3

The length of the shadow of an object is proportional to the height of the object. Based on this information, find the height of the tree on the right.



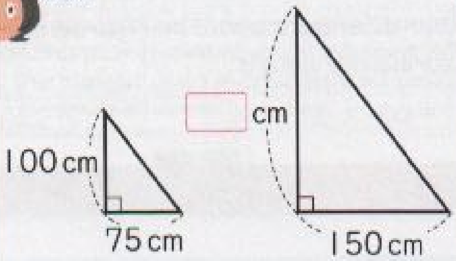
Let's solve the problem by using the proportional relationship.

① Explain the two students' ideas.

Riku

	Stick	Tree
Height x(cm)	100	○ times ☐
Length of shadow y(cm)	75	150 ○ times

Ami



○ times

100 : 75 = ☐ : 150

○ times

Proportional relationships are useful in solving many math problems about things around you.

Haruto

If two quantities are proportional to each other, you can also use the concept of ratio.

5 Practice

1 Is y proportional to x in each of the tables below?

①

x (L)	2	4	6	8	10
y (kg)	4	6	8	10	12

②

x (m)	0.4	0.6	0.8	1	1.2
y (m)	1	1.5	2	2.5	3

2 In each of the tables below, y is proportional to x. Fill in the blanks with numbers.

Also, express y as a math sentence with x.

①

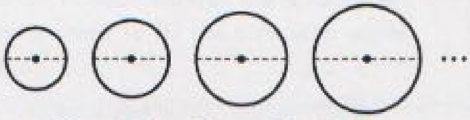
x (m)	2	3	4	5	6
y(yen)		45			

②

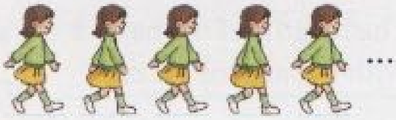
x (cm)	2			8	10
y (cm²)	5	10	15		

3 In each of the following situations, find a quantity that is proportional to the quantity in the ☐.

① When the diameter of a circle changes

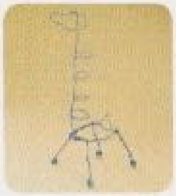


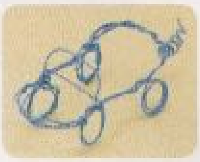
② When you walk for some minutes at the speed of 70 m per minute



③ When the length of the base of a parallelogram with the height of 4 cm changes

4 We made toys with a wire that weighed 20 g per 3 m. Assuming that the weight of the wire is proportional to the length of the wire, answer the following questions:





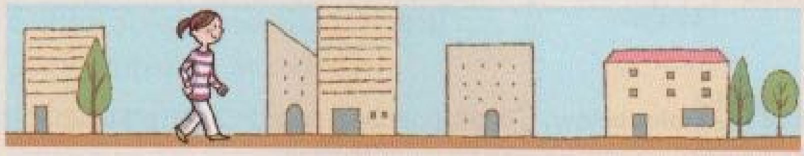
① The upper-right toy weighed 54 g. How many m of wire was used?

② The lower-right toy was made of 7.2 m of the wire. How many g does this toy weigh?

6 Inverse Proportional Relationships

In situations Ⓐ to Ⓒ, the value of y changes as the value of x does. How does y change?

Ⓐ The time, y hours, taken to walk 6 km at the speed of x km per hour



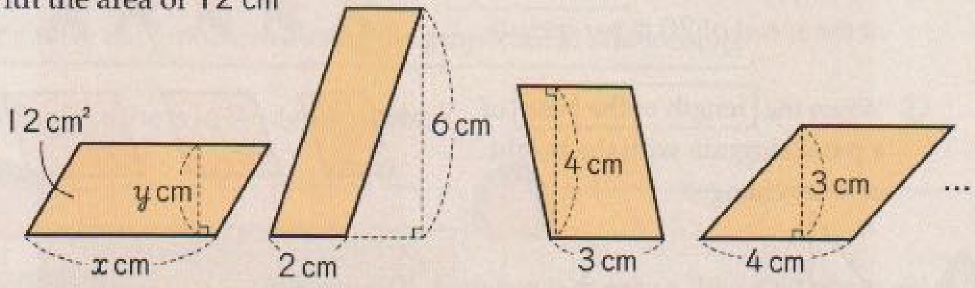
$\boxed{\text{Speed per hour}} \times \boxed{\text{Time}} = \boxed{\text{Distance}}$

$x \times y = 6$

The distance, 6 km, is fixed.

Speed x(km per hour)	1	2	3	4	5	6
Time y(hours)	6	3	2	1.5	1.2	1

Ⓑ The lengths of the base, x cm, and the height, y cm, of a parallelogram with the area of 12 cm²



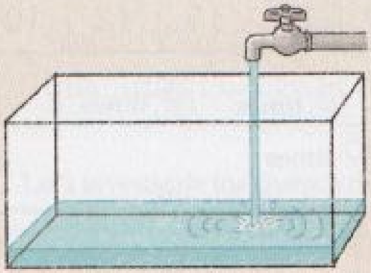
$\boxed{\text{Base}} \times \boxed{\text{Height}} = \boxed{\text{Area}}$

$x \times y = 12$

The area, 12 cm², is fixed.

Base x(cm)	1	2	3	4	5	6
Height y(cm)	12	6	4	3	2.4	2

© The depth, x cm, of water being poured in each minute and the amount of time, y minutes, it take to fill the tank that is 60 cm deep.



The height of the tank, 60 cm, is fixed.

Depth of water being poured in each minute x(cm)	1	2	3	4	5	6
Amount of time y(minutes)	60	30	20	15	12	10

Riku

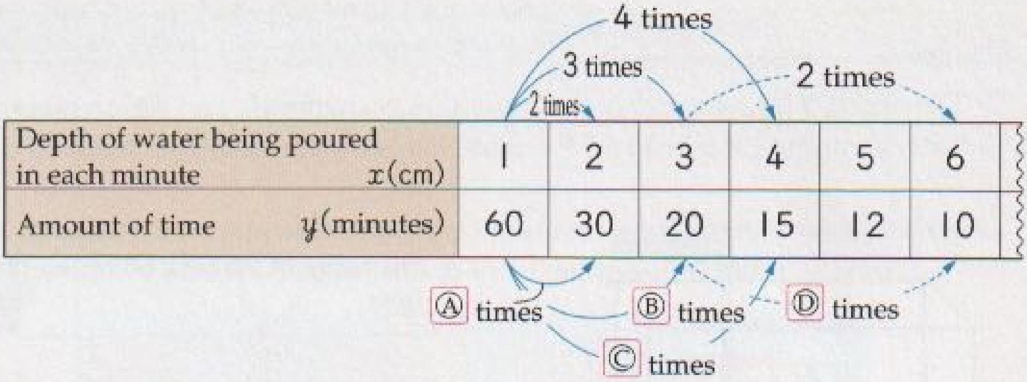
If you increase the depth of water being poured in each minute, the amount of time it takes to fill the tank with water will...

1

If you change the depth of water being poured in each minute in © from 1 cm to 2 cm, 3 cm ... , how will the amount of time it takes to fill the tank change?

Let's investigate the relationship between the two quantities.

① When the depth of wather being poured in each minute becomes 2, 3, ... times as much, how will the amount of time it takes to fill the tank change? Add ⌒ to the table above and find it out.



Depth of water being poured in each minute x(cm)	1	2	3	4	5	6
Amount of time y(minutes)	60	30	20	15	12	10

② What numbers should go into Ⓐ , Ⓑ , Ⓒ , and Ⓓ above?

When the depth of water being poured in each minute, becomes 2, 3, ... times as much, the amount of time required to fill the tank, y minutes, will become $\frac{1}{2}$, $\frac{1}{3}$, ... times as much.

When two quantities x and y change so that as the value of x becomes 2, 3, ... times as much, the corresponding value of y becomes $\frac{1}{2}$, $\frac{1}{3}$, ... times as much, we say that, "y is **inversely proportional** to x."

When the height of the tank is fixed, **the amount of time it takes to fill the tank, y minutes, is inversely proportional to the depth of water being poured in each minute.**

Ami

Quick Review

〈What is a proportional relationship?〉

As the value of x becomes 2, 3, ... times as much, the corresponding value of y also becomes 2, 3, ... times as much.

③ In Ⓐ and Ⓑ on page 154, is y inversely proportional to x? Look at the table and find it out.

1 The table below shows the length and width of a rectangle around which is 16 cm long. Is the width, y cm, inversely proportional to the length, x cm?

Length x(cm)	1	2	3	4	5	6
Width y(cm)	7	6	5	4	3	2

When the value of x changes from 2 to 6, the value of x becomes 3 times as much, while the value of y becomes $\frac{1}{3}$ times as much, but...

Kota

I want to investigate the inversely proportional relationship in detail while looking back on what we learned about the directly proportional relationship.

Characteristics of inverse proportional relationships

2 In © on page 155, the amount of time required to fill the tank with water is inversely proportional to the depth of water being poured in each minute. Investigate how the two quantities change in detail.

Let's investigate the characteristics of two quantities that are inversely proportional.

In a proportional relationship, when the value of x becomes $\frac{1}{2}$, $\frac{1}{3}$, ... times as much...


Haruto

Depth of water being poured in each minute x(cm)	1	2	3	4	5	6
Amount of time y(minutes)	60	30	20	15	12	10

Summary

When y is inversely proportional to x, as the value of x becomes $\frac{1}{2}$, $\frac{1}{3}$, ... times as much, the corresponding value of y becomes 2, 3, ... times as much.

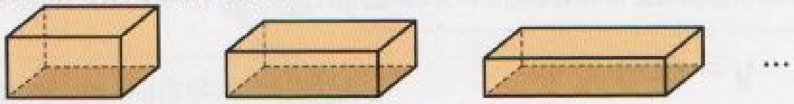
In the inversely proportional relationship, when the value of x is made ■ times as much, the value of y is made $\frac{1}{\blacksquare}$ times as much. They are recipriocal of each other.

Quick Review

〈Characteristics of proportional relationships〉

When the value of x becomes $\frac{1}{2}$, $\frac{1}{3}$, ... times as much, the corresponding value of y also becomes $\frac{1}{2}$, $\frac{1}{3}$, ... times as much.

2 About a rectangular prism with the volume of 100 cm³, its height, y cm, is inversely proportional to the area of its base, x cm².
Fill in the blanks with numbers.



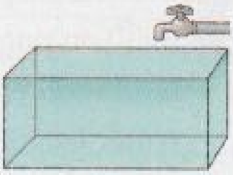
Area of the Base x(cm²)	5	10			40	
Height y(cm)		10	5	4		2


Shiho

I want to express the inversely proportional relationship as a math sentence.

Math sentences for inversely proportional relationships

3 Express the inversely proportional relationship in © on page 155 as a math sentence.



Let's find a property of the inversely proportional relationship and express the property as a math sentence with x and y.

① Look at the table vertically and think about the relationship between the value of x and the corresponding value of y.

Depth of water being poured in each minute x(cm)	1	2	3	4	5	6
Amount of time y(minutes)	60	30	20	15	12	10

② Fill in the □ with the appropriate numbers.

Depth of water being poured in each minute × Amount of time = Depth of the tank ···Ⓐ

$x \times y = \square$

The product of x and y is 60. This shows that the depth of the tank, 60 cm, does not change.

Riku

If the depth of the tank in math sentence Ⓐ is fixed, the other two quantities are in the inversely proportional relationship.

③ Based on the math sentence in ②, express y as a math sentence with x.

$y = 60 \div x$

Now that y is expressed in this way, you can calculate the value of y when the value of x is given.

④ Find the values of y when the values of x are 2.5, 8, and 10.

Summary

If y is inversely proportional to x, **the product of the value of x and the corresponding value of y is constant.**

$x \times y = \text{constant}$

You can express y as the following math sentence with x:

$y =$ **constant** $\div x$

Quick Review
(Math sentence for proportional relationship)
$y = \text{constant} \times x$

⑤ Express y in Ⓐ and Ⓑ on page 154 as math sentences with x.

3 The table below shows the combinations of the speed per hour of a car and the time it takes for the car to travel from City D to City E at the given speed.

Speed x(km)	10	20	30	40	50	60
Time y(hr)	12	6	4	3	2.4	2

① Is the time taken, y hours, inversely proportional to the speed, x km per hour?
Also, explain the reason why.

② What does the product of the speed, x kilometers per hour, and the corresponding time, y hours, represent? What is the value?

Since it is a product of speed and time ...

③ Express y as a math sentence with x.

④ Find the value of y when the value of x is 15.

⑤ Find the value of x when the value of y is 1.5.

I want to express the inversely proportional relationship in a graph.

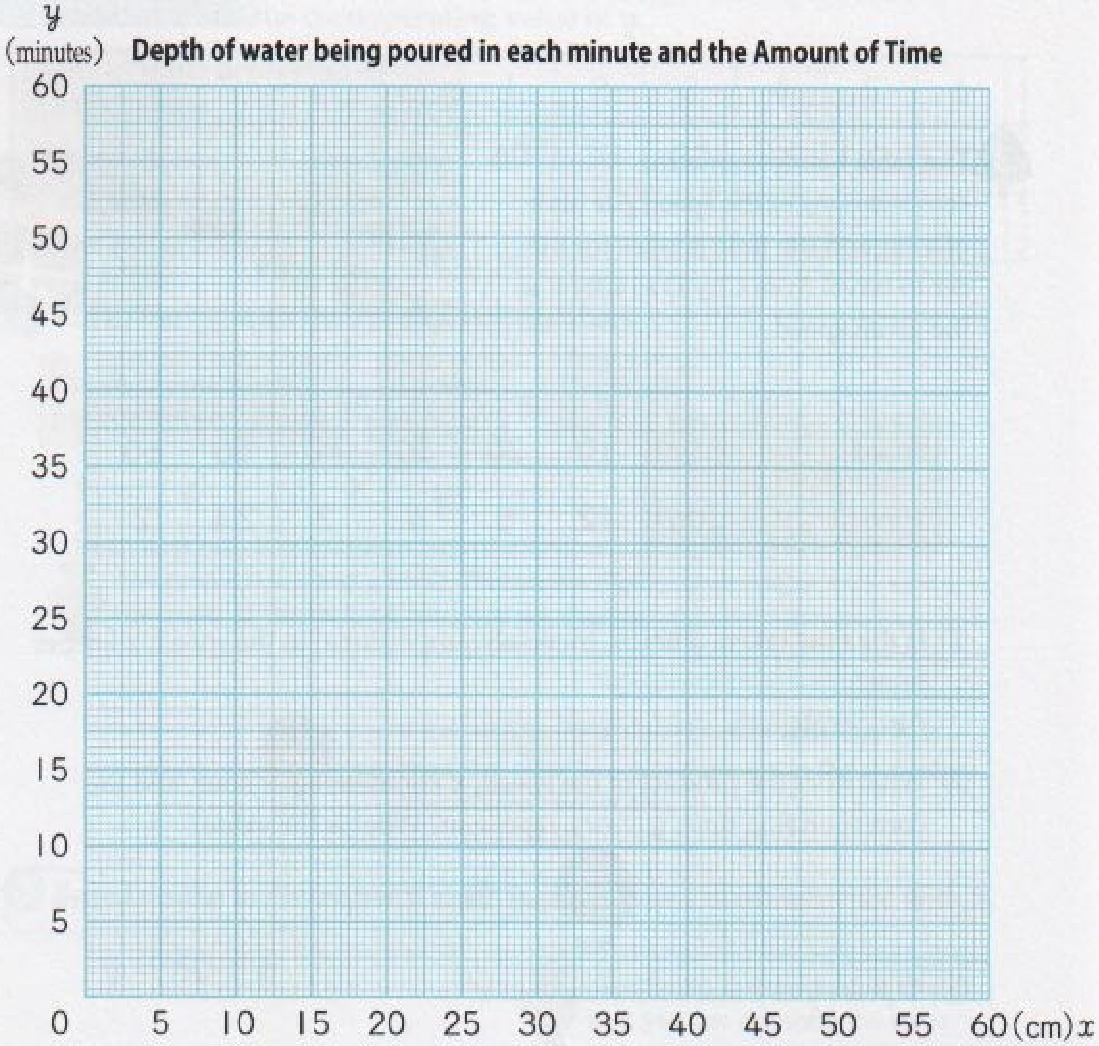
Graphs of inversely proportional relationships

4 Graph the inversely proportional relationship of © on page 155.

Depth of water being poured in each minute x(cm)	1	2	3	4	5	6	10	20	30	40	50	60
Amount of time y(minutes)	60	30	20	15	12	10	6	3	2	1.5	1.2	1

Let's graph the inversely proportional relationship between the two quantities and investigate the characteristics of the graph.

① Plot the values of x and the corresponding values of y in the graph below.

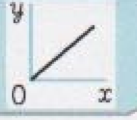


② Compared with graphs of proportional relationships, what characteristics do graphs of the inversely proportional relationship have?

Unlike graphs of proportional relationships,
graphs of the inversely proportional relationships...


Misaki

Quick Review
〈Graphs of proportional relationships〉

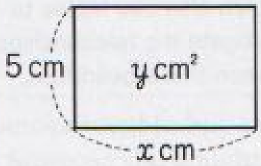


Wrapping Up Your Learning: Proportional Relationships and Inverse Proportional Relationships

Check Your Understanding

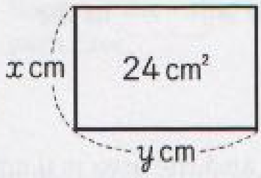
1 Answer the following questions about the relationship between the two quantities, x and y, in ⓐ and ⓑ below.

ⓐ The length, x cm, and the area, y cm², of a rectangle that is 5 cm in width.



Length x(cm)	1	2	3	4	5	6
Area y(cm²)	5	10	15	20	25	30

ⓑ The width, x cm, and the length, y cm, of a rectangle with the area of 24 cm²



Width x(cm)	1	2	3	4	5	6
Length y(cm)	24	12	8	6	4.8	4

① Is y proportional or inversely proportional to x? Also, explain the reason why.

Can you tell if a quantity is proportional or inversely proportional to another?

Page 142 **2**
Page 155 **1**

② Express y as a math sentence with x.

Do you understand a math sentence representing proportional and inversely proportional relationships.

Page 141 **1**
Page 158 **3**

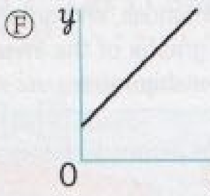
③ Which graph among Ⓕ to Ⓗ shows the relationship between the two quantities, x and y, in ⓐ?

Can you identify graphs of proportional relationships?

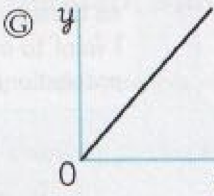
Page 143 **1**

Also, give the characteristics of graphs of proportional relationships.

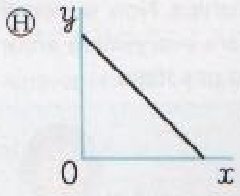
Ⓕ



Ⓖ



Ⓗ



Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

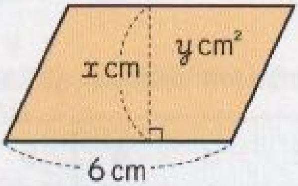
Focus on the Relationships between Quantities and Connect the Proportional Relationship to the Inversely Proportional Relationship

Investigate the relationships between the area, the base, and the height of a parallelogram by using math sentences.

Area of parallelogram = Base × Height

You can also use tables to investigate the relationships between these quantities.

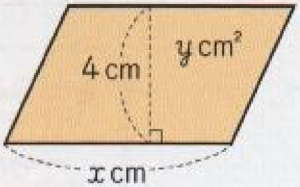
① When the base is fixed ... at 6 cm,



if the height is x cm and the area is y cm², then, $y = \boxed{} \times x$.

Therefore, the area is $\boxed{}$ to the height.

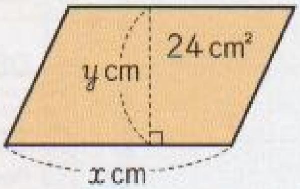
② When the height is fixed ... at 4 cm,



if the base is x cm and the area is y cm², then, $y = x \times \boxed{}$.

Therefore, the area is $\boxed{}$ to the length of the base.

③ When the area is fixed ... at 24 cm²,



if the base is x cm and the height is y cm, then, $\boxed{} = x \times y$. Therefore, the height is $\boxed{}$ to the length of the base.

Depending on the given quantity, the relationship between the other two quantities is proportional or inversely proportional.

Look back on what you have learned in "Let's Further Investigate Proportional Relationships" and discuss.

We have studied proportional and inversely proportional relationships. Now we know that these relationships are everywhere around us, and we know how to use them.

I want to investigate various relationships between quantities by expressing them as tables, math sentences, and graphs. I want to examine graphs of the inversely proportional relationship more.

Challenge Yourself → Page264

Junior High School In junior high school, you will study various relationships between x and y based on what you have learned about the proportional and inversely proportional relationships.

Do You Remember?

Answers → Page 269

1 ① $\frac{3}{8}+\frac{7}{12}$ ② $\frac{8}{9}-\frac{5}{6}$ ③ $\frac{3}{10}\times\frac{5}{12}$ ④ $\frac{2}{9}\div\frac{4}{15}$

⑤ $\left(\frac{3}{4}-\frac{2}{5}\right)\times 20$ ⑥ $\frac{2}{7}\times 3+\frac{2}{7}\times 11$ ⑦ $\frac{5}{6}\div 4\times 3.2$

Order and Properties of Operations/Simplifying Fractions/Finding a Common Denominator
Page 272③, Page 273⑧⑩

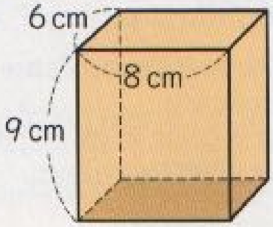
2 Momoka went to buy pencils and an eraser.

① Suppose she bought x pencils that cost 80 yen each and one eraser that cost 120 yen. Write a math sentence to express the total cost.

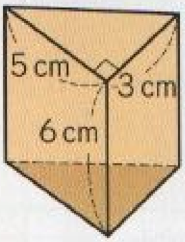
② Suppose the value of x in ① is 3 or 8. Find the total cost in each case.

3 Find the volume of the solid figure as shown below.

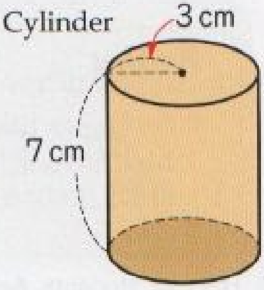
① Cube



② Triangular prism



③ Cylinder



Warm-up

4 We are going to make 3-digit whole numbers using the three cards $\boxed{1}$, $\boxed{3}$, and $\boxed{6}$.

① What is the largest number we can make?

② What is the second largest number we can make?

③ How many even numbers can we make?

Playing with Numbers and Calculations

Let's Complete the Math Sentences

We are going to make addition math sentences with the answer of 100 by using each of the cards $\boxed{1}$ to $\boxed{9}$ once.

Complete the math sentences below by placing the rest of the cards in $\boxed{}$.

① $\boxed{9}\boxed{4}+\dfrac{\boxed{1}\overset{\text{Ⓐ}}{\boxed{}}\overset{\text{Ⓑ}}{\boxed{}}\boxed{8}}{\boxed{2}\boxed{6}\overset{\text{Ⓒ}}{\boxed{}}}$ ② $\boxed{3}+\dfrac{\boxed{6}\overset{\text{Ⓕ}}{\boxed{}}\boxed{2}\overset{\text{Ⓖ}}{\boxed{}}\boxed{8}}{\overset{\text{Ⓗ}}{\boxed{}}\boxed{1}\overset{\text{Ⓘ}}{\boxed{}}}$

What situations are possible?

Umihiko's school is going to hold a sports competition.

Sports Competition

Date: XXX. XX at the Schoolyard and the Gym

Relay

Basketball

Long Rope Jumpi

Tokyo Olympics and Paralympics are coming. Let's play sports at our school!

Let's play sports to make our lives more enjoyable and fulfilling!

Relay

- Each team has four runners.

★ Each team determines the order of the runners beforehand.

Basketball

- Four teams, A, B, C, and D, are going to compete.

★ Each team is going to play against each of the other teams once.

The four students, Aoi, Ikuto, Umihiko, and Eri, are going to make a team.

I want to be the first runner.

What kinds of match-ups are possible?

Discuss possible ways to order runners in a relay and possible basketball game match-ups.

In a relay, the four runners can run in the order of Aoi, Eri, Ikuto, and Umihiko. Other possible orders are...

As for basketball games, A can be matched against B, and C against D. Other possible match-ups are...

I wonder how many different situations there are.
I want to find out wisely.

Ways of Ordering and Combining

11 Let's Investigate Systematically

These four students are going to make a team and run in the relay.

There are only four runners on the team, but there seem to be many ways to orders of them.

I wonder how many ways there are.

1 Ways of ordering

1 The four students, Aoi, Ikuto, Umihiko, and Eri, are going to make a relay team and each of them is going to run once. Investigate possible orders of the runners.

It'd be tough to think randomly.
It'd be also a bother to write down their names every time.

First	Second	Third	Fourth
Aoi	Ikuto	Umihiko	Eri
Eri	Aoi	Umihiko	Ikuto
Eri	Ikuto	Aoi	Umihiko
Umihiko	Ikuto	Eri	Aoi
		⋮	

It will make it simpler to think about the problem if you use symbols like the ones shown below.

Aoi	··· A
Ikuto	··· B
Umihiko	··· C
Eri	··· D

Let's think about how to avoid an overlap or an omission.

First, decide who will be the first runner, and then do the rest systematically...

① Suppose Aoi is the first runner, and investigate possible orders of the other three.

Misaki and Haruto thought about the different orders when A comes first.

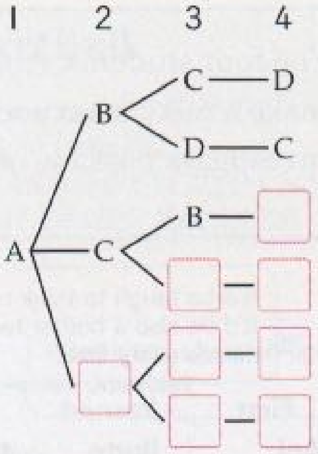
Replace "First" by "1" and...

② Explain Misaki's idea.

1	2	3	4
A	B	C	D
A	B	D	C
A	C	B	
A	C		
A	D		
A			

If the first runner is A, the second runner will be B, C, or D. If the second runner is B...

③ Explain Haruto's idea.

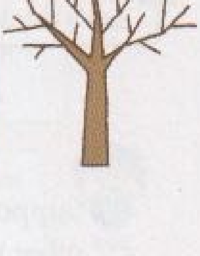


④ Compare and contrast these two students' ideas and discuss what you noticed.

Misaki's idea makes it easier to see the results.

With Haruto's idea, the amount of writing...

A diagram with all possible cases expressed like branches of a tree, like the diagram drawn by Haruto, is called a "tree diagram."



⑤ If the first runner is A, how many different orders will there be?

⑥ Using a diagram like the one drawn by Haruto, investigate how many different orders there will be if the first runner is B, C, or D.

⑦ How many different orders of the four runners are there altogether?

Look at the results of your investigation to see if the order you originally thought is in the results.

Shiho

I was thinking of order D → B → C → A. It is in the diagram and the table we created.

Summary

To investigate the number of ways to order things, you should **investigate systematically by using a diagram or a table.**

You may also want to replace words by symbols.

Additional Problems
→ Page 254 AC

Kota

I want to investigate the number of ways to order things in many situations.

2

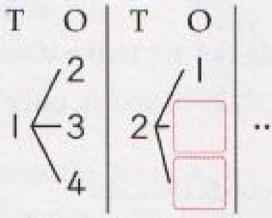
We are going to make 2-digit whole numbers using two of the four cards $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, and $\boxed{4}$. What whole numbers can we make?

Be careful to avoid an overlap or an omission.

① Explain the following two students' ideas.

If we label the tens place as T and the ones place as O...

Ami



Riku

T	O
1	2
1	3
1	4

T	O
2	1
2	
2	

...

② How many different 2-digit numbers can you make?

We should investigate systematically by using a diagram or a table.

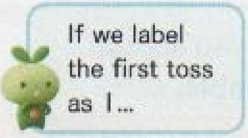
Haruto

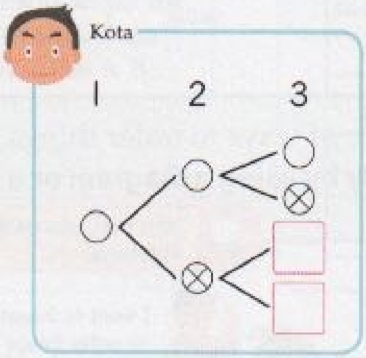
3

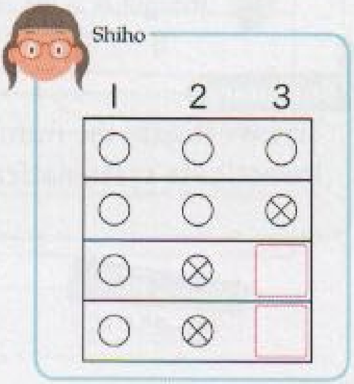
We toss a coin 3 times.
In how many different ways do we get heads and tails?

Let's investigate by avoiding an overlap or an omission.

Kota and Shiho used ○ for heads and ⊗ for tails to draw a table and a diagram.







1	2	3
○	○	○
○	○	⊗
○	⊗	
○	⊗	

① Investigate how heads and tails come up when the first toss is a tail, using a diagram and table.

② In how many different ways can we get heads and tails?

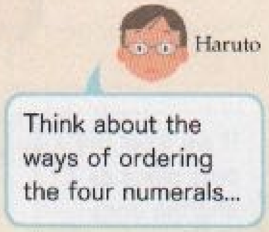
We should investigate systematically by using a diagram or a table.

Mathlin Newsletter

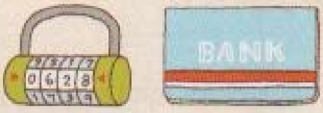
How Many Different Passwords?

We are making 4-digit passwords using some numerals.

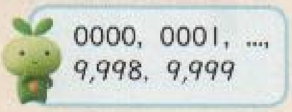
① How many different passwords can we make if we use each of the four numerals, 1, 2, 3, and 4, once?



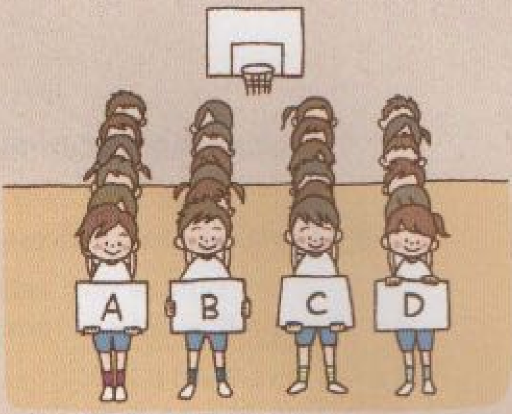
In daily life, there are many passwords that are made up of numerals from 0 to 9.



② If you use the numerals from 0 to 9, and if you can use each numeral as many times as you want, 10,000 different passwords are possible. Think about why you can make so many.



Four teams, A, B, C, and D are going to play each other in basketball.

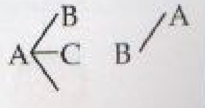


2 Ways of combining

1 Four teams, A, B, C, and D are playing basketball games. If each team plays each of the other teams once, investigate what match-ups there will be.

Let's think about how to avoid an overlap or an omission.

① Name the opponents for Team A.
Also, name the opponents for Teams B, C, and D.



Haruto and Shiho made the following tables to work on the problem.

② Explain Haruto's idea.

A vs. B and B vs. A are the same game...

Team A's Games	A・B	A・C	A・D
Team B's Games	~~B・A~~	B・C	B・D
Team C's Games			
Team D's Games			

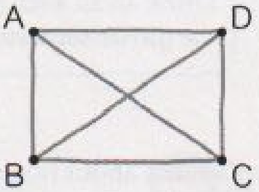
③ Explain Shiho's idea.

	A	B	C	D
A		○	○	○
B			○	○
C				○
D				

What match-up does each ○ represent?

Misaki drew the following diagram to work on the problem.

④ Explain Misaki's idea.



What does each segment represent?

⑤ Compare and contrast the three students' ideas and discuss your observations.

Since Haruto's idea requires us to write the match-ups for each team, if there were a lot of teams...

Since Shiho's idea requires only half of the table, the games that overlap...

Since Misaki's idea represents each match-up with a side or a diagonal of a geometric figure...

Diagonal
Page 274 ⑰

⑥ How many different match-ups of the four teams are there altogether?

Summary

When you investigate possible combinations, you should **investigate systematically by using a diagram or a table like you did with different ways to order things**.

As a combination, "A against B" is the same as "B against A."

Haruto

I want to investigate combinations in many situations.

1 A girl is buying 2 flavors of ice cream from 5 choices: vanilla, chocolate, strawberry, orange, and grape.
What are some possible combinations?
How many different combinations of flavors are possible?

Additional Problems
→Page254 AD

2 Identify situations around you where you can use what you have learned about the number of ways to order things and to make combinations. Also, find out what orders or combinations are possible.

Four students are going to give presentations about their reflections on today's study.

You are going to choose three different types of snack from the following four types: curry-flavored crackers, gummy candies, ramune candies, and doughnuts.

I wonder in how many different orders they can give presentations.

Ami

I wonder what combinations are possible.

Misaki

I wonder how many different combinations are possible.

Kota

Wrapping Up Your Learning: Ways of Ordering and Combining

Use What You Have Learned

Based on what she has studied, Aoi is thinking about what courses she can combine into a meal at a restaurant.

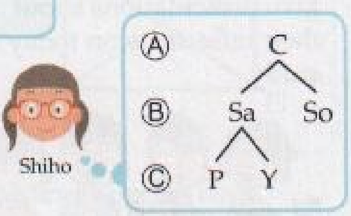
Choose one from each of the categories, Ⓐ, Ⓑ, and Ⓒ.

Which ones should I choose?

① How many choices are there in each of the categories, Ⓐ, Ⓑ and Ⓒ?

② How many different lunch sets can be made?

First, guess how many different combinations are possible, and then find the answer. Did you guess right?



③ Aoi is going to choose one beverage from two choices: orange juice or oolong tea.
When Aoi makes choices from each of the four categories, Ⓐ, Ⓑ, Ⓒ, and beverage, how many different combinations are possible altogether?

Using what we've found in ②...

Wrapping Up Your Learning: Ways of Ordering and Combining

Check Your Understanding

☐ 1 Three people will line up side -by- side to take a picture. How many different ways can they line up? Find it out by drawing a diagram like the one drawn by Haruto on page 166.

Express the first, second, and third people from the left as ①, ②, and ③. Also, use symbols for the three people.

◀ Can you investigate how to order things?
Page 165 1

☐ 2 There are four coins on the right. List all possible amounts of money you can make using two of the four coins.

◀ Can you investigate possible combinations?
Page 169 1

Wrapping Up Your Learning: Ways of Ordering and Combining

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Situation and Think about How to Avoid an Overlap or an Omission

Select all the techniques that you think are useful when you examine ways of ordering and combining.

Ⓐ Listing ways of ordering and combining as they come to your mind, and examine them.

Ⓑ Expressing ways of ordering and combining in a diagram or a table, and examine them systematically.

Ⓒ Abbreviating the conditions and the names of the things you examine for simplification.

Look back on what you have learned in "Let's Investigate Systematically" and discuss.

Now we know how to examine ways of ordering and combining in various situations.

Misaki

I want to examine the situation of combining ice cream flavors with many more choices of flavor.

Kota

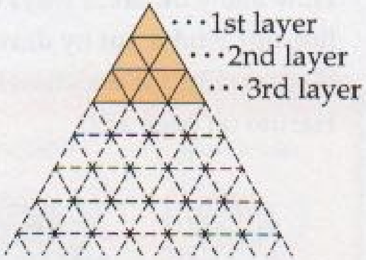
Extending Mathematical Thinking

Focus on Relationships

● Think by Using Diagrams, Tables, and Math Sentences ●

1

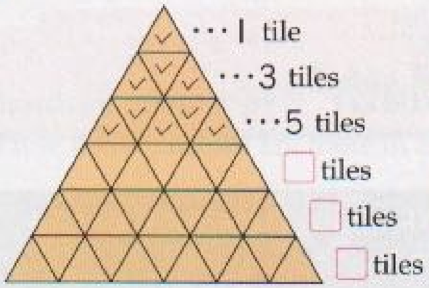
As shown on the right, we are arranging tiles in the shape of an equilateral triangle. How many equilateral-triangle tiles will be in the 21st layer?



I can tell if we actually draw up to the 21st layer, but...

I wonder if we can find out any relationship by just looking at a few layers.

① Let the layer number be x and the number of tiles be y, and find the number of tiles in the 1st, 2nd, ..., 6th layers.



Draw a picture and count the tiles. Then, record the results in a table. Is there any pattern?

Layer number	1	2	3	4	5	6
Number of tiles	1	3	5			

② Is the number of tiles, y, proportional to the layer number, x?

③ Look at the table Haruto created and investigate the relationship between the layer number and the number of tiles.

As the layer number increases by 1, what happens to the number of tiles?

Layer number	1	2	3	4	5
Number of tiles	1	3	5		

If I compare the layer number and the number of tiles ...

Layer number	1	2	3	4
Number of tiles	1	3	5	

④ Based on the relationship you have found, find the number of tiles in the 21st layer.

Ami

Layer number	1	2	3	4	5	6		21
Number of tiles	1	3	5	7	9	11		☐

increases by 2 tiles

$$1+\underbrace{2+2+\cdots\cdots+2}_{(21-1)\text{ sets of }2}=1+2\times(21-1)$$

$$=\boxed{}$$

Answer $\boxed{}$ tiles

Kota

Layer number	1_{0}	2_{1}	3_{2}	4_{3}	5_{4}	6_{5}		21_{20}
Number of tiles	1	3	5	7	9	11		☐

Layer number Number of tiles

1st layer ··· $1+0=1$

2nd layer ··· $2+1=3$

3rd layer ··· $3+2=5$

4th layer ··· $4+3=7$

⋮

If you add the layer number and the number that is 1 less than the layer number, you get the number of tiles.

$21+20=\boxed{}$ Answer $\boxed{}$ tiles

⑤ Explain the two students' ideas.

Ami is looking at the table horizontally, isn't she? What about Kota?

⑥ In Ami's last math sentence, $1+2\times(21-1)$, what do 1, 2, and $(21-1)$ represent?

⑦ Using Ami's idea, find the number of tiles in the 50th layer.

Math Sentence ▸ $1+2\times(\boxed{}-1)=\boxed{}$ Answer $\boxed{}$ tiles

⑧ Based on Ami's idea, use x and y to write a math sentence representing the relationship between the layer number, x, and the number of tiles, y.

$1+2\times(\boxed{}-1)=\boxed{}$

Calculate the number of tiles in different layers using this math sentence.

Shiho

I wonder if we can use x and y to write a math sentence for Kota's idea, too.

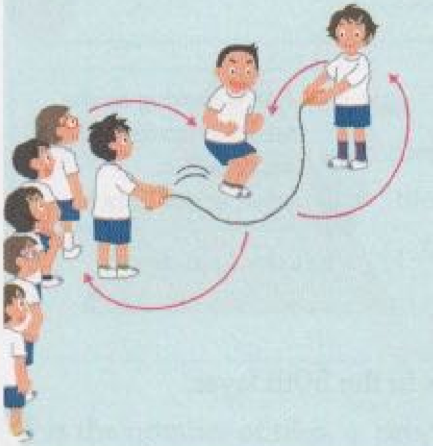
Let's predict whether Kota's class will be the champion or not

Kota's school has three classrooms in the 6th grade. The 6th grade is planning to hold a long rope jumping competition among the classrooms before graduation.

Kota

Let's make this competition one of our great memories of our elementary school days!

The students' council has set the rules of long rope jumping as follows:



Rules

- One student enters, jumps, and exits the rope, followed by one student after another. When all the students in a classroom are on the other side of the rope, they do the same, moving in a figure 8 pattern (see the red arrows in the diagram). Classrooms compete in the number of jumps made in one minute.
- Even if a student fails to jump, the class may continue jumping rope.
  The classroom that made the most jumps will be the champion.

Classroom A, to which Kota belongs, is practicing long rope jumping to prepare for this competition.

Kota

We want to be the champion in the competition! I wonder if our class will make it.

Misaki

What data do we need to predict whether we will be the champion or not?

Riku

Why don't we collect records of Classroom A's practice?

On every day of practice, Kota's classroom recorded the number of jumps they made in one minute. The table below shows the data from their 15 days of practice.

Number of Jumps
(Kota's) Classroom A Made

Day 1	① 61
Day 2	② 60
Day 3	③ 57
Day 4	④ 62
Day 5	⑤ 55
Day 6	⑥ 56
Day 7	⑦ 64
Day 8	⑧ 63
Day 9	⑨ 67
Day 10	⑩ 62
Day 11	⑪ 68
Day 12	⑫ 63
Day 13	⑬ 70
Day 14	⑭ 62
Day 15	⑮ 66

What can you tell from these data?

What is the largest number of jumps?

Are there two or more days when we made the same number of jumps?

How many jumps did we make on average?

Average
Page 273 ⑫

Summarize what you can tell from the data.

Largest number of jumps	Day	
Smallest number of jumps	Day	

We can learn so many things from a single set of data.

Are the data above enough to predict whether Classroom A, to which Kota belongs, will be the champion?

These data tell us much about Classroom A's performance. It's great they made as many as 70 jumps once, but these data are not enough...

We need to check these data of other classrooms...

How to Analyze Data

12 Let's Investigate the Characteristics of Data and Make Judgments

The tables below show the data of the long rope jumping records of Classrooms A, B, and C in the 6th grade of Kota's school.

The data of Classroom B cover 14 days of records, while the data of Classroom C cover 17 days or records.

Numbers of Jumps Classroom A Made

Day 1	① 61
Day 2	② 60
Day 3	③ 57
Day 4	④ 62
Day 5	⑤ 55
Day 6	⑥ 56
Day 7	⑦ 64
Day 8	⑧ 63
Day 9	⑨ 67
Day 10	⑩ 62
Day 11	⑪ 68
Day 12	⑫ 63
Day 13	⑬ 70
Day 14	⑭ 62
Day 15	⑮ 66

Numbers of Jumps Classroom B Made

① 54
② 55
③ 53
④ 56
⑤ 65
⑥ 65
⑦ 70
⑧ 67
⑨ 68
⑩ 70
⑪ 56
⑫ 56
⑬ 71
⑭ 67

Numbers of Jumps Classroom C Made

① 56
② 60
③ 60
④ 55
⑤ 59
⑥ 58
⑦ 56
⑧ 57
⑨ 63
⑩ 40
⑪ 67
⑫ 70
⑬ 65
⑭ 73
⑮ 70
⑯ 61
⑰ 70

1 Approaches to the Solution

Average and Spread

1 From the data above, predict which of the three classrooms, A, B, or C, will be the champion in the long rope jumping competition.

Can we say that Classroom A make more jumps than the other classrooms?

Let's think about how to compare them.

① How can we compare them?

Pay attention to the same number of jumps...

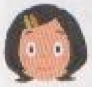
Compare them in the largest number of jumps...

Compare them in the sum of the numbers of jumps...

Compare them in the smallest number of jumps...

Compare them in the number of jumps they made on average...

② If you compare the classrooms in the ways we thought about in ①, which classroom is predicted to be the champion? Discuss how to compare the classrooms.



If you compare the classes in the largest number of jumps, Classroom C is the best. Even if you compare the classrooms in the smallest number of jumps...

The data collected from the classrooms are different in the number of days, so we can't simply compare the classrooms in the sum of the numbers of jumps...

③ Compare Classrooms A, B, and C in the number of jumps they made on average.

Round the answer to the nearest whole number.

The average of values for a group is called the **mean** of the data for the group.

Summary

To compare data from different groups, you may sometimes **use the mean of data for each group.**

Even if the number of data are different among groups, you can compare the groups using the mean.

Either Classroom A or B is likely to become the champion because their means are high.

The mean for Classroom C is not so low compared with the other classrooms...

Can we expect that each classroom will make about the mean number of jumps at the competition? If you compare the mean for Classroom B, 62, with the actual data of the classroom...

Is it really okay to predict a champion based on means only?

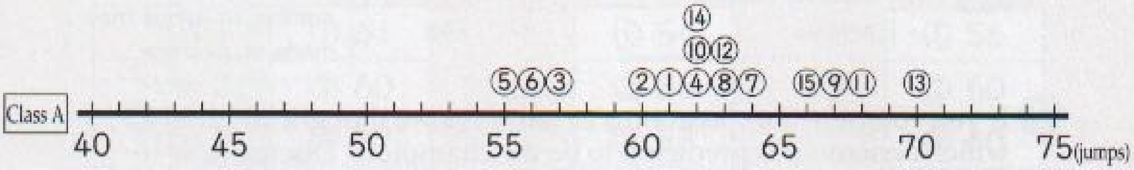
Line Plot

2

For each of Classrooms A, B, and C, investigate how the numbers of jumps are spread out. Then, predict which classroom will be the champion.

For the numbers of jumps made by the three classes, see page 178 or 277.

It would be easy to investigate it if you represent the numbers of jumps on a number line.



A number line with values plotted as dots above it is called a **line plot.**

Haruto

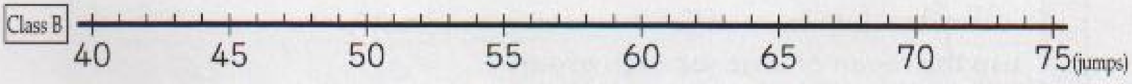
Numbers for the same value are written on top of another.

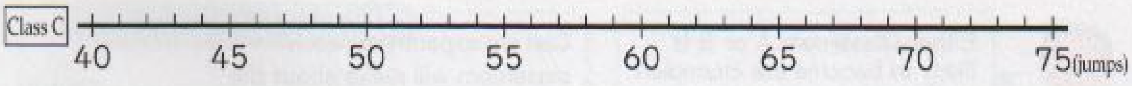
You may replace ①, ②... by ●.

① Represent the numbers of jumps each of classrooms B and C made in a line plot.

There are number lines on page 277, too.

(Write)





Let's think about what you should focus on to investigate how the numbers of jumps are spread out.

2 What are the largest and smallest numbers of jumps each of the three classrooms made?

3 On each line plot, draw an arrow ↑ at the tick mark indicating the mean of the numbers of jumps.

4 Can we say that the numbers of jumps will always gather around the mean?

Dots are stacked higher at the numbers of jumps that were made more often.

The value that appears most often in a set of data is called the **mode.**

If there are two or more values that appear most often, all these values are modes.

Look at the line plot representing the numbers of jumps Classroom A made on page 180. Since the value that appears most often is 62, the mode of the numbers of jumps Classroom A made is 62.

Classroom A made 62 jumps on 3 days. That is the number of jumps made most often. The mode is 62 (jumps), not 3 (days).

5 What is the mode of the numbers of jumps Classroom B made? What is the mode of the numbers of jumps Classroom C made?

You could find the modes from the tables on page 178, but if you simply look at the dot plots...

If you compare the three classes in their modes...

Summary

If you **plot data on a line plot, you can observe how they are spread out, which you cannot observe by only finding the mean of the data.** When you compare data from different groups, you can **use the mode** of each group.

The mode of Classroom C is the highest, so I think Classroom C will be the champion.

Classroom C often made 70 or more jumps, but it once made only 40 jumps...

I wonder if we can organize the data so that the spread of the values can be easily seen with numbers.

Frequency Distribution Table

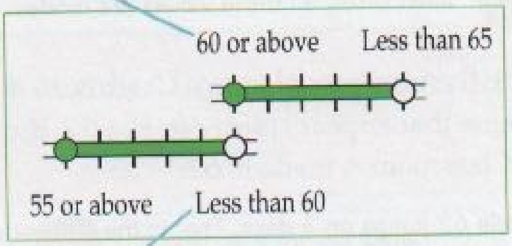
3 Organize the data of the numbers of jumps Classrooms A, B, and C made into a table so that the overall spread of the data can be more easily seen with numbers.

① We are going to organize the numbers of jumps by putting them into 5-jump intervals to find on how many days the number of jumps fell into each of the intervals.
Into which interval will 60 jumps fall?

Numbers of Jumps Classroom A Made

Number of Jumps	Number of Days
40 or above and less than 45	
45 or above and less than 50	
50 or above and less than 55	
55 or above and less than 60	
60 or above and less than 65	
65 or above and less than 70	
70 or above and less than 75	
Total	

"60 or above" means either equal to 60 or greater than 60.



"Less than 60" does not include 60.

Let's organize Classroom A's data, to begin with.

② Write the appropriate number of days for each interval in the table above.

You may want to use tally marks first, and then write numbers.

Class ················	An interval of values to organize data
Class interval ······	The width of interval for each class
Frequency ········	The number of values in each class
Frequency distribution table ···	A table where data are sorted into classes, like the one shown above

③ Look at the frequency distribution table for Classroom A on page 182. What is the class interval?
Which class has the frequency of 3 days?

④ Represent the numbers of jumps each of Classrooms B and C made in a frequency distribution table.

Numbers of Jumps Classroom B Made

Number of Jumps	Number of Days
40 or above and less than 45	
45 or above and less than 50	
50 or above and less than 55	
55 or above and less than 60	
60 or above and less than 65	
65 or above and less than 70	
70 or above and less than 75	
Total	

Numbers of Jumps Classroom C Made

Number of Jumps	Number of Days
40 or above and less than 45	
45 or above and less than 50	
50 or above and less than 55	
55 or above and less than 60	
60 or above and less than 65	
65 or above and less than 70	
70 or above and less than 75	
Total	

Let's make many different observations based on the frequency distribution tables for the three classrooms on pages 182-183.

⑤ For each of Classrooms A, B, and C, what is the frequency of "55 or above and less than 60" jumps?

⑥ For each of Classroom A, B, and C, what is the sum of the frequencies of 65 jumps or above?
About what % is the rate of that sum compared to the whole sum of all the frequencies?

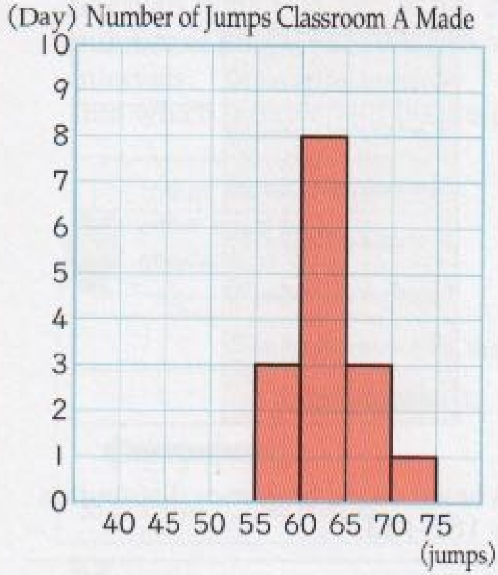
⑦ In the data of Classroom A's 15 days practice, into which classes do the 4th, 8th, and 12th smallest numbers of jumps fall?

If you show the spread of data in a frequency distribution table, **you can observe characteristics of the data, which you cannot observe by only finding the mean of the data.**

I wonder if there is a graph that can visually show how the values are spread.

Histograms

4 We are going to make a graph, like the one as shown below, from the frequency distribution tables on the previous page. By looking at the graph, investigate how the numbers of jumps Classrooms A, B, and C are spread.

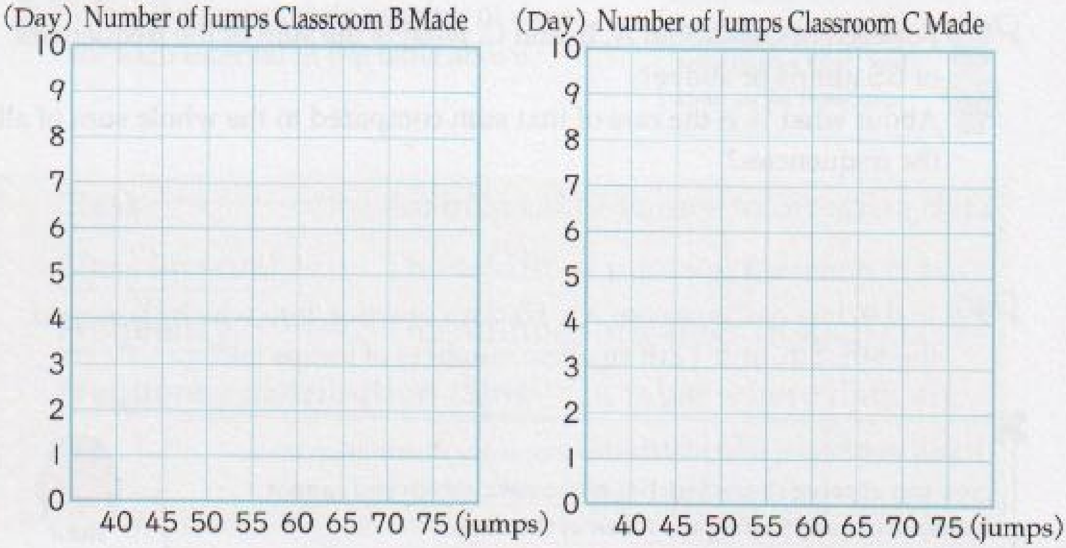


A graph like this is called a **histogram**.

How are histograms different from bar graphs?

A line plot shows the spread of data as well as the specific values of those data. A histogram groups these data into intervals of 5 jumps and is useful in examining how entries are spread on the whole.

① Represent the numbers of jumps each of Classrooms B and C made in a histogram.



Let's make different observations from the histograms.

〈Draw Graph〉

② For each of Classrooms A, B, and C, which class has the largest frequency?

③ For each of Classrooms A, B, and C, into which class does the mean fall?

We found the means on page 179.

④ Look at each of the histograms for Classrooms A, B, and C, and describe the characteristics of how the numbers of jumps are spread.

I wonder if the class in which the mean falls has always the highest frequency.

I wonder how the graphs are shaped.

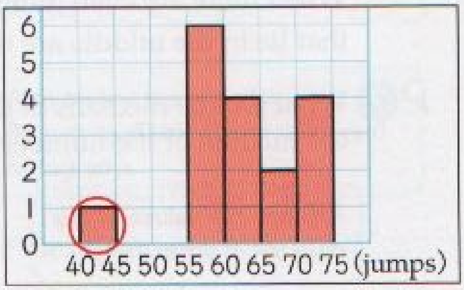
A histogram is useful in seeing the overall spread of data at a glance.

Additional Problems → Page255 AE

I've found something interesting in the histogram for Classroom C.

When Misaki looked at the histogram for Classroom C, she paid attention to the following:

The bar marked with the circle represents 40 jumps. The value for this day is so far from those of the others.



That day, the students who usually turned the rope were absent, and teachers turned the rope instead.

If there is a value that is so far from the others, like 40 jumps above, the value may affect the mean. So, you may want to disregard the value.

⑤ Disregard the 40 jumps, then find the mean and compare it to the original mean of the numbers of jumps Classroom C made.

When you investigate or describe the characteristics of a set of data, you may want to use the value that lies in the middle of the arranged values.

The value that lies in the middle of the arranged values is called the **median**.

A value that is so far from the others is unlikely to affect the median.

Let's find the median.

We have arranged the values of the data set for Classroom A in order from the least.

Class A 55 56 57 60 61 62 62 62 63 63 64 66 67 68 70

⑥ What is the median of the numbers of jumps Classroom A made?

Kota

The total number of values is... Count them from the least...

Shiho

Maybe we can find the median in the line plot, too.

When there are odd numbers of values, the value that lies in the exact middle is the median.
When there are even numbers of values, the mean of the two values that lie in the middle are the medians.

⑦ What are the medians of the numbers of jumps Classroom B made? What is the median of the numbers of jumps Classroom C made?

Arrange the values of the data set in order...

Misaki

When there are even numbers of values, use the two values that lie in the middle and $(\square+\square)\div 2 = \ldots$

Riku

It's important to **pay attention to whether the number of values is odd or even**.

Haruto

If you compare the medians of Classrooms A, B, and C...

When you investigate or describe the characteristics of a set of data, you may often want to take a single value as representative of the data set. We can then compare several data sets using those values from each data set. Such a value is called a **representative value**. The mean, the mode, and the median are all representative values.

You will learn about programming on page 242.

Shiho

I think we can predict the champion with clear reasoning based on what we've studied.

In order to predict which class will be the champion in the long rope jumping competition, we have compared Classrooms A, B, and C in many ways based on the numbers of jumps they made during practice.

5 About the numbers of jumps Classrooms A, B, and C made, organize various comparison methods and results into the table below.

① Write the results in the table below.

	Classroom A	Classroom B	Classroom C
Largest number of jumps			
Smallest number of jumps			
Mean			
Mode			
Median			
Rate of the frequencies of 65 or more jumps (%)			
Class with the largest frequency in the frequency distribution table and in the histogram			

Let's come up with our idea and predict which classroom will be the champion, A, B, or C.

Also review the line plot, frequency distribution tables, and histograms.

② Which classroom do you predict will be the champion, A, B, or C?

(Your name) 's Prediction

● Prediction for the champion: Classroom A/B/C

● Reason:

Based on various **characteristics** of the data, **make a judgment with clear reasoning**.

Ami

Additional Problems → Page255 AF

Kota

It looks like we have some more days to practice before the big day. We'll pay attention to the characteristics of the data and make adjustments to our practice plan.

In addition to recognizing the champion in the long rope jumping competition, the students' council has decided to give awards to the classrooms that practiced hard based on the previous practice data.

Becoming the champion in the competition would be the best, but...

6

What awards would you give to each of Classrooms A, B, and C?
Make awards based on the tables, line plots, frequency distribution tables, and histograms on the previous pages.

All the classrooms look like they are practicing hard, so...

I'm wondering if I want to make an award based on how the values are spread out.

Among the representative values, I'll focus on the mode...

I want to give a "Hardest Workers Award" to the classroom that practiced the most days...

Misaki

Riku

Shiho

Let's find something that stands out in the data and make awards.

① Write down the awards you have made in the table below.

	Award name	Reason for the award
Classroom A		
Classroom B		
Classroom C		

You may investigate the data in a different way, too.

If you compare the three classrooms in only the data for the last five days, not the whole data...

I'm wondering if I want to investigate changes. Maybe I should make a broken line graph from the data...

Ami

Haruto

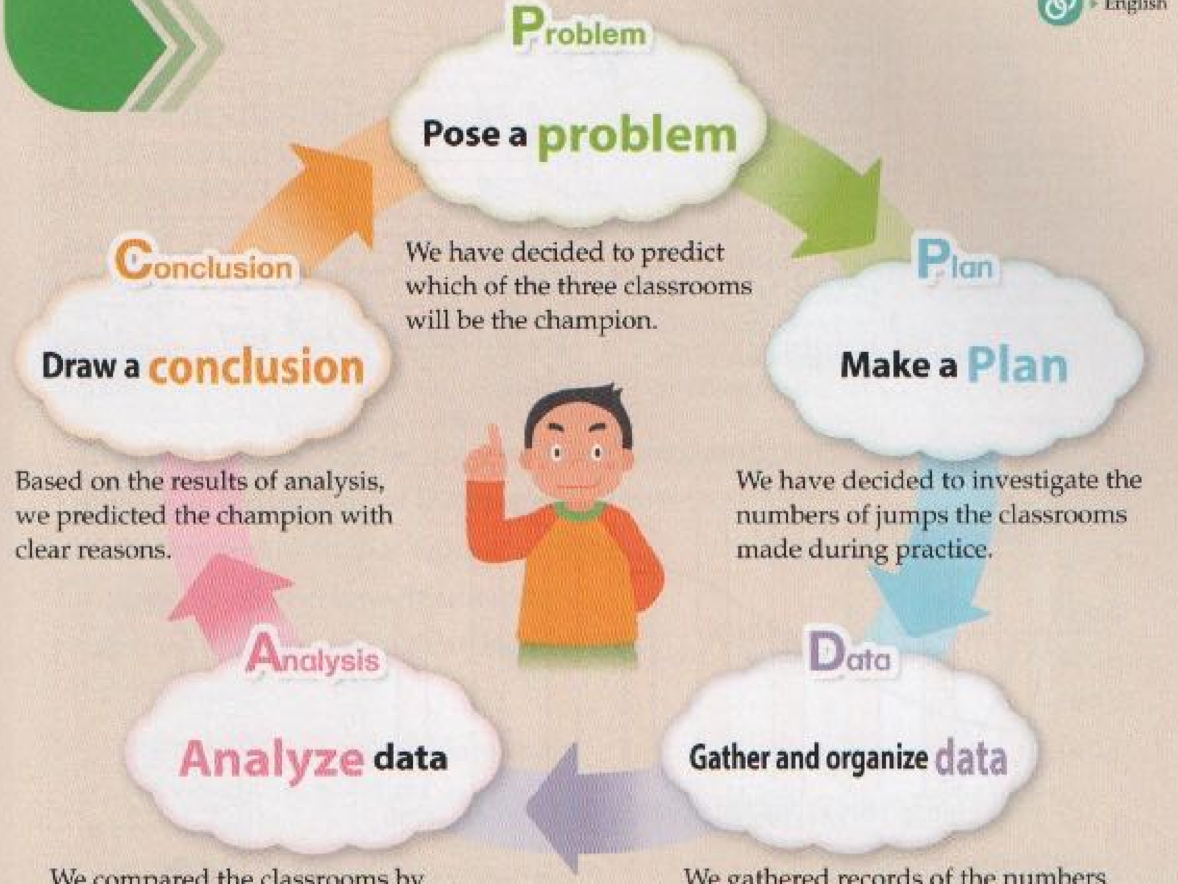
We're going to create original awards **with clear reasoning based on the data.**

Kota

Let's look back on the problem solving strategies we have learned so far.

6th grade students at Kota's school are going to enter a long rope jumping competition.

▸ English



Problem

Pose a problem

We have decided to predict which of the three classrooms will be the champion.

Plan

Make a Plan

We have decided to investigate the numbers of jumps the classrooms made during practice.

Data

Gather and organize data

We gathered records of the numbers of jumps made not only by Classroom A, which Kota belongs to, but also by Classrooms B and C during practice.

Analysis

Analyze data

We compared the classrooms by representing the data in line plots, frequency distribution tables, and histograms, and by finding representative values.

Mean, Mode, and Median

Conclusion

Draw a conclusion

Based on the results of analysis, we predicted the champion with clear reasons.

Drawing a conclusion was not the end of the process. We used the results of analysis and also tried other methods of analysis. We even posed a new problem about making awards.

Kota

We made use of the results of analysis in our later practice for the long rope jumping competition.

Misaki

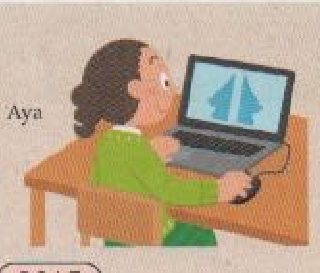
At first, we only used the mean. Then, we used additional ways to analyze the data, such as other representative values, the line plot, and the frequency distribution table.

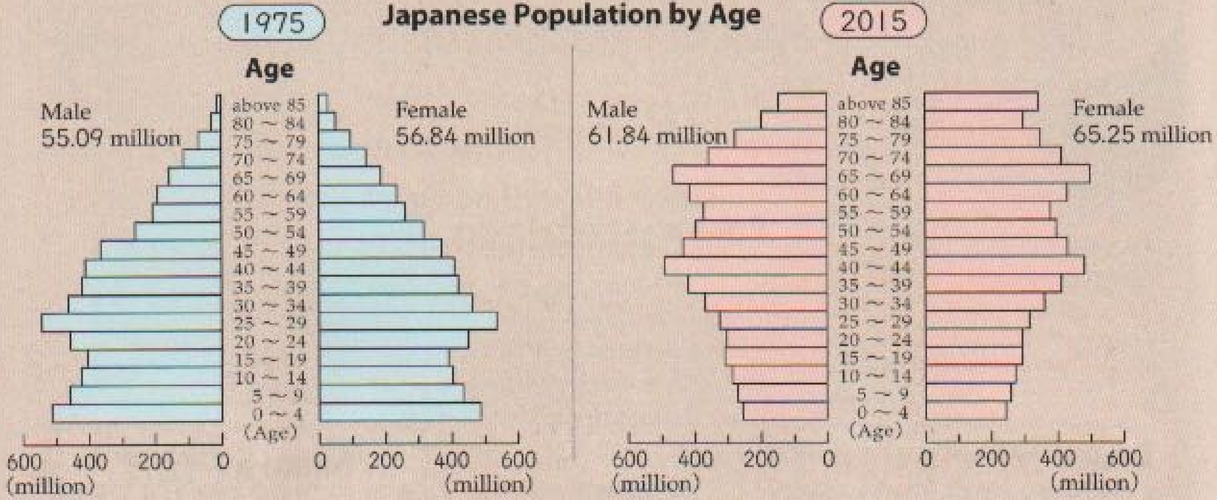
Shiho

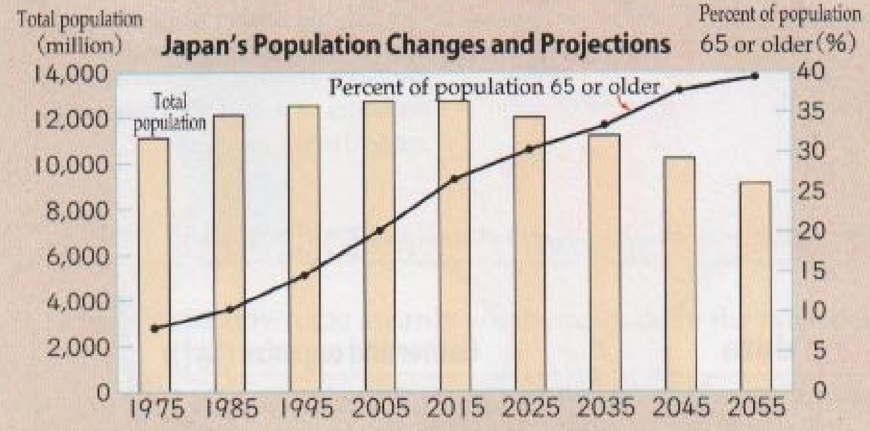
If a problem arises around us, I'd like to use this kind of method to address it.

2 Various Graphs

Aya was investigating the changing age demographics in Japanese society and found the graphs below.




Japanese Population by Age


Japan's Population Changes and Projections

How are these graphs different from the ones we have studied so far?

(Developed by Tokyo Shoseki based on the website of the Statistics Bureau of Japan)

1 Let's investigate these graphs above.

Let's make different observations from the graphs above.

① What were the age classes with the largest number of people in 1975 and 2015?

In 2015, there were 4.65 million men and 4.98 million women in the 65-69 age group, and 4.91 million men and 4.81 million women in the 40-44 age group.

② What can you conclude when you compare the age spreads of the Japanese population in 1975 and 2015?

③ What was the approximate population of Japan in 1975? About what % of the population was 65 or older?

④ Describe the projected change from 1975 to 2055 in the % of population of people aged 65 or older.

Share other observations you made from the graphs on the previous page.

There are various graphs depending on what you want to represent.

Haruto

(Investigate)

The Statistics Bureau, the Ministry of Internal Affairs and Communications has a webpage "Naruhodo Tokei Gakuen" (School for the Study of Statistics), which provides a wide variety of statistic data. Use the data to find answers for your daily matters.

Mathlin Newsletter

Various Graphs

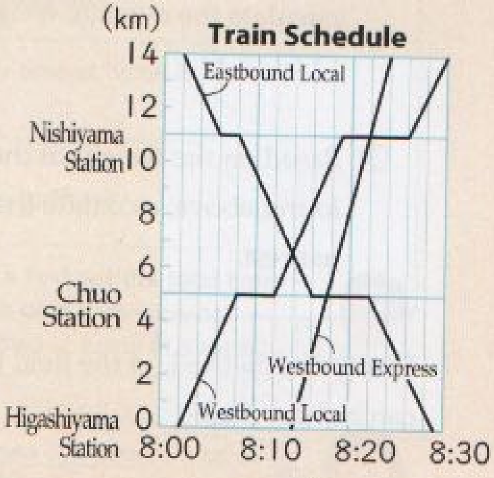
Advanced 8th Grade

The graph on the right shows how a train system operates. The vertical axis represents the location of the stations while the horizontal axis represents time. A graph like this is known as a train schedule diagram. From this diagram, find the following.

① the time the westbound local train arrives at Chuo Station

Misaki

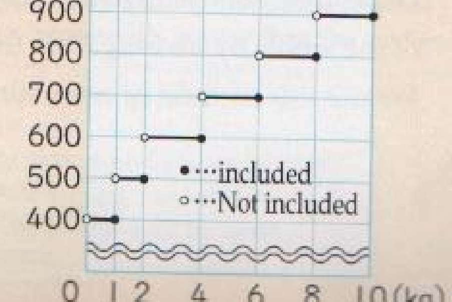
The departure time from Higashiyama station is 8:02 ...



② the total time the eastbound local train stays at Nishiyama station

③ the time and location of when the westbound express train passes the westbound local train

The graph on the right shows the shipping cost for packages for a certain carrier. Look at the graph and find the shipping costs for each of 1.2 kg and 6 kg packages.



Riku

The 6 kg package is in interval ...

Mathlin Newsletter

Features of the Whole and Features of a Part

Advanced 9th Grade

Investigate how we can study the amount of harvest from a field.

Ami

If the field is large, it will be difficult to gather data about the whole field.

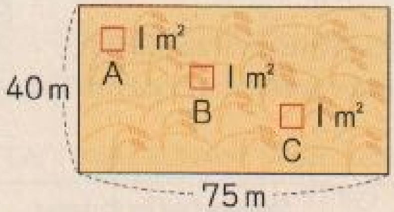
There are methods, like the one shown below, that base the estimate on the harvest of part of the field.

① Randomly select 3 plots that have an area of $1\ \text{m}^2$ each.

② Weigh the harvest from each plot and calculate the average weight.

③ Based on the weight of the harvest of $1\ \text{m}^2$ found above, calculate the total amount of harvest.

Using this method, in the field below, let's think about how many kg of rice can be harvested.



	A	B	C
Weight of Rice (kg)	0.51	0.46	0.47

Depending on what we investigate, sometimes we can estimate the features of the whole by investigating the features of smaller parts.
We use this method of estimating in many different ways in our daily lives.

For example, to predict whether a candidate will win an election, the entire results are estimated after only some voters are surveyed.

Wrapping Up Your Learning: How to Analyze Data

Use What You Have Learned

Misaki and her friends are re-examining their daily lives.

Let's make a habit of studying at home so that we can keep up with our studies after going on to junior high school.

Sleep is also important. I wonder how many hours my classmates sleep a day.

Reading means a lot, too. Reading books...

What about the time spent to help out around the house or to work out?

Pick one thing you want to investigate about your class.

Based on what you have studied, pose a problem and try to find out about it like these students do as follows.

For example, suppose you investigate study time at home.

① Pose a Problem. What do you want to find out?

We'll investigate whether my study time at home is longer than my classmates' or not.

② Make a Plan. What data do you need to gather?

I'm afraid a person's study time at home may change from day to day...

Let's find out the total time each of our classmates studied at home in a week.

③ Gather and organize Data.

Express the data as a table or a graph...

④ Analyze the data. What kind of characteristics can you find out?

Find representative value...

⑤ Draw a Conclusion.

It's safe to say that my study time at home is long, because it's greater than the median.

⑥ Review the process.

Should we have examined the data of students in other classrooms?

My study time at home is longer than my classmates', but I sometimes don't study much at home. I'll be careful.

Wrapping Up Your Learning: How to Analyze Data

Check Your Understanding

1 Classroom A in the 6th grade has decided to survey all the students' commuting time. The table below lists the commuting time for the students in the classroom. Answer the following questions.

Can you make and interpret line plots, frequency distribution tables, and histograms that represent the spread of values?

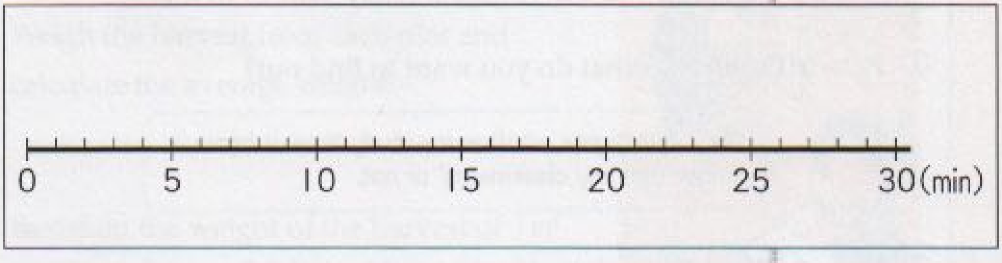
Commuting Time of Students in Classroom A in 6th Grade (min)

9	13	7	8	20	12	6	13	18	8
10	4	14	13	9	11	23	26	7	3
9	14	2	5						

① Represent the values in a line plot.

Represent each value as one ●.

① Page 180 2



② Find the mean, the mode, and the median.

On the line plot above, draw an arrow ↑ at each of the tick marks indicating the mean, the mode, and the median.
There can be more than one mode.

② Page 178 1
Page 180 2
Page 184 4

③ Complete the frequency distribution table by writing the number of students in each class.

How many min is the class interval?

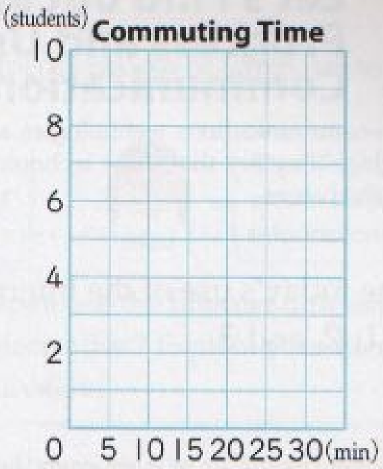
④ Which class has the largest frequency?
About what % is the rate of the frequency compared to the sum of all the frequencies?

③④
Page 182 3

Commuting Time

Time (min)	Number of students
0 or above and less than 5	
5 ~10	
10 ~15	
15 ~20	
20 ~25	
25 ~30	
Total	

⑤ Represent the data in a histogram.



⑤⑥
Page 184 4

⑥ Put a ◯ mark to the value(s) that can be found from the histogram. Put a × mark to the value(s) that cannot be found from the histogram.

Ⓐ the number of students whose commute time is 15 minutes or longer, but less than 25 minutes ( )
Ⓑ the average commuting time ( )
Ⓒ the rate of students whose commute time is less than 10 minutes ( )

Wrapping Up Your Learning: How to Analyze Data

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on a Problem and Think about How to Solve It

While looking at page 189, look back on the ways of solving problems by using data. From Ⓐ to Ⓒ below, select the correct statement(s).

Ⓐ Whenever you analyze data, all you need to do is to find the mean.
Ⓑ When you draw a conclusion, give a clear reason for it based on the results of analysis.
Ⓒ There is always only one conclusion. So, once you have reached one, you do not need to think more.

Look back on what you have learned in "Let's Investigate the Characteristics of Data and Make Judgments" and discuss.

Now we're familiar with ways of solving problems by using data. I want to work on various problems using clear reasoning based on data.


Haruto

When we studied bar graphs, we learned about changing the amount of one increment. I want to know what happens to the histogram if you change the class interval on it.


Ami

Junior High School
You'll study it in detail in junior high school.

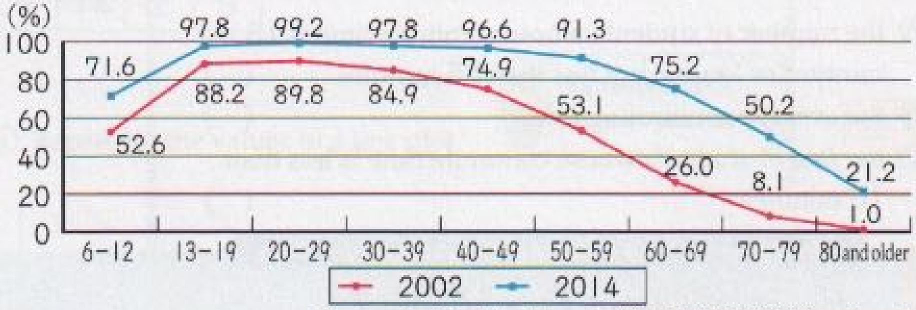
Let's Find out about the Progress and Use of Information-Communications Technologies

Information-communications technologies are progressing everyday and used in many fields. They say that these technologies should bring great change to our lives in the future.

1

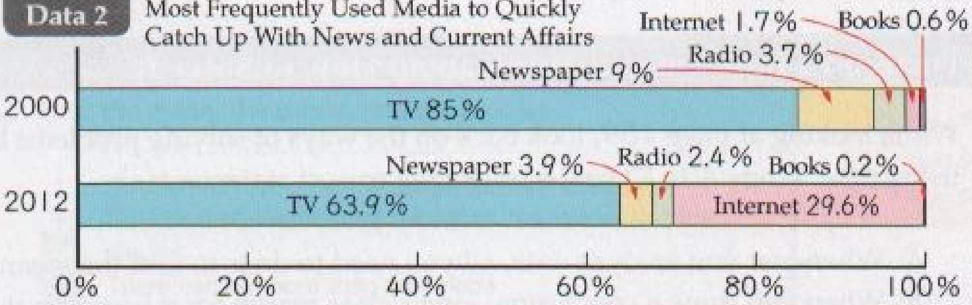
Investigating today's use of the Internet, Riku and his friends found data 1, 2, and 3.

Data 1 Internet Usage Rate by Age Group (Japan)



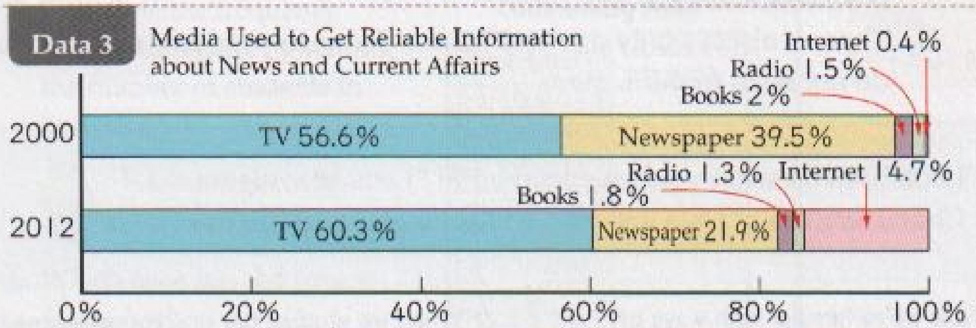
(Developed by Tokyo Shoseki based on 2015 WHITE PAPER Information and Communications in Japan (the Ministry of Internal Affairs and Communications))

Data 2 Most Frequently Used Media to Quickly Catch Up With News and Current Affairs



(Developed by Tokyo Shoseki based on 2015 WHITE PAPER Information and Communications in Japan (the Ministry of Internal Affairs and Communications))

Data 3 Media Used to Get Reliable Information about News and Current Affairs



(Developed by Tokyo Shoseki based on 2015 WHITE PAPER Information and Communications in Japan (the Ministry of Internal Affairs and Communications))

① Explain what data 1 to 3 above tell you about the Internet usage rate.

② What can you say about the media people of Japan use to get information?

Riku

Compared with 2000, in 2012, the number of Internet users...

Shiho

That's right, but depending on the purposes of getting information...

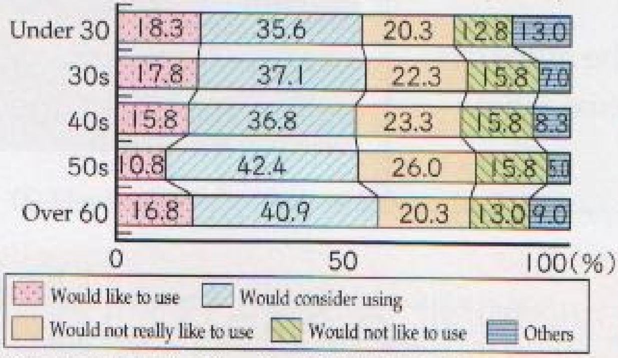
③ Can you say that people of Japan use the Internet differently depending on the purposes of getting information? Explain your idea to others and listen to other students' explanation.

A self-driving car can travel automatically without being driven by a human driver, but, for its safe travel, it needs to use information-communications technologies, for example, receiving and sending a lot of information. So, towards a full-scale rollout of self-driving cars, experts have been pursuing research on information-communications technologies.

2

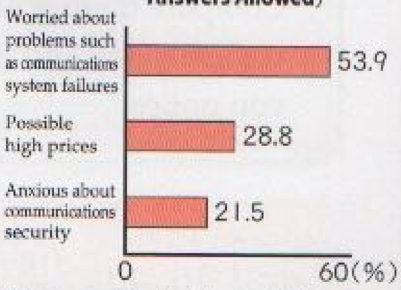
Ami and her friends did research about people's view on self-driving cars and found Data 4 and 5.

Data 4 **Survey of Willingness to Use Self-driving Cars by Age Group (400 Respondents from Each Age Group)**



(Developed by Tokyo Shoseki based on 2015 WHITE PAPER Information and Communications in Japan (the Ministry of Internal Affairs and Communications))

Data 5 **Major Reasons for Not Wanting to Use Self-driving Cars (Multiple Answers Allowed)**



(Developed by Tokyo Shoseki based on 2015 WHITE PAPER Information and Communications in Japan (the Ministry of Internal Affairs and Communications))

① Imagine you were a developer of self-driving cars.
As far as Data 4 and 5 indicate, which age groups should you develop self-driving cars for?
For better sales, what do you think you should focus on during development? Discuss what you think and the reasons why you think so with your classmates.

Ami

According to Data 4, the age group that has the most people who want to use self-driving cars is...

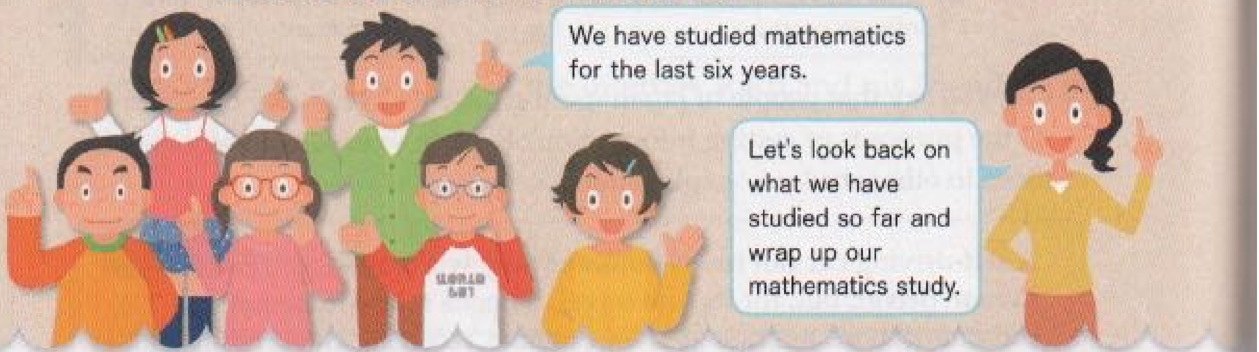
Haruto

According to the reasons for not wanting to use self-driving cars, what these people are anxious about are...

Finishing Touches

13

Let's Wrap Up Our Study of Elementary School Mathematics

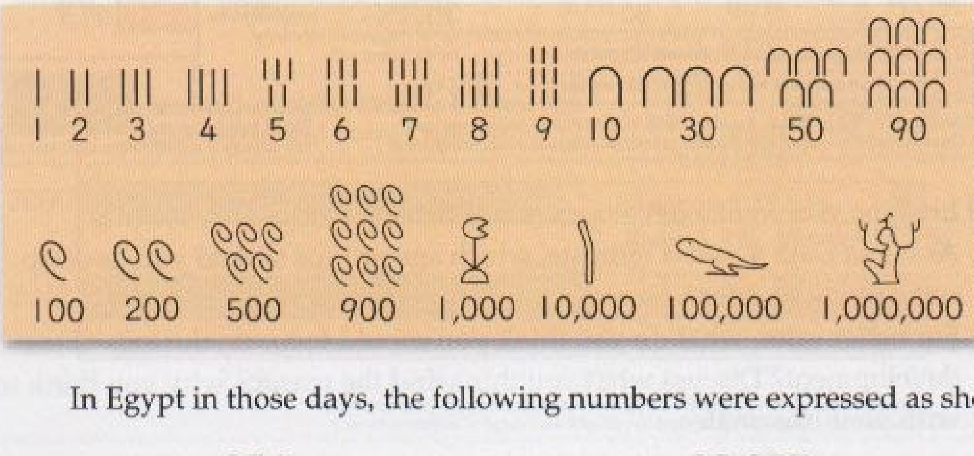


1 Numbers and Calculations

Answers → Page 266

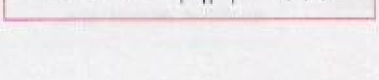
1 How to Express Numbers and the Structure of Numbers

1 The picture below shows the numerals used in Egypt some 5,000 years ago.
Compare them with the numerals we use today and discuss what you notice.



In Egypt in those days, the following numbers were expressed as shown below:

356



23,057

Ami

It looks hard to memorize so many numerals...

It's tedious to write some numbers, for example, 50 as ∩∩∩∩∩.

Kota

In a whole number or a decimal number, each place has its own value, and the numeral in the place tells us how many of that value are in the number. So, with 10 numerals from 0 to 9 and a decimal point, we can express numbers of any size.

Haruto

☐ **1** Write the following numbers:

① The number made of 2 hundred millions, 8 millions, and 5 ten thousands.

② How many 1,000's are in 357,000?

③ Six million multiplied by 100.

④ $\frac{1}{100}$ of 470 thousand.

4th Grade
Structure of Whole Numbers
Page 272④

☐ **2** Write the following numbers:

① The number made of four 10's, two 1's, one 0.1's, nine 0.01's, and five 0.001's.

② The number made of 129 pieces of 0.1's.

③ 12.3 multiplied by 100.

④ $\frac{1}{100}$ of 0.98.

⑤ Two $\frac{1}{3}$'s.

⑥ How many $\frac{1}{10}$'s are in 2?

①~③ 4th Grade
④ 5th Grade
Structure of Decimal Numbers
Page 272④

⑤⑥ 3rd Grade

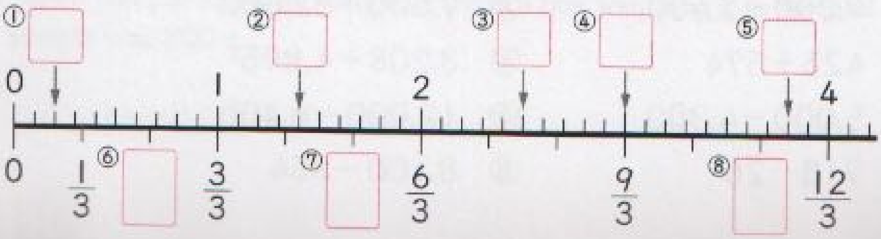
☐ **3** Write the fractions as decimal numbers and the decimal numbers as fractions.

① $\frac{1}{8}$ ② $\frac{9}{5}$ ③ 0.7
④ 2.03

5th Grade
Relationships between Fractions and Decimals
Page 272⑥

☐ **4** Write the appropriate whole or decimal number in each of boxes ① to ⑤. Write the approximate fractions in ⑥ to ⑧.

5th Grade



2 Addition and Subtraction

2 Fill in the □ with appropriate numerals to make math sentences based on the idea of using a base amount and the calculation of 3 + 2.

(Example) □□ + □□


Shiho

Think about $\boxed{3}\boxed{0}+\boxed{2}\boxed{0}$ with the base amount of 10. Since 3 + 2 = 5, there are five 10's. So, the answer is 50.

(1) □□□ + □□□ … The base amount is □

(2) $\boxed{0}.\square + \square.\square$ … The base amount is □

(3) $\dfrac{\square}{5}+\dfrac{\square}{5}$ … The base amount is □

① Replace 3 + 2 in **2** by 7 − 3 and make math sentences in the same way.

What are the base amounts?

If you pay attention to the base amount, you can consider complicated math sentences (with numbers with multiple digits or other than whole numbers) as the addition or subtraction of 1-digit whole numbers.


Riku

5 Do the following addition and subtraction calculations with whole numbers. 3rd Grade

① 9,000 + 2,400
② 7,500 + 32,000
③ 426 + 574
④ 3,208 + 4,895
⑤ 5,000 − 4,300
⑥ 12,000 − 6,400
⑦ 713 − 26
⑧ 8,100 − 354

6 Do the following addition and subtraction calculations with decimal numbers.

① $4.8+2.3$　② $16.6+3.4$　①② 3rd Grade

③ $50.8+7.34$　④ $2.53+8.6$　③④ 4th Grade

⑤ $7.6-5.3$　⑥ $4.8-2$　⑤⑥ 3rd Grade

⑦ $9.152-8.72$　⑧ $2.5-1.86$　⑦⑧ 4th Grade

7 Do the following addition and subtraction calculations with fractions.

①④ 4th Grade

②③⑤⑥ 5th Grade

① $\frac{3}{7}+\frac{6}{7}$　② $\frac{4}{5}+\frac{1}{3}$　③ $1\frac{1}{6}+2\frac{3}{4}$

④ $\frac{9}{8}-\frac{3}{8}$　⑤ $\frac{4}{7}-\frac{1}{5}$　⑥ $3\frac{5}{6}-2\frac{4}{9}$

8 Write the appropriate number in each $\square$.

4th Grade

① $(198+84)+16=198+(\square+16)$

② $0.8+7.6=\square+0.8$

9 Calculate the following.

① $197+236+64$　② $1{,}000-(350-200)$　①~⑥ 4th Grade

③ $6.3+1.75+3.7$　④ $17.3+(12-9.2)$

⑤ $3.4-(2.9-1.1)$　⑥ $28.3-(13.6+1.4)$

⑦ $\frac{3}{2}-\left(\frac{2}{3}+\frac{1}{6}\right)$　⑧ $\frac{7}{3}-\left(\frac{13}{12}-\frac{3}{4}\right)$　⑦⑧ 5th Grade

10 Relationships between quantities are described in the text below. Express them as math sentences with x and y.

6th Grade

① When you have run x m along a 1,500 m jogging path, you still have 700 m to go.

② When you bought a snack for x yen and juice for 150 yen, the total cost was y yen.

③ When you put an object weighing y g in a box weighing x g, the total weight was 800 g.

3 Multiplication and Division

Problem 1

We bought 1.6 m of ribbon. The cost was 80 yen.
How much does 1 m of this ribbon cost?

Problem 2

1 meter of ribbon costs 80 yen.
I bought 1.6 m of the ribbon, how much was the cost?

3 Cards Ⓐ to Ⓓ below are about either Problem 1 or 2 above. Which problem is each of these cards about, 1 or 2?

〈Math Sentences〉

Ⓐ 80×1.6

Ⓑ $80 \div 1.6$

〈How to Calculate〉

Ⓒ

The cost is proportional to the length.
Find the cost for 16 m of the ribbon, and then find $\frac{1}{10}$ of the cost.

$80 \times 1.6 = \square$

↓ 10 times $\frac{1}{10}$

$80 \times 16 = \square$

Ⓓ

The cost is proportional to the length.
So, it is the same as 16 m of the ribbon costing 800 yen.

$80 \div 1.6 = \square$

↓ 10 times ↓ 10 times Equal

$800 \div 16 = \square$

When we multiplied or divided a number by a decimal number, we thought about how to calculate using whole numbers.

Misaki

11 Calculate the following. For each division problem, divide completely.

① 537×46　　② 326×418

③ $204 \div 6$　　④ $360 \div 45$

⑤ 2.6×8　　⑥ 1.7×3.6

⑦ 7.04×5.2　　⑧ 24.5×0.34

⑨ $8.4 \div 7$　　⑩ $9.1 \div 2.6$

⑪ $46.4 \div 14.5$　　⑫ $2.1 \div 0.42$

⑬ $\frac{2}{3} \times 6$　　⑭ $\frac{3}{5} \times \frac{10}{9}$　　⑮ $\frac{7}{13} \times 1\frac{6}{7}$

⑯ $\frac{3}{4} \div \frac{1}{3}$　　⑰ $\frac{6}{7} \div 3$　　⑱ $1\frac{5}{8} \div 2\frac{1}{4}$

① 3rd Grade

②~④ 4th Grade

⑤⑨ 4th Grade

⑥~⑧ 5th Grade
Multiplication Algorithm of Decimal Numbers
Page 273 ⑦

⑩~⑫ 5th Grade
Division Algorithm of Decimal Numbers
Page 273 ⑨

⑬~⑱ 6th Grade

12 Calculate the following.

① $96 - 72 \div 8$　　② $(4.5 - 2) \div 0.125$

③ $\frac{7}{3} \div 28 \times 0.8$　　④ $2 - \frac{4}{5} \times \frac{3}{8} \div 7.2$

① 4th Grade

② 5th Grade

③④ 6th Grade

13 Think of ways to make the calculation simpler, and then calculate.

Use the properties of operations.

① $7 \times 25 \times 4$　　② $1.25 \times 7.2 \times 8$

③ $2.4 \times 9.3 - 7.3 \times 2.4$　　④ $\left(\frac{6}{5} - \frac{3}{4}\right) \times 400$

⑤ 3×998　　⑥ 1.01×23

①⑤⑥ 4th Grade

②③ 5th Grade

④ 6th Grade

14 Relationships between quantities are described in the text below. Express them as math sentences with x and y.

6th Grade

① When you bought x bottles of juice that cost 150 yen per bottle, the total cost was 750 yen.

② A building that is 1.5 times as high as a building that is x m tall is y m tall.

4 Properties and Processing of Numbers

4

Pick a number and give three hints to your classmates so that they can guess it.

〈Hint ①〉
The number is greater than or equal to 100, and it is less than 120.

〈Hint ③〉
The number is a multiple of 8.

〈Hint ②〉
Rounding the numeral in the lowest place makes the number 100.

1 Based on the three hints above, guess the number Shiho picked.

〈Hint ①〉
The number is less than or equal to 20.

〈Hint ③〉

〈Hint ②〉
The number has four factors.

2 The number Kota picked is 15. What property or processing of numbers should you mention in hint ③ so that his classmates can narrow down possible answers to 15?

If he mentions whether the number is odd or even...

3 Make three hints with words that describe some properties or processing of numbers like the ones listed on the right. Give these hints to your friends and quiz each other.

"Even number," "odd number," "multiple," "factor," "rounding," "greater than or equal to," "less than or equal to," "less than"

Paying attention to the properties of numbers, we have divided numbers into groups and solved problems.

We found it useful to pay attention to the approximate size of a number and round the number or estimate the result of calculation.

☐ **15** Are these numbers even or odd?

① 26　② 43　③ 187　④ 6,590

5th Grade

☐ **16** Find the least common multiple of the numbers in each ( ).

① (2, 8)　② (3, 5)　③ (8, 10)

5th Grade

☐ **17** Find the greatest common factor of the numbers in each ( ).

① (14, 21)　② (16, 32)　③ (18, 24)

5th Grade

4th Grade
Approximate Numbers
Page 272 ⑤

☐ **18** Round the numbers to the place indicated in the ( ).

① 24,351 (thousands place)　② 97,320 (ten thousands place)

③ 403,820 (ten thousands place)　④ 859,542 (hundred thousands place)

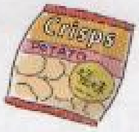
☐ **19** Describe the range of numbers that will be rounded to 48,000 when rounded to the nearest thousand using "_or greater" and "less than."

4th Grade

☐ **20** When buying the three items below, Ami and Kota thought in the following ways. If you think in the same way as they did, how can you calculate the total cost?
From Ⓐ to Ⓓ, pick math sentences that match Ami's and Kota's ideas separately.

4th Grade


137yen



88yen


263yen


Ami


Kota

Ⓐ 130 + 80 + 260　Ⓑ 140 + 90 + 260

Ⓒ 137 + 88 + 263　Ⓓ 140 + 90 + 270

Review Your Sense about Numbers and Calculation

When we studied numbers and calculations, we sometimes focused on the structures and groups of numbers to express numbers and think how to calculate.
Let's look back on how we have expanded and deepened our study.

Focus on the Structure of Numbers and Think about How to Express Numbers

Whole and Decimal Numbers

1st through 5th Grades

4	2 .	1	9	5
Tens place	Ones place	Tenths place	Hundredths place	Thousandths place

- 42.195 is made of four 10's, two 1's, one 0.1, nine 0.01's, and five 0.001's.
- Each place holds a numeral between 0 and 9. (When a place value reaches 10, 1 is regrouped to the next higher place.)

Fractions

2nd through 5th Grades

$\frac{2}{3}$ is:

- Made of two $\frac{1}{3}$'s
- Equal to $\frac{4}{6}, \frac{6}{9}, \dots$

There are many fractions of the same size.

- The quotient of 2 ÷ 3 $2 \div 3 = \frac{2}{3}$

Focus on the Groups of Numbers and Think about How to Calculate

Addition

1st Grade ▶▶ **2nd Grade** ▶▶ **3rd Grade**

1st Grade:
$3+2=5$
$30+20=50$
3 + 2 if you use 10 as the base amount

2nd Grade:
$300+200=500$
3 + 2 if you use 100 as the base amount

3rd Grade:
$0.3+0.2=0.5$
3 + 2 if you use 0.1 as the base amount

$\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$
3 + 2 if you use $\frac{1}{7}$ as the base amount

Multiplication

2nd Grade ▶▶ **3rd Grade** ▶▶ **4th Grade** ▶▶ **6th Grade**

2nd Grade:
$3 \times 2=6$

3rd Grade:
$300 \times 2=600$
3 × 2 if you use 100 as the base amount

4th Grade:
$0.3 \times 2=0.6$
3 × 2 if you use 0.1 as the base amount

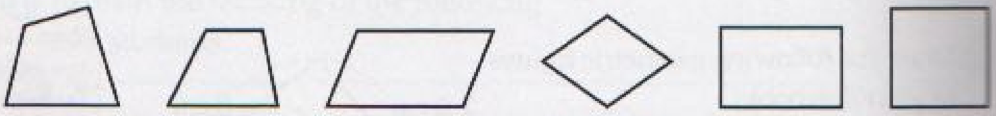
6th Grade:
$\frac{3}{7} \times 2=\frac{6}{7}$
3 × 2 if you use $\frac{1}{7}$ as the base amount

2 Geometric Figures

Answers→ Page 266

1 the properties of geometric figures

1 Let's take a new look at the quadrilaterals we have studied.



① What is the measure of each angle of a rectangle?

② Is there any other quadrilateral that always has the same property that you answered in ①?


Riku

How about the square?

③ How are the two pairs of opposite sides of a parallelogram related?

④ Is there any other quadrilateral that always has the same property that you answered in ③?


Misaki

How about the rhombus?

When we investigated the properties of geometric figures, we focused on their sides and angles.


Shiho

2nd through 4th Grades
Various Quadrilaterals/
Diagonal
Page 274

1 For each geometric figure, if the characteristic is always true, put a ○ in the cell.

Name Characteristic	Quadrilateral	Trapezoid	Parallelogram	Rhombus	Rectangle	Square
Two pairs of opposite sides are parallel						
All four sides have equal lengths						
All four angles are right angles						
The lengths of the two diagonals are equal						
The two diagonals are perpendicular to each other						

2 Which of the following figures are line symmetric? Point symmetric? Answer using the appropriate letters.

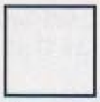
6th Grade

Ⓐ

Isosceles triangle

Ⓑ

Parallelogram

Ⓒ

Square

Ⓓ

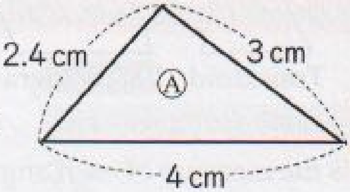
Rhombus

3 Draw the following geometric figures in your notebook:

① A triangle congruent to Ⓐ

② A 2 times enlarged drawing of Ⓐ

③ A $\frac{1}{2}$ reduced drawing of Ⓐ



① 5th Grade
How to Draw Congruent Triangles
Page 274 ⑲

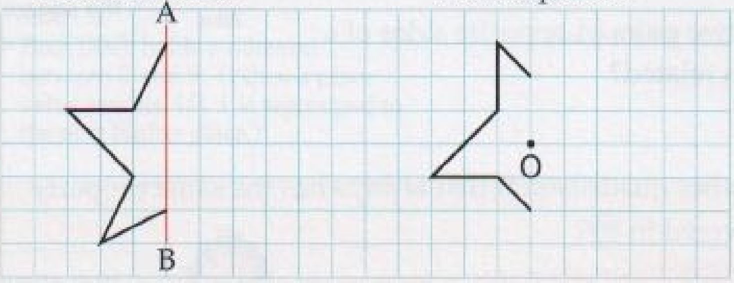
②③ 6th Grade

4 Complete the following symmetric figures:

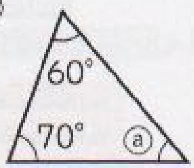
6th Grade

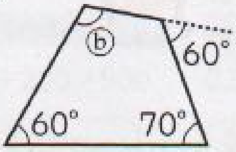
① A geometric figure that is line symmetric around axis AB

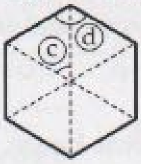
② A geometric figure that is point symmetric around point O



5 What are the measures of angles ⓐ through ⓓ of the geometric figures below?

①


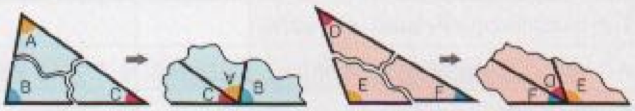
②


③ Regular hexagon


5th Grade
Sum of the Angles of a Triangle or a Quadrilateral
Page 275 ㉑

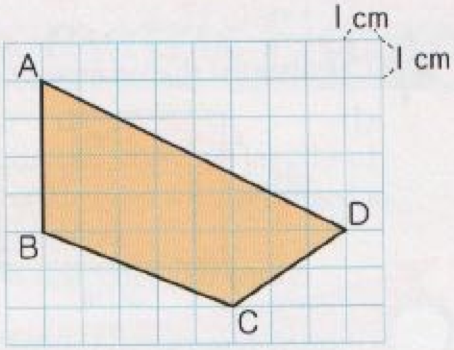
From Elementary School Math ≫ to Junior High School Math

Can you explain why the sum of the angles of a triangle is 180°?



In elementary school, we discovered that the sum of the angles of a triangle is always 180° when we examined some triangles by putting the three angles of a triangle around a point or actually measuring the three angles.
In junior high school, you will also learn how to properly explain the reason why the sum is always 180° based on what you know.

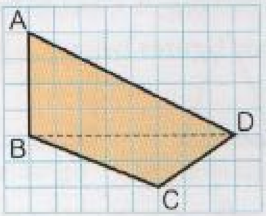
2 Area and Volume



2

Find the area of quadrilateral ABCD on the right.

① Explain the thinking of the following two students.

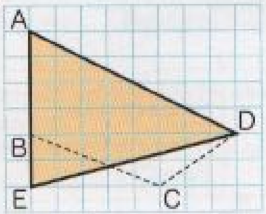


Divide the quadrilateral into triangles ABD and BCD.

triangle ABD　8×4÷2 = ☐ (cm²)

triangle BCD　8×2÷2 = ☐ (cm²)

Answer ☐ cm²



Move vertex C of triangle BCD to E without changing the area of the triangle to make triangle AED.

triangle AED　6×8÷2 = ☐ (cm²)

Answer ☐ cm²

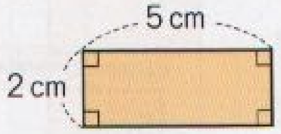
When we found the area of geometric figures, we focused on the characteristics of their shapes. We thought about a geometric figure as the combination of familiar geometric figures or changed the shape of the geometric figure.

③－⑥ 5th Grade
The Formulas for Calculating Area of Triangles and Quadrilaterals
Page 275 ㉓

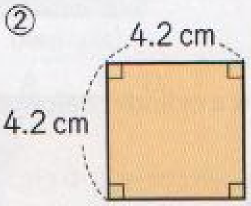
①② 4th Grade

☐☐ 6 Calculate the area of the following figures.

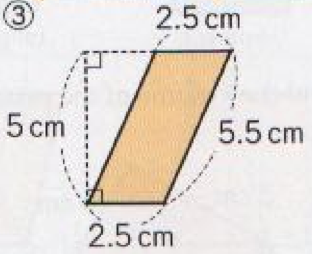
①



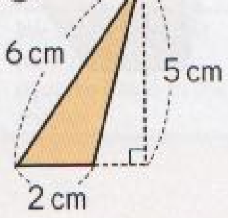
②



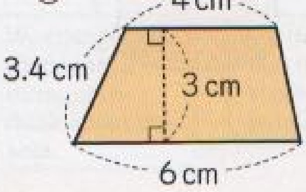
③



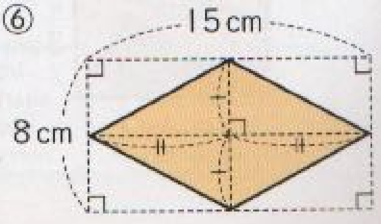
④



⑤



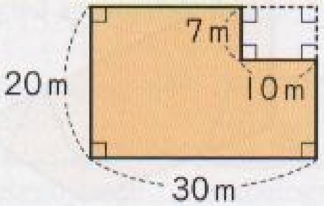
⑥



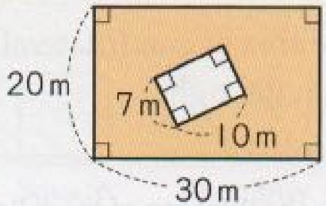
7 Calculate the shaded areas of the following geometric figures.

4th Grade

①



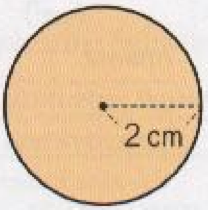
②



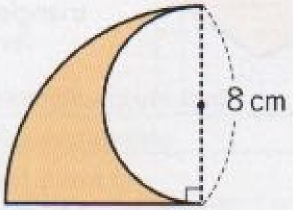
8 Calculate the area of the shaded parts in the geometric figures below. Find the length around the parts, too.

5th and 6th Grades

①



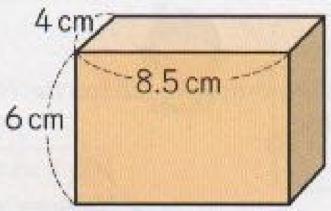
②



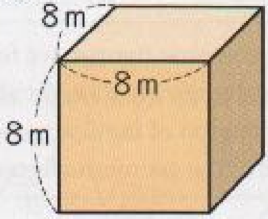
9 Find the volume of the solid figures below.

5th and 6th Grades
The Formulas for Calculating Volume of Cubes and Cuboids
Page 275 ㉕

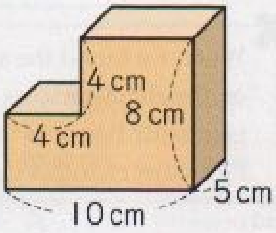
①



②



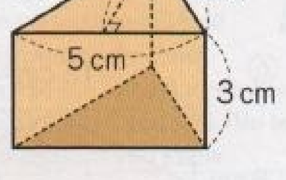
③



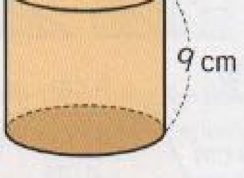
10 Find the volume of the prisms and a cylinder below.

6th Grade

①



②



③

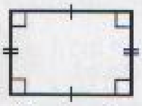
Review Your Sense about Geometric Figures

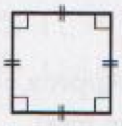
When we studied geometric figures, we sometimes focused on lengths of their sides, measures of their angles, and how their sides were arranged to think about their properties and how to find their area.
Let's look back on how we have expanded and deepened our study.

Focus on the Sides and Angles of Geometric Figures and Think about the Properties and Relationships These Figures Have

2nd Grade ▶▶ **3rd Grade** ▶▶ **4th Grade** ▶▶ **5th Grade** ▶▶ **6th Grade**

2nd Grade

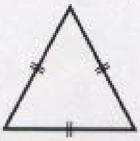

Rectangle


Square

We focused on the number and lengths of sides and right angles.

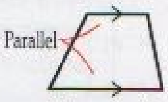
3rd Grade


Isosceles triangle

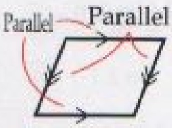

Equilateral triangle

We focused on the lengths of sides.

4th Grade



Trapezoid

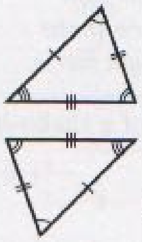


Parallelogram

We focused on how the sides were arranged.

5th Grade

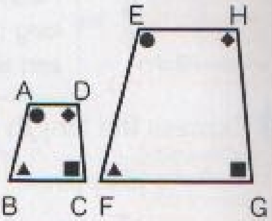
Congruent Shapes



We focused on the lengths and the measures of corresponding sides and corresponding angles.

6th Grade

Enlarged and Reduced Drawings



AB : EF = 1 : 2
BC : FG = 1 : 2

We focused on the ratio of the lengths of corresponding sides and the measures of corresponding angles.

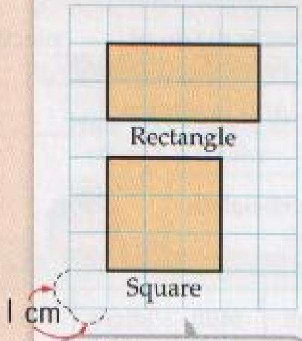
When we thought about the properties of solid figures, we focused on the lengths of their edges and the shapes of their faces.


Ami

Focus on the Characteristics of Shapes and Think about How to Calculate Their Areas

4th Grade ▶▶ **5th Grade**

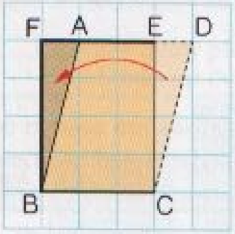
4th Grade



We thought about how many 1 cm² squares fitted in the shapes.

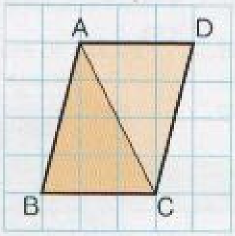
5th Grade

Parallelogram
(Example)



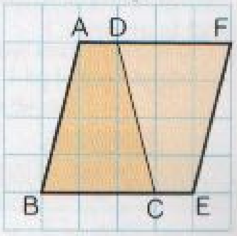
We changed the shape into a rectangle to think about the area.

Triangle
(Example)



We thought that the shape was half of a parallelogram.

Trapezoid
(Example)



We thought that the shape was half of a parallelogram.

3 Measurement

1 Comparing Quantities and Units

1 The Japanese flute on the right is called *shaku-hachi*. Let's think about its length.

Shaku-hachi takes its name from its length, one *shaku* and *hachi* (= eight) *sun*. *Shaku* and *sun* are old Japanese units of length.

Shiho

I wonder how long one *shaku* and eight *sun* is.

1 *sun* = approx. 3.03 cm
1 *shaku* = approx. 30.3 cm (10 *sun*).

① Express the length of a *shaku-hachi* in cm.

1 *shaku* is approx. ☐ cm.

8 *sun*, made of 8 pieces of 1 *sun*, is approx. ☐ cm.

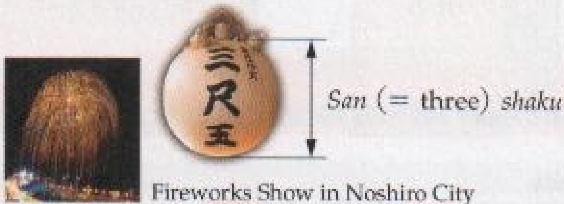
Approx. ☐ cm in total.

It was the same in those days as it is now that you can express length by telling how many pieces of a base amount the length is made of.

Haruto

1 There are other objects named after old Japanese units of length.

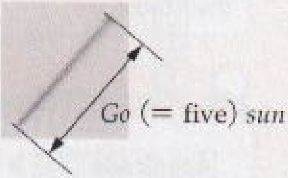
① *Sanshaku-dama* fireworks



Fireworks Show in Noshiro City

San shaku is made of ☐ pieces of 1 ☐ and is approx. ☐ cm.

② *Gosun-kugi* nails



Go sun is made of ☐ pieces of 1 ☐ and is approx. ☐ cm.

These prefixes are used in combination with m (meters) and g (grams).

Riku

Prefixes in Units at a Glance

Prefix	Giga G	Mega M	Kilo k	Hecto h	Deca da		Deci d	Centi c	Milli m	Micro μ	Nano n
Meaning	1 billion times	1 million times	1 thousand times	1 hundred times	10 times	1	$\frac{1}{10}$ times	$\frac{1}{100}$ times	$\frac{1}{1,000}$ times	$\frac{1}{1,000,000}$ times	$\frac{1}{1,000,000,000}$ times

2 Write the appropriate unit in each ( ).

① Length of a ballpoint pen …… 15 ( )

② Width of the blackboard in the classroom ……5 ( )

③ Area of Hokkaido island …… Approx. 80,000 ( )

④ Area of a page of your math textbook …… 480 ( )

⑤ Weight of an egg …… 60 ( )

⑥ Weight an elevator can hold …… 450 ( )

①② 2nd Grade

③④ 4nd Grade

⑤⑥ 3nd Grade

3 Write the appropriate number in each ☐.

① 1 cm = ☐ mm ② 1 m = ☐ cm

③ 1 m^2 = ☐ cm^2. ④ 1 a = ☐ m^2

⑤ 1 ha = ☐ a ⑥ 1 km^2 = ☐ m^2

⑦ 1 L = ☐ cm^3 ⑧ 1 m^3 = ☐ cm^3

⑨ 1 kg = ☐ g ⑩ 1 t = ☐ kg

①② 2nd Grade

③ – ④ 4nd Grade

⑦⑧ 5nd Grade

⑨⑩ 3nd Grade

Review Your Sense about **Measurement**

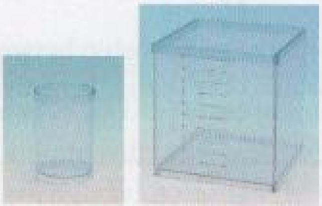
When we studied measurement, we sometimes focused on the quantities and base amounts of objects to express and compare quantities.
Let's look back on how we have expanded and deepened our study.

Focus on the Quantities and Base Amounts of Objects and Think about How to Express and Compare Quantities

2nd Grade (Length) ▶▶ **2nd Grade (Amount of water)** ▶▶ **3rd Grade (Weight)**

How many pieces of 1 cm or 1 m fill the length?

How many pieces of 1 dL or 1 L take up the space?



How many pieces of 1 g or 1 kg is the weight made of?

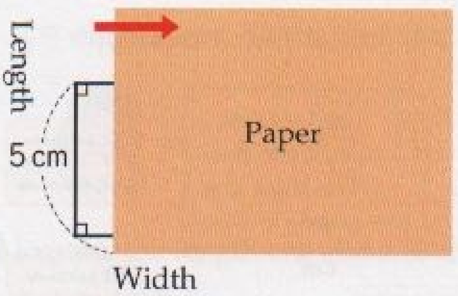
We also expressed area and volume by telling how many 1 cm^2 or 1 cm^3 it was made of.

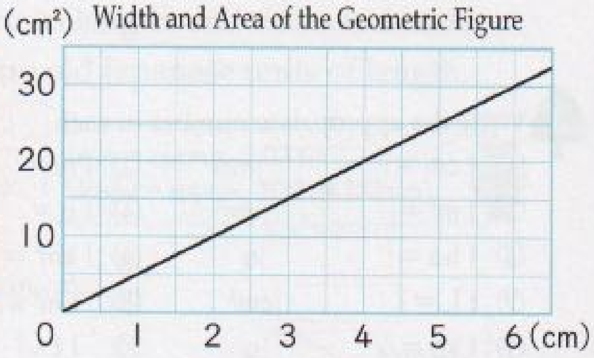
4 Changes and Relationships

Answers→ Page 267

1 Changes and Proportional and Inversely Proportional Relationships

1 Think about a case where you place a piece of paper on top of a certain geometric figure and slide the paper in the direction as indicated by the arrow in the diagram below.

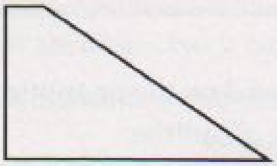




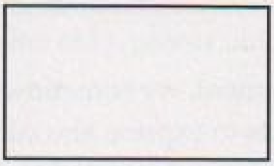
① As you slide the paper, the visible part of the geometric figure changes. The graph above shows the relationship between the width and area of the visible part.

- Select the geometric figure hidden under the paper.

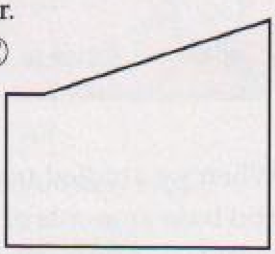
Ⓐ



Ⓑ



Ⓒ



② In problem **1**, what quantities change?
What quantity does not change?

We were able to solve many problems by identifying two quantities that changed in relationship to each other and investigating how they changed.

Misaki

1 Complete the table below with the relationship between the width of the visible part and the length around the figure. When you shifted the paper in problem **1**.

4th Grade

Width (cm)	1	2	3	4	5	6
Length around the figure (cm)	12	14				

2 Using sticks of the same length, we will make and arrange triangles side by side as shown below.

4th Grade

① Let the number of triangles be x and the number of sticks be y, and complete the table below with the relationship between x and y.

Number of triangles x	1	2	3	4	5	6
Number of sticks y	3	5				

② When 10 triangles are arranged, how many sticks are used?

3 Tables Ⓐ and Ⓑ represent the relationship between x and y.

6th Grade

Ⓐ Length and Width of Rectangles with an Area of 20 cm²

Length x(cm)	20	10	5	4	2	1
Width y(cm)	1	2	4	5	10	20

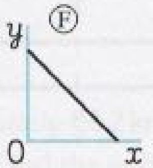
Ⓑ Time and Distance of a Car Traveling at 40 km per hour

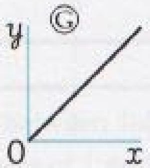
Time x(hours)	1	2	3	4	5	6
Distance y(km)	40	80	120	160	200	240

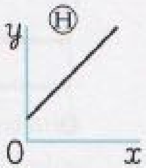
① Which table represents a proportional relationship between x and y? Which table represents an inversely proportional relationship between x and y?

② For Ⓐ and Ⓑ, express y as a math sentence with x.

③ Among graphs Ⓕ to Ⓗ on the right, select the one that represent the relationship between x and y in Ⓑ.







4 In which of the following situations, is y proportional to x? In wich situations, is y inversely proportional to x? Use tables to investigate.

6th Grade

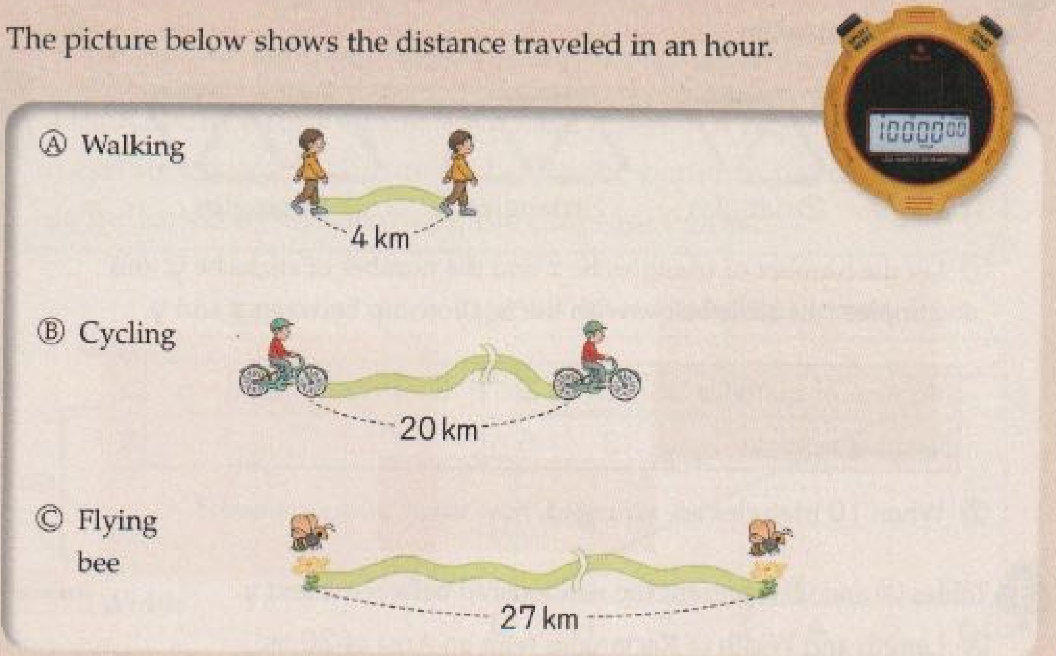
Ⓐ The area, y cm², of a square with x cm sides

Ⓑ The area, y cm², of a triangle with the base of 4 cm and the height of x cm

Ⓒ The time, y minutes, taken to walk 20 km at the speed of x km per minute

2 Speed and Per Unit Quantity

The picture below shows the distance traveled in an hour.



2 Look at the picture above and review what speed means.

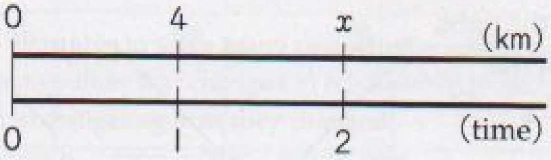
5th Grade
Speed
Page 273 ⑪

① Among Ⓐ, Ⓑ, and Ⓒ above, who/which travels the fastest?

② Explain the reason of your answer to ①.

How far each of them travels in 1 hour?

③ If the boy keeps walking for 2 hours, how many km does he travel?



About the rider and the bee, answer the same question as above.

Speed can be expressed by the distance traveled in a unit amount of time.

Speed is a quantity made by combination of two quantities: time and distance.

5 A household uses 2,800 L of tap water a week.

5th Grade

① How many L of tap water does the household use on average per day?

② How many L of tap water does the household use a year?
Assume the year has 365 days.

6 The table below lists the areas of rooms A and B and the number of people in each of the rooms.

5th Grade

Which room is more crowded?

Room Area and Number of People

	Area (m^2)	Number of People
A	30	12
B	48	20

7 Answer the following questions:

5th Grade

① It took you 4 minutes to run 500 m. How many m per minute did you run?

② It took a horse 30 seconds to run 420 m. How many m per second did it run?

③ A bicycle is traveling at 200 m per minute. How many m does it travel in 30 minutes?

④ A car is traveling at 40 km per hour. How many km does it travel in 1 hours 15 minutes?

⑤ A man is walking at 60 m per minute. How long does it take him to walk 1.8 km?

8 The bullet train, Nozomi, goes approximately 552 km between Tokyo and Shin-Osaka in 2 hours and 25 minutes. Find the answers of 1 and 2 below, rounded to the nearest whole number.

5th Grade

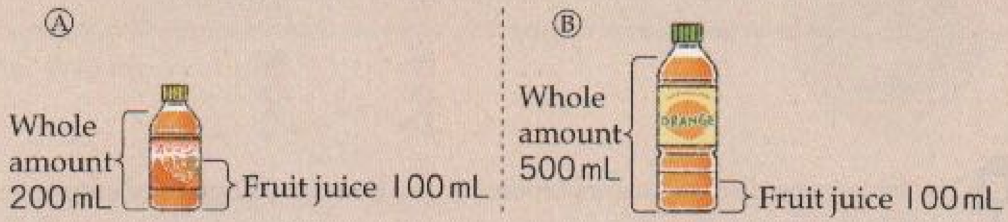
① What is the speed in km per hour?

② What is the speed in m per second?

1 hour = 60 minutes = 3,600 seconds
1 km = 1,000 m

3 Rates

There are two types of beverage that contains fruit juice.



3

Which beverage contains a higher rate of fruit juice, Ⓐ or Ⓑ?

5th Grade
Rates
Page 273 ⑭


Ami

Both types of beverage contain 100 mL of fruit juice.

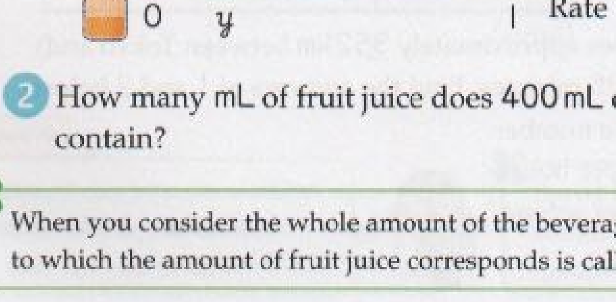

Haruto

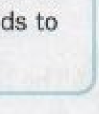
The whole amounts are different...


Misaki

Exactly half of Ⓐ is fruit juice.

① If you consider the whole amount of Ⓐ as 1, what does the amount of fruit juice in Ⓐ correspond to? If you consider the whole amount of Ⓑ as 1, what does the amount of fruit juice in Ⓑ correspond to?




Kota

If you consider 200 mL, the whole amount of Ⓐ, as 1, 100 mL is half of it and corresponds to □.

② How many mL of fruit juice does 400 mL of beverage Ⓐ above contain?

When you consider the whole amount of the beverage as 1, the number to which the amount of fruit juice corresponds is called the rate.


Riku

9 Change the following rates from decimal numbers to percentages.

5th Grade

① 0.01 ② 0.1 ③ 1.05 ④ 1.5

10 Write the appropriate number in each □.

5th Grade

① 7% of 30L is □ L.

② 2kg is □ % of 5kg.

③ 25% of □ m is 1.5m.

④ If you buy an item that costs 2,000 yen with a 20% discount, that will be □ yen.

11 A museum had 750 visitors last week and 900 visitors this week.

5th Grade

① What % is this week's number of visitors compared to last week's?

② Children accounted for 75% of this week's visitors.
How many children visited the museum this week?

③ The number of visitors for this week is equal to 30% of the number of visitors for last month.
How many visitors did the museum have last month?

12 Answer the following questions:

6th Grade

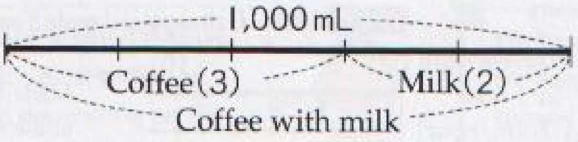
① Find the value of ratio 2 : 7.

② Simplify ratio 15 : 9.

③ Find the value of x when 28 : 4 = 35 : x.

13 We are going to mix coffee and milk in the ratio of 3 : 2 to make 1,000 mL of coffee with milk. How many mL of milk do you need?

6th Grade



Review Your Sense about Changes and Relationships

When we studied changes and relationships, we sometimes focused on two quantities that changed in relationship to each other and investigated how these quantities changed and were related to each other to solve problems. Let's look back on how we have expanded and deepened our study.

Focus on Two Quantities that Change in Relationship to Each Other and Investigate How They Change

4th Grade Investigating Changes ▶▶ 5th and 6th Grades Proportional Relationship

Tables

+1 +1 +1 +1

○	1	2	3	4	5
□	11	10	9	8	7

−1 −1 −1 −1

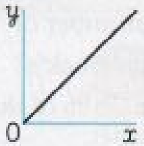
Using tables, graphs, and math sentences, we were able to show changes clearly.

×2 ×3

x	1	2	3	4	5	6
y	4	8	12	16	20	24

×2 ×3

Graph



The graph of 2 quantities that are in a proportional relationship is a line that passes through 0.

Math Sentence

$y = 4 \times x$

When $x = 1$, the value of y is equal to 4, the constant.

Focus on the Relationship between Quantities and Investigate It

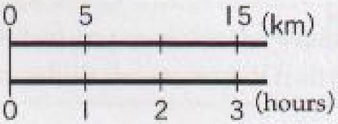
5th Grade Speed

Focus on time and distance, and express speed as the distance traveled per hour or minute.

Boy A Travels 8 km in 2 hours

Travels 4 km in 1 hour (4 km/hour)

Boy B Travels 15 km in 3 hours



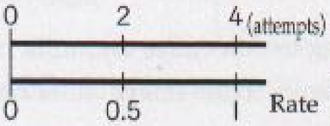
Travels 5 km in 1 hour (5 km/hour)

B travels faster than A.

5th Grade Rates

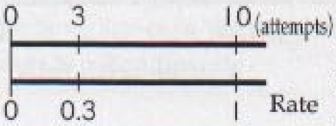
Focus on a base amount (total attempts) and a quantity being compared (baskets made). Consider the base amount as 1, and express what the quantity being compared corresponds to.

Boy C Made 4 attempts and made 2 baskets



Made baskets at the rate of 0.5

Boy D Made 10 attempts and made 3 baskets



Made baskets at the rate of 0.3

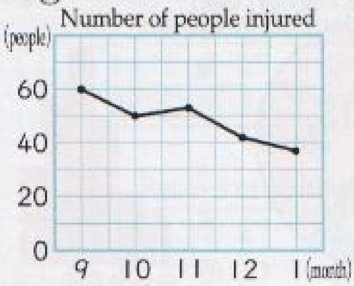
C was better at making baskets than D.

5 Utilization of Data

Answers→ Page 267

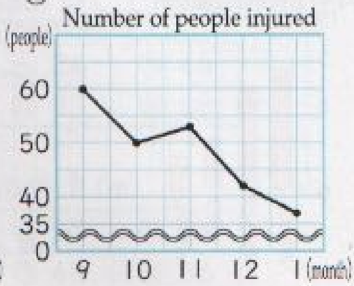
The school health committee has been working on campaigns to reduce injuries since last September.
To summarize their results, we counted the monthly number of injured students who visited the nurse's office between September and January, and represented the data in graphs.

Ⓐ



Ⓑ

Ⓒ



1 Look at the graph above and answer the following questions.

1. Which graph shows the monthly number of injured people the most clearly, Ⓐ, Ⓑ, or Ⓒ?
2. What is the difference in the number of injured people between October and November?
3. Kota looks at graph Ⓐ and says as follows.
   What do you think about what Kota says?
   Discuss with your classmates.

Kota

The number of injured students who visited the nurse's office did not change much between September and January.

Ami

If you compare September with January in the number of visitors...

Riku

Graph Ⓒ is the same as graph Ⓐ except each increment represents 5 people. If you look at how the number of people changed in Ⓒ...

We have learned that we should select a type of graph depending on the purpose of use.

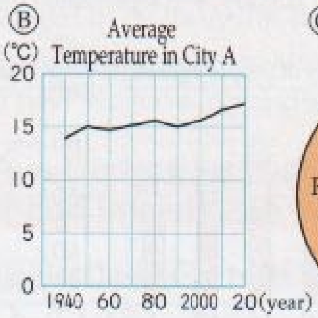
Shiho

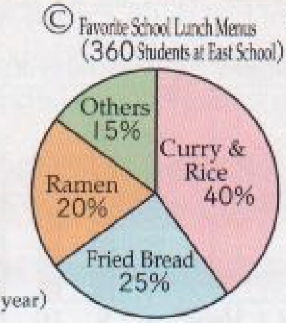
We have learned that when we examine a graph we should pay attention not only to its appearance but also to the size of each increment.

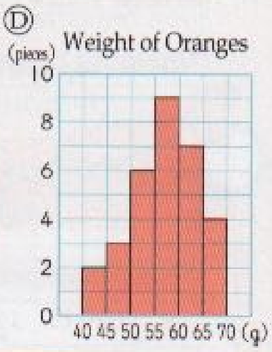
Misaki

1 We have studied the following types of graph:

3rd through 6th Grades







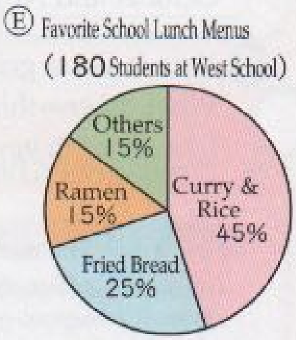
① What types of graphs are Ⓐ through Ⓓ ?

② Which graph from Ⓐ through Ⓓ is appropriate for each purpose in (1) through (4)?

(1) To investigate how quantities change.
(2) To compare quantities.
(3) To express the rates of parts to the whole.
(4) To examine the way the data are spread.

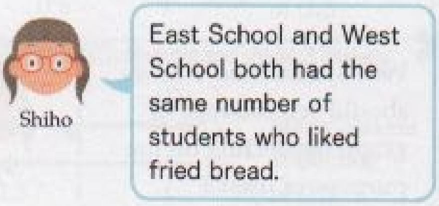
③ West School, too, did a survey about favorite school lunch menus. The results are as shown on the right.

Based on graphs Ⓒ and Ⓔ, select the correct statement(s) from below.



(1) At both East School and West School, curry & rice is the most popular.
(2) West School serves more curry & rice per student than East School does.

④ Shiho looked at graphs Ⓒ and Ⓔ and said as follows.
Is what she said correct?
Also, explain why or why not.



2 The tables and graphs below show records of softball throwing by students from Classrooms A and B. Based on these records, we are going to think which classroom was better at softball throwing.

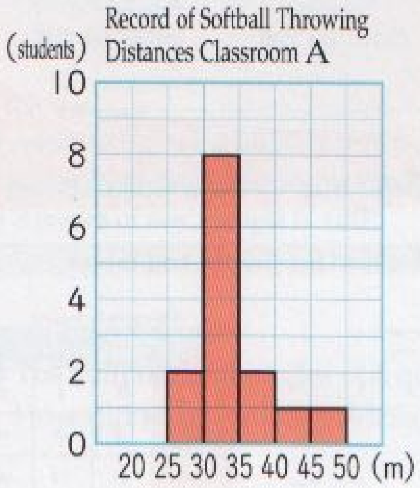
6th Grade

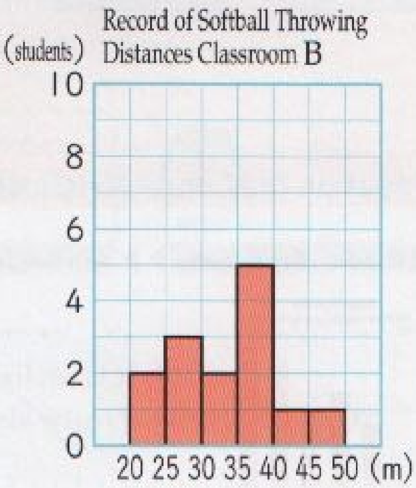
Record of Softball Throwing Distances Classroom A (m)

① 34	② 25	③ 40	④ 31	⑤ 32	⑥ 34	⑦ 45
⑧ 28	⑨ 36	⑩ 34	⑪ 34	⑫ 32	⑬ 33	⑭ 38

Record of Softball Throwing Distances Classroom B (m)

① 34	② 27	③ 42	④ 36	⑤ 23	⑥ 27	⑦ 35
⑧ 26	⑨ 37	⑩ 24	⑪ 31	⑫ 36	⑬ 48	⑭ 36


Record of Softball Throwing Distances Classroom A


Record of Softball Throwing Distances Classroom B

I wonder if I want to find the means and compare them.

Maybe I should find the modes and compare them.

① Find the mean for each of Classrooms A and B.

② Can we say that the records will always gather around the mean?

③ Find the modes for each of Classrooms A and B.

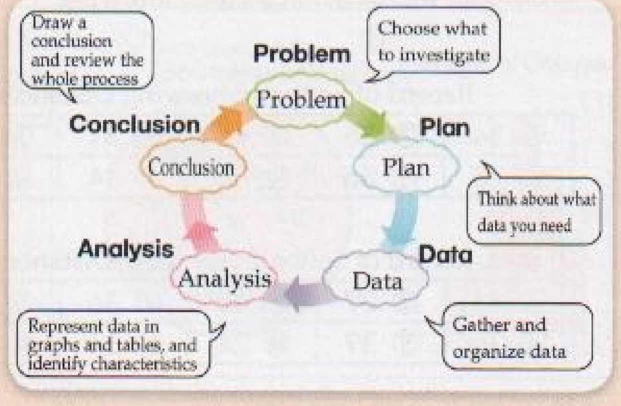
④ Find the median for each of Classrooms A and B.

If you compare Classrooms A and B in representative values, which can you say performed better?

⑤ For Classroom A, which class has the largest frequency? For Classroom B, which class has the largest frequency?

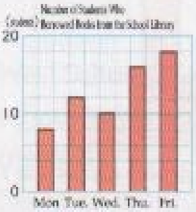
Review Your Sense about Utilization of Data

When we studied utilization of data, we sometimes solved problems by collecting, organizing, and representing data depending on the purpose of use, as shown on the right.
Let's look back on how we have expanded and deepened our study.



Focus on the Characteristics of Data and Draw a Conclusion

3rd Grade Bar Graphs ▶▶ 4th Grade Broken Line Graphs and Tables



We used bar graphs to show which category had more or less at a glance.

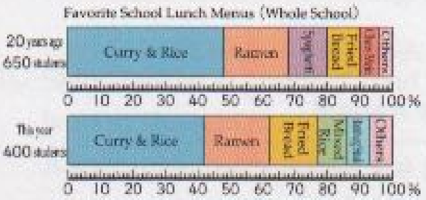
We used broken line graphs to examine the characteristics of changes.

Library Use by Students

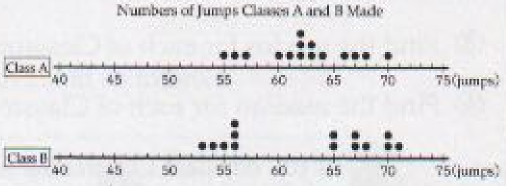
		This Week		Total
		Borrowed a book	Did not borrowed a book	
Last Week	Borrowed a book	8	4	12
	Did not borrowed a book	3	15	18
Total		11	19	30

We organized data into a table.

5th Grade Percentage Bar Graphs and Pie Charts ▶▶ 6th Grade Line plots and Histograms



We used percentage bar graphs and pie charts to show the rates of different parts to the whole and to compare the rates of different parts.



You can easily observe how the values are spread out.

Based on the modes, you can say that Classroom A made more jumps.

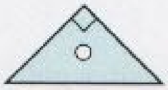
If you combine graphs, a bar graph and a broken line graph, for example, you can relate two quantities to each other and make observations.

6 Methods of Reasoning and Expressions

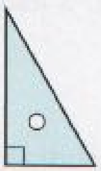
3 Think Logically

1 Review what we learned about the sum of the angles of geometric figures.

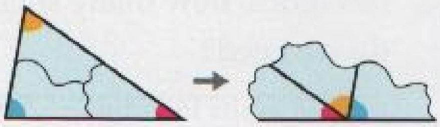
① The sum of the 3 angles of a triangle is 180°.
How did we examine triangles when we found this fact?



Set squares

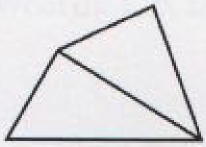


Set squares



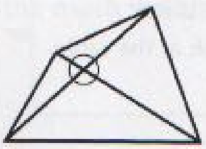
We examined the measures of the 3 angles of several triangles, and found out that the sum of the 3 angles of any triangle is 180°.

② The sum of the 4 angles of a quadrilateral is 360°.
How did we examine quadrilaterals when we found this fact?



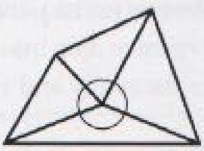
Split the quadrilateral into 2 triangles by a diagonal.

$180\times2=360$



Split the quadrilateral into 4 triangles by diagonals.

$180\times4-360=360$

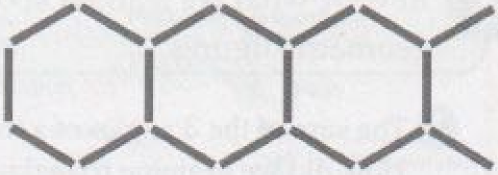


Place a point in the quadrilateral and split the quadrilateral into 4 triangles.

$180\times4-360=360$

By using the familiar fact that the sum of the 3 angles of a triangle is 180°, we found that the sum of the 4 angles of a quadrilateral is 360°.

2 Using sticks of the same length, we will make and arrange regular hexagons side by side as shown on the right.
When we make 30 regular hexagons, how many sticks do we need?



Kota

I can tell if we actually draw 30 regular hexagons...

Ami

I wonder if we can find patterns while making fewer regular hexagons.

① When you wanted to find a pattern like Ami did, what kinds of ways were there to find one?

+1 +1 +1 +1 +1

Number of regular hexagons x	1	2	3	4	5	6
Number of sticks y	6	11	16	21	26	31

+5 +5 +5 +5 +5

We can find a pattern more easily if we organize data into a table and look at the table horizontally and vertically.

Misaki

② Shiho looked at the table above and found the number of sticks necessary to make 30 regular hexagons with the following math sentence:

Shiho

$6+5\times(30-1)=151$

Answer 151 sticks

What do 6, 5, and $(30-1)$ in the math sentence written by Shiho represent?

Haruto

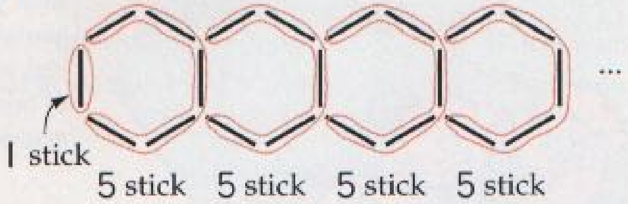
6 is the number of sticks for the first regular hexagon...

③ Based on Shiho's idea, express the relationship between the number of regular hexagons (x) and the number of sticks (y) in a math sentence.

The numbers 6, 5, and 1 are constant...

④ Using the math sentence you wrote in ③, find the value of y (the number of sticks) when x (the number of regular hexagons) is 101.

⑤ Using a diagram, Kota thought about the number of sticks necessary to make 30 regular hexagons as follows:



$1+5\times30=151$ Answer 151 sticks

What do 1, 5, and 30 in the math sentence written by Kota represent?

⑥ Based on Kota's idea, express the relationship between the number of regular hexagons (x) and the number of sticks (y) in a math sentence.

⑦ Using the math sentence you wrote in ⑥, find the value of y (the number of sticks) when x (the number of regular hexagons) is 200.

Math sentences are not just for finding their answers but can also be used to express your ideas, quantities, and the relationships between quantities. You can call math sences the "language of math."

Mathematical Graduation Trip

At the conclusion of your 6 years of study of elementary school mathematics, let's visit the world of mathematics. Pick one of the four courses you'd like, and let's go.

Junior High School Orientation Course

Page 229

In junior high school, you will further study mathematics based on what you have studied in elementary school. You will get a glimpse of how the world of mathematics will unfold.

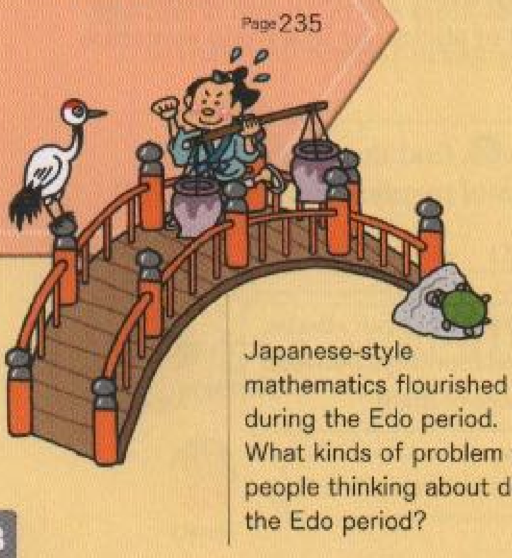
International Course

Page 232

Mathematical thinking is universal, but procedures like algorithms may be different.

Wasan (Historical Japanese Math) Course

Page 235



Japanese-style mathematics flourished during the Edo period. What kinds of problem were people thinking about during the Edo period?

Riddles/Puzzles Course

Page 238

Many riddles and puzzles use ideas from mathematics.

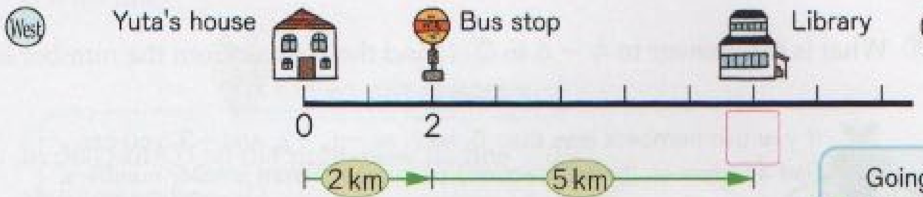
Junior High School Orientation Course

① Numbers less than 0

Advanced Mathematics 1st Grade of Junior High

Set the 0 on a number line to Yuta's house and show other places as shown below.

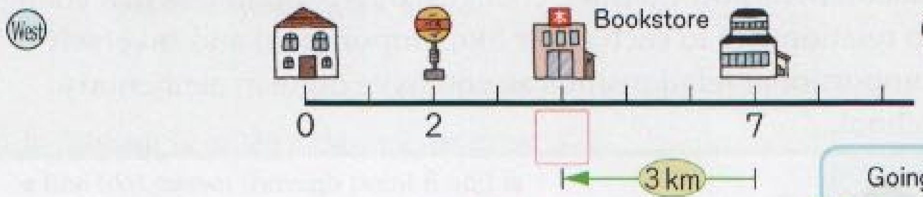
① Yuta got on a bus at the bus stop located 2 km to the east of his house, and went to the library located 5 km to the east of the bus stop.
What is the number that corresponds to the library on the number line?



$2+\square=\square$

Going east is like doing addition.

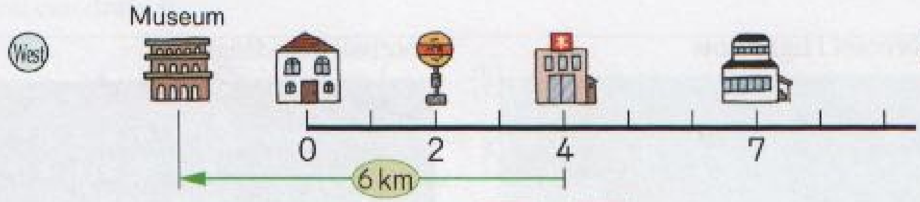
② Next, Yuta went to the bookstore located 3 km to the west of the library.
What is the number that corresponds to the bookstore on the number line?



$7-\square=\square$

Going west is like doing subtraction.

③ Then, Yuta went to the museum located 6 km to the west of the bookstore.
What is the number that corresponds to the museum on the number line?



$4-\square=\square$

The math sentences would be 4 − 6...

Extend the number line to the left from 0...

Can we subtract a number from a smaller one?

Numbers to the left of 0 by 1, 2, 3 ... are represented as −1, −2, −3 ... and read minus 1, minus 2, minus 3...
The number −1 is less than 0 by 1.

④ By how much are −2 and −3 less than 0?

⑤ What is the answer to 4 − 6 in ③? Find the answer from the number line above.

If you use numbers less than 0, such as −1, −2, and −3, you can find answers to the subtractions of numbers from smaller numbers.

Junior High School

In junior high school, you will explore the world of numbers further to the numbers less than 0. You will also think about calculations and examine changes in two quantities that change in relationship to each other like proportional and inversely proportional relationships as you have done in elementary school.

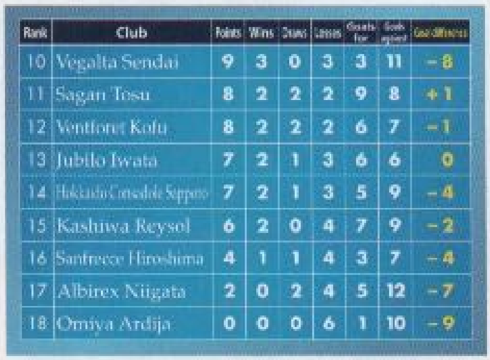
Misaki

I wonder how we can do multiplication or division with numbers less than 0.

⑥ Identify situations around you where numbers less than 0 are used.

Forecast High/Low

Football Standings



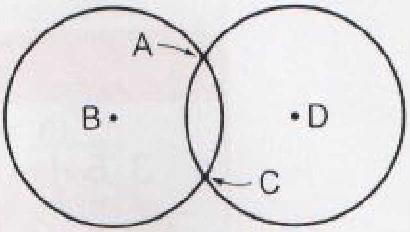
Rank	Club	Points	Wins	Draws	Losses	Goals for	Goals against	Goal difference
10	Vegalta Sendai	9	3	0	3	3	11	−8
11	Sagan Tosu	8	2	2	2	9	8	+1
12	Ventforet Kofu	8	2	2	2	6	7	−1
13	Jubilo Iwata	7	2	1	3	6	6	0
14	Hokkaido Consadole Sapporo	7	2	1	3	5	9	−4
15	Kashiwa Reysol	6	2	0	4	7	9	−2
16	Sanfrecce Hiroshima	4	1	1	4	3	7	−4
17	Albirex Niigata	2	0	2	4	5	12	−7
18	Omiya Ardija	0	0	0	6	1	10	−9

〔Developed by Tokyo Shoseki based on the website of the Japan Professional Football League〕

Advanced Mathematics 1st Grade of Junior High

② Utilization of the Properties of Geometric Figures

There are two circles of equal radius intersecting each other as shown on the right.

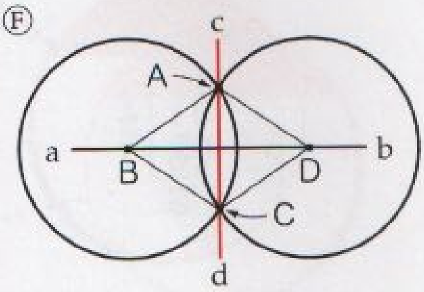


① What kind of quadrilateral will you make if you connect the four points A, B, C, and D with line?

Explain your reasoning.

② In diagram Ⓕ on the right, how do line ab intersect line cd?

Ⓕ



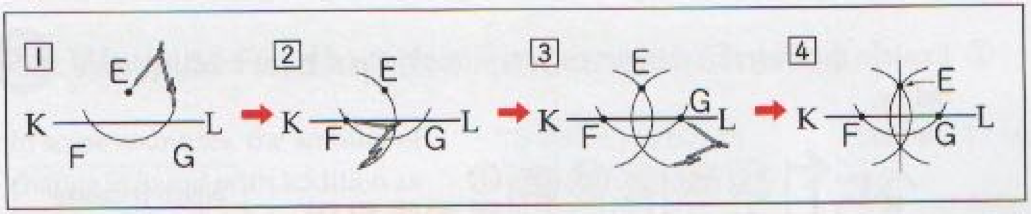
Shiho

Quadrilateral ABCD is line symmetric.
If you consider line ab as the axis of symmetry, line cd is...

③ In diagram Ⓖ on the right, you can draw a line that passes through point E and is perpendicular to line KL as shown below with a pair of compasses and a ruler.

Ⓖ

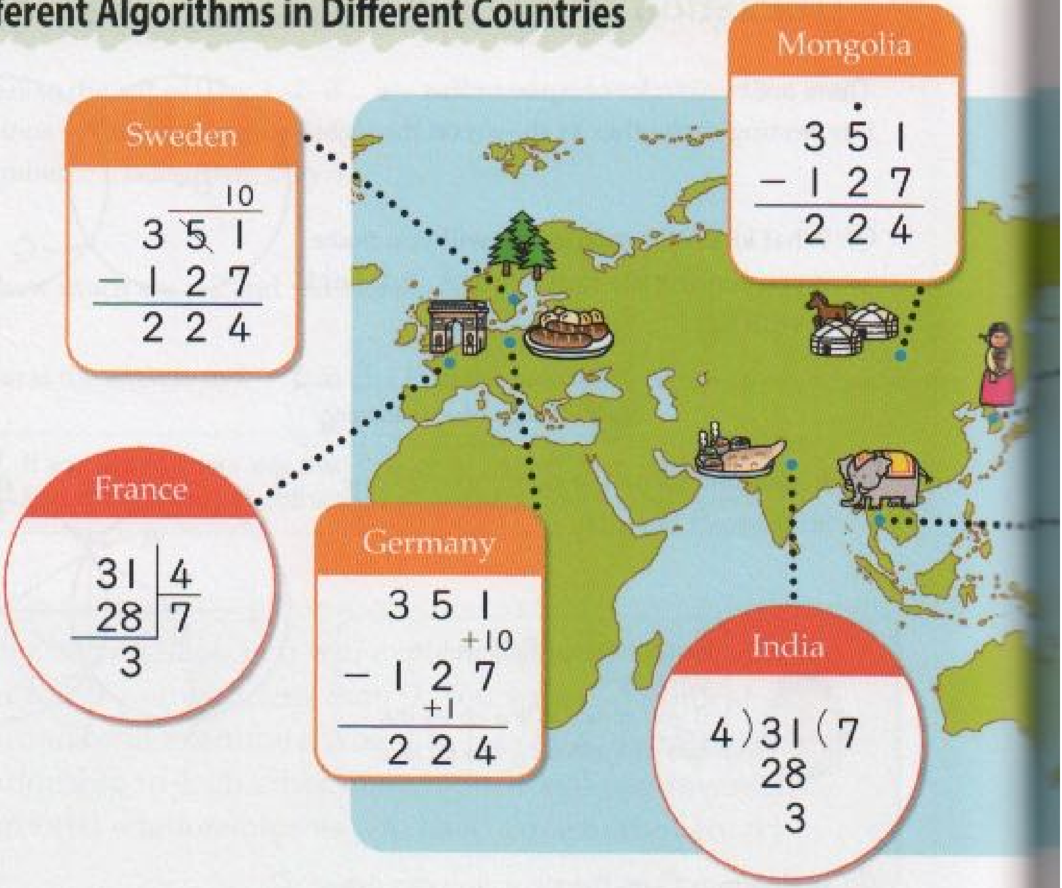
Draw the line yourself. Also, based on ① and ②, think about the reason why you can draw it.



Junior High School

In junior high school, you will draw and examine geometric figures based on the properties of geometric figures you studied in elementary school.

1 Different Algorithms in Different Countries

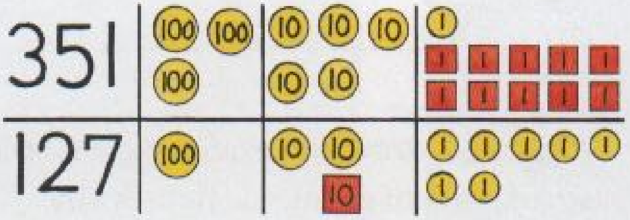


① Look at the subtraction algorithms shown above, think about how they work, and explain.

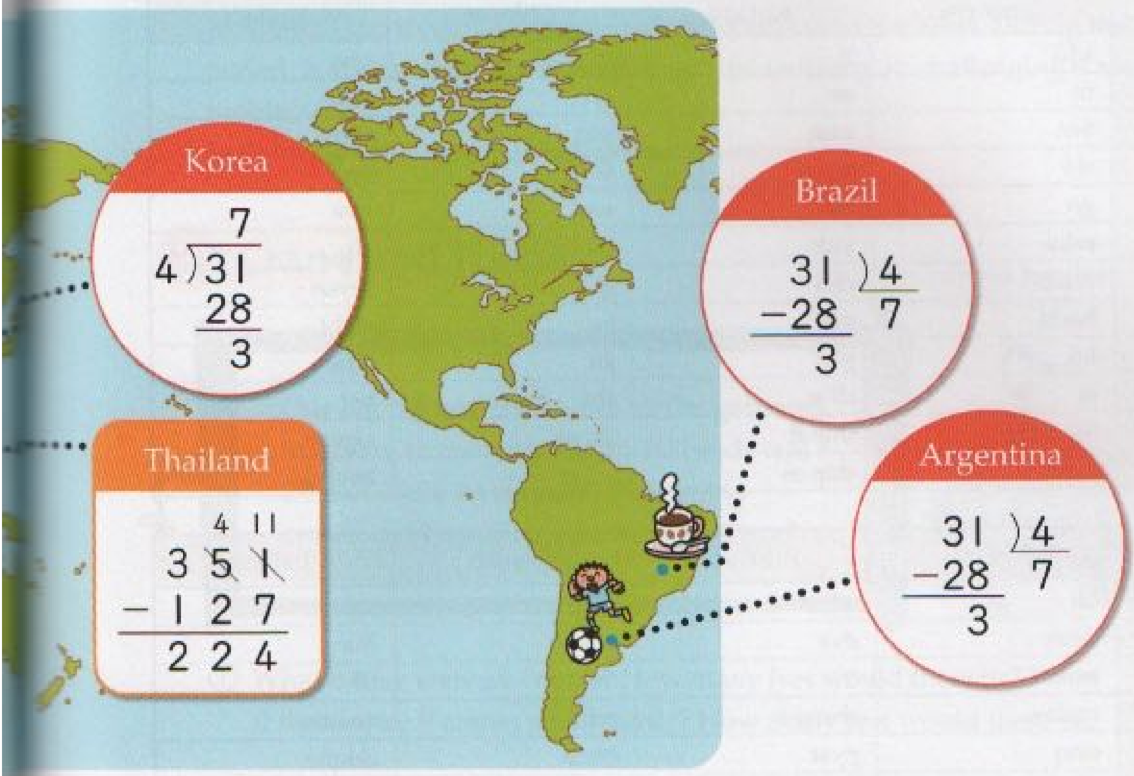
The $\underline{10}$ in the Swedish algorithm is moved from the 5 in the tens place to the ones place ...

In the German algorithm, they write + 1 under 2, but ...

② Explain the German subtraction algorithm by looking at the diagram below.



Which property of operations is being used?



$$\begin{array}{r} 7 \\ 4\overline{)31} \\ 28 \\ \hline 3 \end{array}$$

$$\begin{array}{rl} 31 & \underline{)4} \\ -28 & 7 \\ \cline{1-1} 3 & \end{array}$$

$$\begin{array}{rrr} & 4 & 11 \\ 3 & \cancel{5} & \cancel{1} \\ -1 & 2 & 7 \\ \hline 2 & 2 & 4 \end{array}$$

$$\begin{array}{rl} 31 & \underline{)4} \\ -28 & 7 \\ \cline{1-1} 3 & \end{array}$$

③ Look at each of the division algorithms shown above, think about how they work, and explain.

The Korean algorithm looks the same as the Japanese one, and the way it works is ...

Depending on the country, the division algorithms are written differently, but the thinking behind each is ...

2 Ways of Finding the Amount of Change

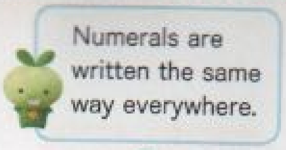
In some countries, the amount of change is found with addition as shown on the right. In the way as shown on the right, find the amount of change you receive when you buy an item that costs 720 yen with a 1,000 yen bill.

Suppose you buy an item that costs \$68 with a \$100 bill.

→

\$68 and \$2 makes \$70.
\$70 and \$30 makes \$100.
So, here's your \$32 change.

3 Number Words in Different Languages

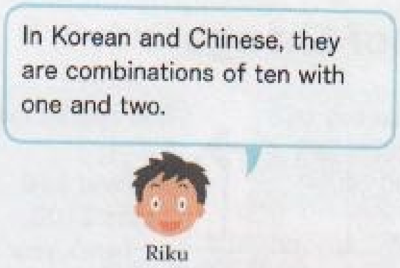


English

	Japanese	Korean	Chinese	English
1	ichi	il	yi	one
2	ni	ee	er	two
3	San	sam	san	three
4	shi	sa	si	four
5	go	oh	wu	five
6	roku	yuk	liu	six
7	shichi	chil	qi	seven
8	hachi	pal	ba	eight
9	ku	goo	jiu	nine
10	ju	ship	shi	ten
11	ju ichi	ship il	shi yi	eleven
12	ju ni	ship ee	shi er	twelve

	French	Russian	Spanish	Italian
1	un	adeen	uno	uno
2	deux	dva	dos	due
3	trois	tri	tres	tre
4	quatre	chetyre	cuatro	quatro
5	cinq	pyat	cinco	cinque
6	six	shest	seis	sei
7	sept	sem	siete	sette
8	huit	vosem	ocho	otto
9	neuf	devyat	nueve	nove
10	dix	desyat	diez	dieci
11	onze	odinnadtsat	once	undici
12	douze	dvenadtsat	doce	dodici

Look at how numbers 11 and 12 are read, and discuss what you observed.

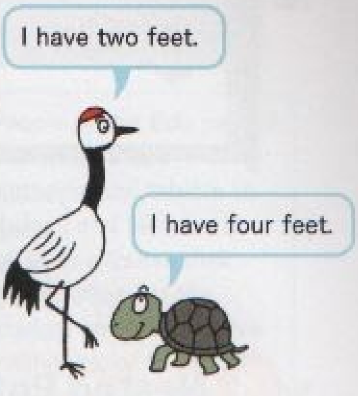


Wasan Course

Wasan is a unique style of mathematics developed in Japan during the Edo period.
In some fields, it was as advanced as mathematics in the West. During the Edo period, it appears that many people enjoyed working on challenging wasan problems.
Let's tackle some wasan problems ourselves.

1 Crane and Turtle

One night some cranes and turtles gathered in a field. In the moon-light, 10 heads and 28 feet could be counted. How many cranes and how many turtles were there?



① What if they were all cranes? How many feet would there be? What if there were 9 cranes and 1 turtle? How many feet would there be?

Number of Cranes	10	9									
Number of Turtles	0	1									
Number of Feet											

② As the number of turtles increase by one, how does the number of feet change?

③ Find how many cranes and how many turtles there were.

An Edo period mathematician named Chisho Imamura created a formula to calculate the number of cranes and made it into a short poem.

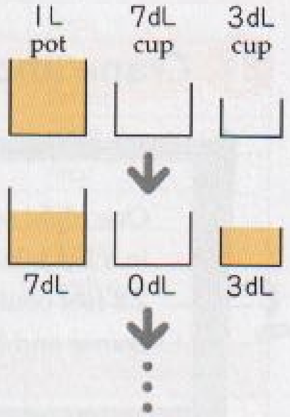
Number of Cranes = Number of Heads × 2 − Number of Feet ÷ 2

To find the number of cranes, double the number of heads, then subtract a half of the number of feet

④ Using the formula, find the answer to the first problem and compare it to the answer found in ③.

② Dividing Oil

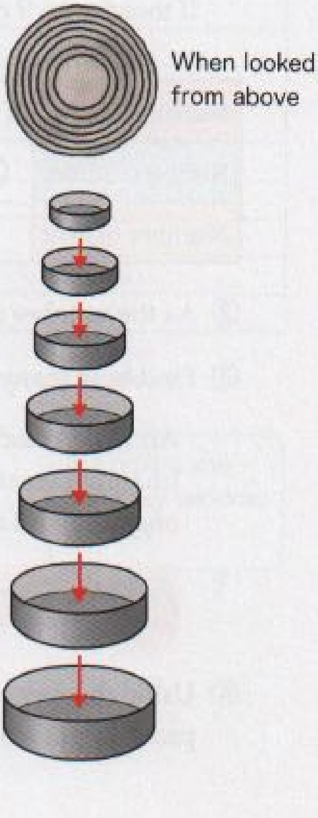
A 1 L pot is filled with oil. You want to split this oil equally with another person so that each will receive 5 dL. Unfortunately, there is only a 7 dL cup and a 3 dL cup available. Using only these two cups, divide the oil so that each person will receive 5 dL.



It may be easier to think about the problem if you draw a diagram like the one shown on the right.

③ Nested Pots

At a shop, there is a special set of 7 nested pots.
Nested pots are specially designed so that the 2nd largest pot will fit inside the largest pot, the 3rd largest pot will fit inside the 2nd largest pot, and so on. The price of the pot that is one size larger costs 250 yen more.
If the total price of the 7 nested pots is 9,800 yen, how much is the smallest pot?



④ Sangaku

Sangaku (a mathematical tablet) is a wooden tablet with a mathematical problem and its solution written on it, and it was dedicated to a shrine or a temple. During the Edo period, this practice became very popular, and even today, more than 900 such tablets remain at various locations.

People of the Edo period dedicated these mathematical tablets to shrines and temples in order to express their appreciation for the wisdom needed to solve a problem, or to report the findings of their research.

The problems on these mathematical tablets are usually very difficult, but there are some that can be solved by elementary school students.
The two problems below are from mathematical tablets, rewritten using modern language and units.

Let's tackle some problems from mathematical tablets.

①

A fox is planting rice seedlings. If 5 seedlings are planted at a time, 1 seedling will be left. But, if 7 seedlings are planted at a time, 2 seedlings will be left.
How many seedlings are there? Answer with the smallest possible number of seedlings.

From Miharumachi Inari Shrine, Tamura Gun, Fukushima Prefecture

②

A merchant bought rice at a price of 1,250 yen for 1 kg. He sold the rice for 1,500 yen for 1 kg, and he made a total profit of 10,000 yen.
How much money did the merchant spend on buying the rice?

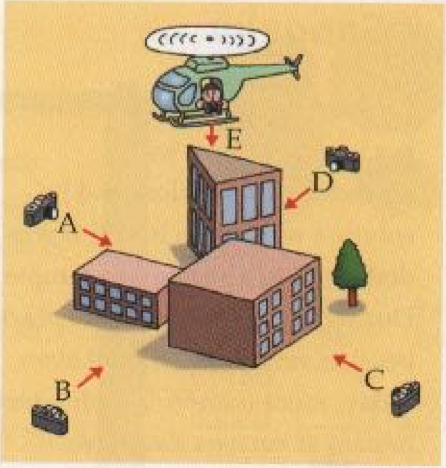
From Tanaka Shrine, Ohta City, Gunma Prefecture

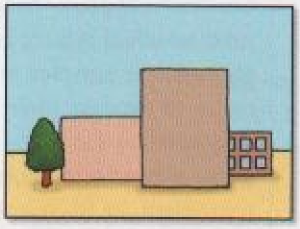
There are some mathematical tablets that were made by children who were about your age.

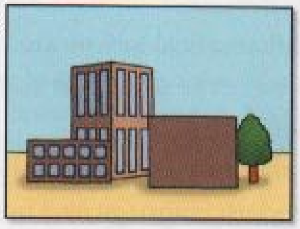
Riddles/Puzzles Course

① Which way do we look?

From which position, A, B, C, D, or E was each of the pictures below, Ⓐ, Ⓑ, Ⓒ, Ⓓ, and Ⓔ taken?



Ⓐ


Ⓑ


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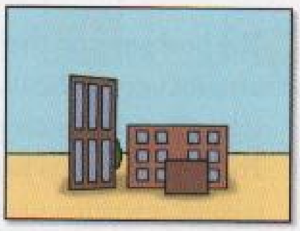

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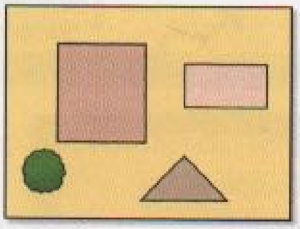
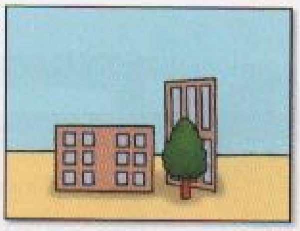
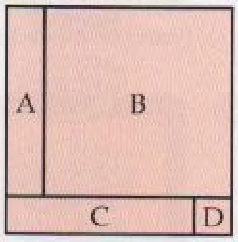


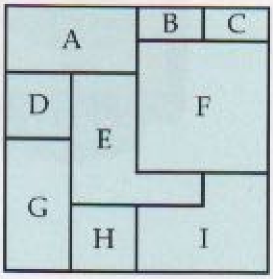
Ⓔ


② In what order?

① After placing 4 overlapping squares of the same size down, the pattern on the right was created.
Name the squares in the order they were laid.

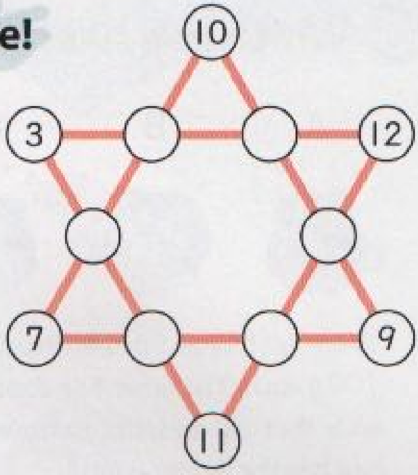


② After placing 9 overlapping squares of the same size down, the pattern on the right was created.
Name the squares in the order they were laid.



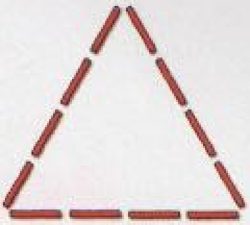
③ Let's tackle a number puzzle!

Place numbers 1 through 12 in the ○ on the right so that the sum of 4 numbers on each line will be the same.

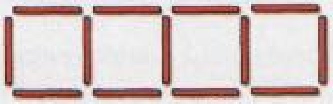


④ Arranging sticks!

① An equilateral triangle is made with 12 sticks, as shown on the right. Move 4 of these sticks and make the area half.

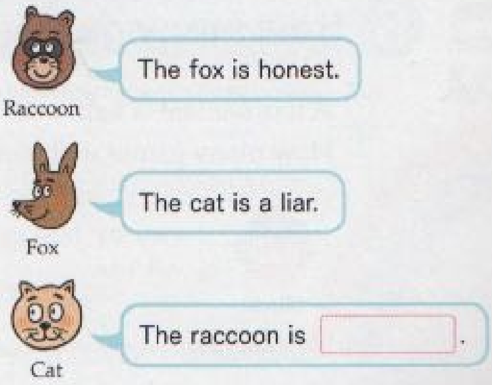


② Using 13 sticks, you can make 4 squares of the same size as shown on the right. How can you arrange 12 of the same sticks to make 4 squares of the same size?



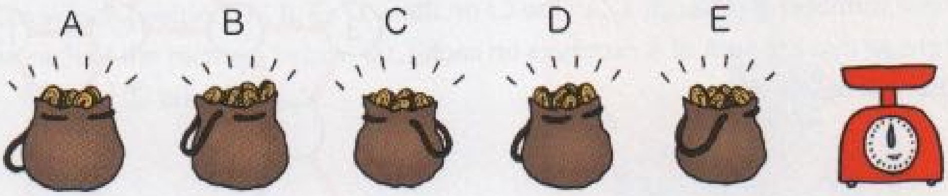
⑤ Honest? Liar?

Each animal is either always honest or always a liar. What did the cat say about the fox?



What if the raccoon is always honest? What if the raccoon is always a liar?

6 Which bag has lighter coins?

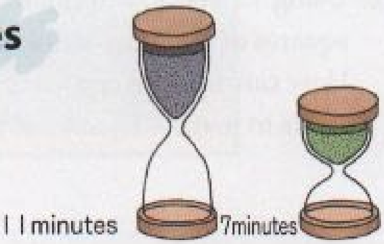


There are 5 bags filled with coins. 4 of the bags contain coins that weigh 100 g each. The other bag contains coins that weigh 99 g each. Using a scale that can measure accurately to 1 g only once, can you find which bag has the lighter coins?

If all the coins in all the bags weighed 100 g each, one coin from A, two coins from B, ... five coins from E would weigh 1,500 g altogether.

7 How to measure 15 minutes

There is an 11-minute egg timer and a 7-minute egg timer. Using only these timers, can you measure 15 minutes? How?



8 How many games in all?

① A tournament is held with 8 teams.
How many games will there be in all?

Misaki

I wonder how many teams will lose.

② A tournament is held with 48 teams.
How many games will there be in all?
Can you find the answer within 5 seconds?

New Mathematics 6 Plus

Additional Problems

Need more practice? Try these!

① Let's Investigate Well-balanced Geometric Figures ···· 246
② Let's Express Quantities and Their Relationships as Math Sentences ···· 246
③ Let's Think about Multiplication of Fractions ········ 247
④ Let's Think about Division of Fractions ········ 249
● Times as Much with Fractions ········ 250
⑤ Let's Investigate How to Express Rates ········ 250
⑥ Let's Investigate Geometric Figures that Have the Same Shape but Different Size ········ 251
⑦ Let's Think about How to Calculate the Area of Circles ···· 252
⑧ Let's Think about How to Calculate the Volume of Prisms and Cylinders ········ 253
⑩ Let's Further Investigate Proportional Relationships ···· 253
⑪ Let's Investigate Systematically ········ 254
⑫ Let's Investigate the Characteristics of Data and Make Judgments ···· 255
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Let's Try WONDERful Problems!

These are problems that will further extend and deepen your knowledge about what you have studied.
Think about these problems and enjoy the challenge.

② Let's Express Quantities and Their Relationships as Math Sentences ········ 258
③ Let's Think about Multiplication of Fractions ········ 259
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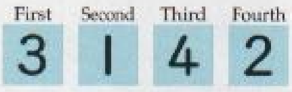
Notice to Teachers and Parents

"New Mathematics 6 Plus" is an optional learning material for students who want or need to work on it.
Not all students need to use "New Mathematics 6 Plus."

Let's Try Programming!

Think about How to Arrange Numbers



Suppose there are four numbers arranged as shown on the right. To arrange these numbers in ascending order using a computer that is capable of doing the following, what instructions do you need to give to the computer?

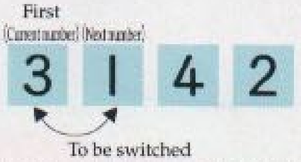
When were finding a median, we rearranged numbers.

(A) Examining the numbers from the first one
(B) Comparing the current number (the one the computer is checking) with the next number in size
(C) Switching the current number with the next number

The computer will arrange numbers in ascending order as shown below.

An example of number rearrangement by the computer

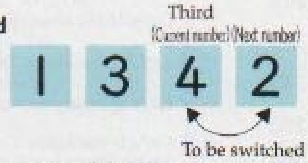
① **The computer examines the first number.**
If the current number is greater than the next, the computer switches the numbers.
If not, the computer does not switch them.



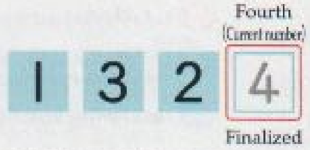
The computer examines each number starting with the first.

② **The computer examines the second number.**
If the current number is greater than the next, the computer switches the numbers.
If not, the computer does not switch them.

③ **The computer examines the third number.**
If the current number is greater than the next, the computer switches the numbers.
If not, the computer does not switch them.

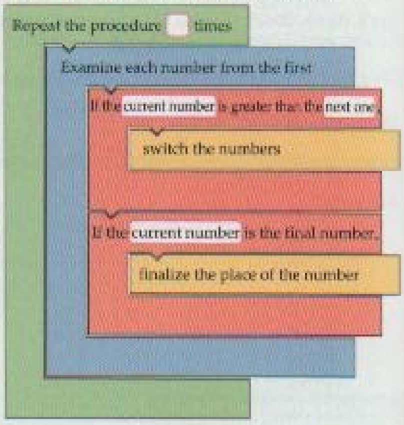


④ **The computer examines the fourth number.**
If the current number is the final number, the computer finalizes the place of the number.



⑤ **The computer repeats the same procedure until it finalizes the places of all the numbers.**

How many times does the computer have to repeat ① through ④?



```
Repeat the procedure ☐ times
    Examine each number from the first
        If the current number is greater than the next one,
            switch the numbers
        If the current number is the final number,
            finalize the place of the number
```

① In Step ④ on the previous page, what can you say about the number whose position was finalized?

② After the computer finishes Step ① through Step ④ once, the number cards are arranged as shown on the right. Then, the computer will repeat Step ① through Step ③ to examine the first three numbers. The place of which number will be finalized this time?

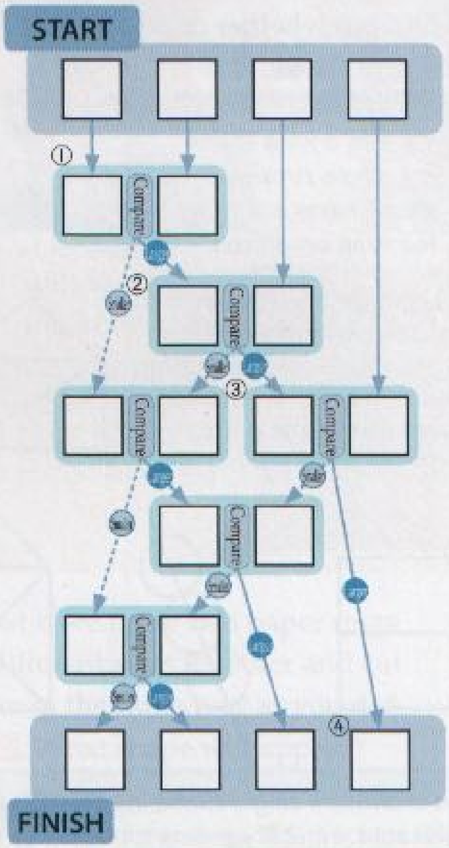
③ About the procedure on the previous page, explain why the numbers can be arranged in ascending order.


Shiho

Every time the computer examines the numbers from the first one to the last, the largest number...

Such a procedure predetermined to solve a problem is called an algorithm.

④ The diagram below shows the algorithm for arranging the numbers on the previous page. Use the diagram to arrange the numbers.



Procedure

- In the first row labeled **START** on the left, write four different numbers of your choice in random order.
- Copy each number in the next blank as indicated by the arrow.
- In each **Compare** box, compare the two numbers.
  Copy the larger number in → the next blank (on the right).
  Write the smaller one in ⇢ the next blank (on the left).

Change the way the numbers are arranged, and try again.

(Try programming)

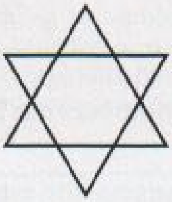
Let's Play with Shapes

Drawing Shapes in a Single Stroke

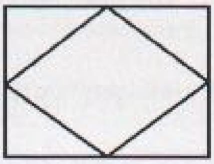
Drawing a shape in a single stroke means to draw it without lifting your pen from the paper or tracing any segment you have drawn.

1 Select the shapes you can draw in a single stroke.

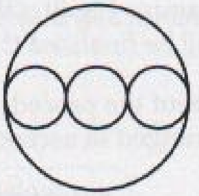
Ⓐ



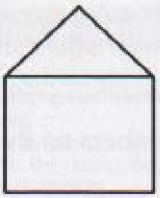
Ⓑ



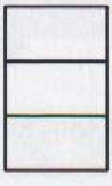
Ⓒ



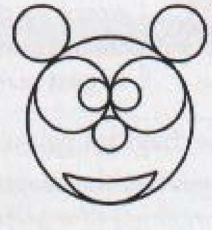
Ⓓ



Ⓔ



Ⓕ



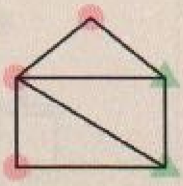
2 Read the following conditions and find out whether or not you can draw each of the shapes above in a single stroke.

You can draw a shape in a single stroke if the shape meets one of the following conditions.

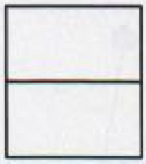
Leonhard Euler

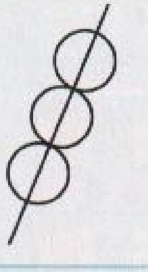
- Each point in the shape is shared by an even number of segments

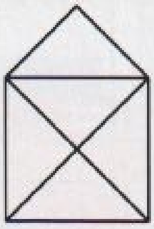
- The shape has exactly two points where an odd number of segments going out. We start our drawing from one of the points with an odd number



3 Come up with some shapes you can draw in a single stroke.

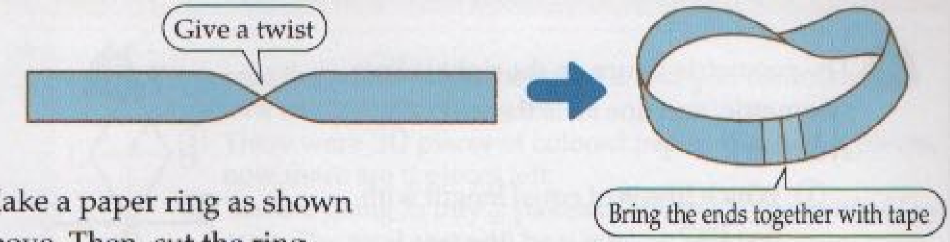




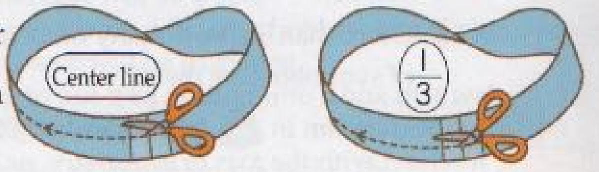


When you draw a shape that can be drawn in a single stroke, can you draw it in a single stroke whichever point you start with? Try to find it out. It'll be fun!

Mysterious Transformation of Rings

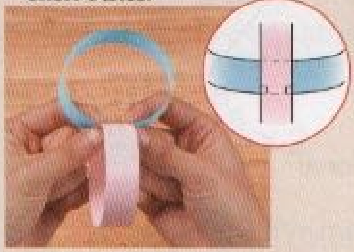


1 Make a paper ring as shown above. Then, cut the ring along the center of the tape or the line that is $\frac{1}{3}$ of the width of the ring from one edge.

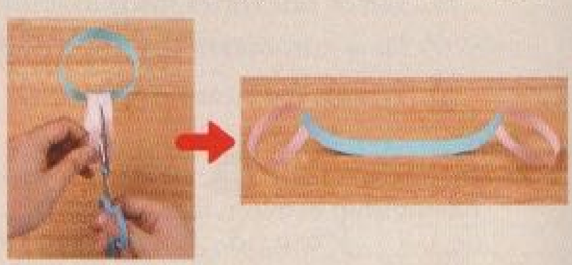


2 As shown below, paste two paper rings of the same size together and cut them.

① Paste two rings together so that they are perpendicular to each other.



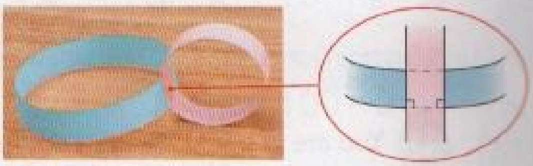
② Cut one of the rings along the center line of the length of one of the rings.



③ Cut the other ring in the same way and open it up.

Where were the sides of the resulting shape in the original two rings?

3 What if we paste two paper rings of different sizes together and cut them in the same way as you did in **2**. What shape will appear?



You may also want to find out what shape will appear if you paste two rings together at a different angle.

Additional Problems

▲ Similar Problems ● Slightly Difficult Problems

1 Let's Investigate Well-balanced Geometric Figures

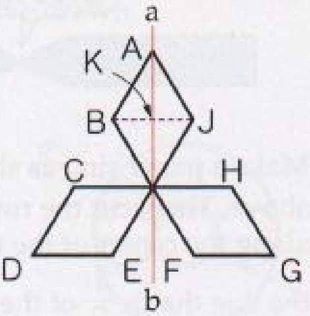
Answers → Page 256

Page 12 2 →

A The geometric figure on the right is line symmetric, and line ab is the axis of symmetry.

① Which line is of equal length with line BK?

② Other than ab, how many other axes of symmetry are there?



A In the diagram in **A**, how does line BK intersect with the axis of symmetry, or line ab?

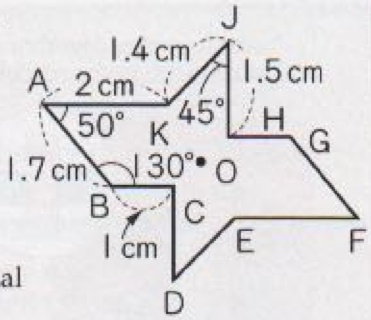
Page 16 1 →

B The figure on the right is point symmetric. Point O is the center of symmetry.

① Which side corresponds to side AB?

② How many cm is side EF?

③ What is the measure of angle D?



B In the diagram in **B**, what is the positional relationship between line AK and BC?

Page 19 1 →

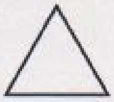
C Among the following geometric figures from Ⓐ through Ⓔ, select all that are both line symmetric and point symmetric.

Ⓐ Right triangle

Ⓑ Parallelogram

Ⓒ Rhombus

Ⓓ Equilateral triangle



Ⓔ Regular hexagon

2 Let's Express Quantities and Their Relationships as Math Sentences

Answers → Page 256

Page 27 1 →

D You are going to divide 1.2 L of juice among x people.

① Express the amount of juice each person will get as a math sentence.

② If you divide the juice among 4 people, how many L of juice will each person get?

Page 27
1 →

D We are going to make a rectangle with the width of x cm out of a string that is 20 cm long.

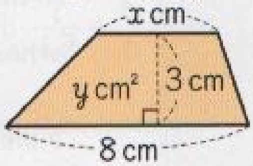
① Express the length of the rectangle as a math sentence.
② Express the area of the rectangle as a math sentence.
③ Find the area of the rectangle when x is 2 cm.

Page 28
2 →

E Represent the relationships between x and y in the following situations using math sentences.

① There were 30 pieces of colored paper. We used x pieces, and now there are y pieces left.
② You are going to buy x pieces of chocolate that costs 40 yen each. The total cost will be y yen.
③ We are going to divide a 60 cm of ribbon into x pieces of equal length. Each piece will be y cm long.
④ We are going to put x kg of sand into a box that weighs 0.2 kg. The total weight will be y kg.

E There is a trapezoid as shown on the right. The top base is x cm long, the bottom base is 8 cm long, the height is 3 cm, and the area is y cm². Express the relationship between x and y as a math sentence.



3 Let's Think about Multiplication of Fractions

Answers→ Page 256

Page 37
2 →

F ① $\frac{5}{8}\times 2$ ② $\frac{1}{9}\times 6$ ③ $\frac{7}{4}\times 12$ ④ $\frac{11}{25}\times 100$

F Place the five numerals 2, 3, 4, 6, and 9 in the $\boxed{}$ on the right to make two correct math sentences.

$$\frac{\boxed{\text{Ⓑ}}}{\boxed{\text{Ⓐ}}}\times\boxed{\text{Ⓒ}}=\frac{\boxed{\text{Ⓔ}}}{\boxed{\text{Ⓓ}}}$$

Note that in each of the math sentences, $\frac{\boxed{\text{Ⓑ}}}{\boxed{\text{Ⓐ}}}$ and $\frac{\boxed{\text{Ⓔ}}}{\boxed{\text{Ⓓ}}}$ must be fractions that cannot be further simplified.

Page 39
4 →

G ① $\frac{3}{4}\div 2$ ② $\frac{3}{5}\div 4$ ③ $\frac{20}{9}\div 15$ ④ $\frac{25}{9}\div 100$

G ① $\frac{3}{5}\times 2\div 3$ ② $\frac{13}{25}\times 100\times 2$

③ $\frac{25}{9}\div 15\times 18$ ④ $\frac{36}{7}\div 6\times 35$

Page 44

1 →

H ① $\frac{1}{3}\times\frac{4}{5}$ ② $\frac{1}{6}\times\frac{7}{8}$ ③ $\frac{5}{4}\times\frac{3}{7}$ ④ $\frac{5}{9}\times\frac{7}{2}$

H Do the following multiplication calculations with fractions. Next, change the fractions into decimal numbers and do multiplication calculations with the decimal numbers. Then, check that the products of the fractions are equal to the products of the decimal numbers.

① $\frac{1}{2}\times\frac{3}{10}$ ② $\frac{5}{8}\times\frac{3}{4}$

Page 45

3 →

I ① $\frac{3}{10}\times\frac{2}{5}$ ② $\frac{1}{2}\times\frac{4}{3}$ ③ $\frac{8}{9}\times\frac{11}{12}$ ④ $\frac{3}{4}\times\frac{5}{6}$

⑤ $\frac{5}{12}\times\frac{4}{15}$ ⑥ $\frac{9}{10}\times\frac{2}{3}$ ⑦ $\frac{21}{8}\times\frac{24}{35}$ ⑧ $\frac{5}{12}\times\frac{26}{9}\times\frac{3}{4}$

I Place the four numerals 2, 3, 4, and 5 in the $\boxed{}$ on the right to make a math sentence whose product is $\frac{3}{10}$.

$$\frac{\boxed{\text{Ⓑ}}}{\boxed{\text{Ⓐ}}}\times\frac{\boxed{\text{Ⓓ}}}{\boxed{\text{Ⓒ}}}=\frac{3}{10}$$

Note that $\frac{\boxed{\text{Ⓑ}}}{\boxed{\text{Ⓐ}}}$ and $\frac{\boxed{\text{Ⓓ}}}{\boxed{\text{Ⓒ}}}$ must be fractions that cannot be further simplified.

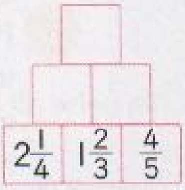
Page 46

3 →

J ① $7\times\frac{3}{4}$ ② $10\times\frac{5}{8}$ ③ $\frac{4}{9}\times2$ ④ $\frac{11}{18}\times6$

⑤ $\frac{3}{5}\times2\frac{5}{8}$ ⑥ $1\frac{1}{2}\times\frac{4}{9}$ ⑦ $1\frac{2}{3}\times1\frac{5}{7}$ ⑧ $2\frac{5}{14}\times1\frac{1}{6}$

J Multiply the two numbers that are next to each other and write the answers in the $\boxed{}$ just above the two numbers.



Page 49

6 →

K ① $\left(\frac{8}{11}\times\frac{3}{7}\right)\times\frac{7}{3}$ ② $\left(\frac{6}{5}+\frac{2}{3}\right)\times15$ ③ $\frac{3}{8}\times6+\frac{3}{8}\times10$

K Using the properties of operations, write the appropriate number in each $\boxed{}$ on the right to complete the math sentence.

$$\begin{aligned}1\frac{2}{3}\times\frac{3}{10}&=(1+\boxed{\text{Ⓐ}})\times\frac{3}{10}\\&=1\times\boxed{\text{Ⓑ}}+\boxed{\text{Ⓒ}}\times\frac{3}{10}\\&=\frac{\boxed{\text{Ⓓ}}}{10}+\frac{\boxed{\text{Ⓔ}}}{10}\\&=\boxed{\text{Ⓕ}}\end{aligned}$$

4 Let's Think about Division of Fractions

Answers → Page 256

Page 59
1 →

L ① $\frac{2}{3} \div \frac{5}{7}$ ② $\frac{1}{2} \div \frac{2}{5}$ ③ $\frac{5}{9} \div \frac{4}{5}$ ④ $\frac{5}{8} \div \frac{4}{15}$

L You were supposed to multiply a certain number by $\frac{4}{5}$, but you multiplied the number by the reciprocal of $\frac{4}{5}$ by mistake and got the answer of $\frac{1}{3}$. Find the number and the correct answer.

Page 60
3 →

M ① $\frac{4}{9} \div \frac{5}{6}$ ② $\frac{1}{8} \div \frac{1}{4}$ ③ $\frac{3}{10} \div \frac{2}{5}$ ④ $\frac{7}{12} \div \frac{8}{15}$

⑤ $\frac{9}{4} \div \frac{21}{8}$ ⑥ $\frac{7}{18} \div \frac{14}{27}$ ⑦ $\frac{3}{14} \div \frac{6}{7}$ ⑧ $\frac{10}{9} \div \frac{5}{12}$

⑨ $\frac{3}{4} \times \frac{1}{9} \div \frac{5}{2}$ ⑩ $\frac{6}{7} \div \frac{5}{14} \times \frac{1}{4}$ ⑪ $\frac{7}{12} \div \frac{5}{6} \div 14$

M Find the fraction that fits in each $\boxed{}$. Note that $\boxed{}$ must be fractions that cannot be further simplified.

① $\boxed{} \div \frac{3}{5} \div \frac{5}{14} = 1$ ② $\frac{3}{10} \div \frac{9}{8} \times \boxed{} = \frac{1}{3}$

Page 60
3 →

N ① $7 \div \frac{3}{4}$ ② $4 \div \frac{8}{9}$ ③ $\frac{2}{5} \div 3$ ④ $\frac{3}{4} \div 6$

⑤ $2\frac{3}{4} \div \frac{3}{8}$ ⑥ $\frac{1}{6} \div 1\frac{2}{7}$ ⑦ $1\frac{2}{5} \div 1\frac{1}{5}$ ⑧ $2\frac{11}{12} \div 4\frac{2}{3}$

N We are going to place the three numerals 2, 3, and 6 in the $\boxed{}$ on the right to complete the math sentence.
Find the appropriate number in each $\boxed{}$.

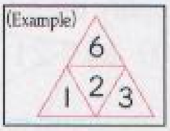
$$\frac{\boxed{Ⓑ}}{\boxed{Ⓐ}} \div 1\frac{1}{\boxed{Ⓒ}} = \frac{4}{7}$$

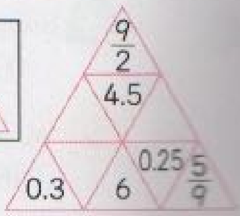
Page 65
6 →

O ① $\frac{4}{7} \times \frac{5}{6} \times 1.4$ ② $\frac{3}{8} \div \frac{7}{12} \times 3.5$ ③ $1.8 \div \frac{10}{9} \div 0.27$

④ $0.45 \times 4 \div 6.3$ ⑤ $5 \div 1.25 \times 0.9$ ⑥ $5.4 \times 0.05 \div 9$

O As shown in (Example), we are going to multiply the three numbers that are side by side and write the answers in the △ just above the three numbers.
Find the appropriate number in each △.





Times as Much with Fractions

Answers→ Page 256

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2 →

P ① How many times as long is $\frac{7}{8}$ m as $\frac{3}{4}$ m?

② If you consider $\frac{2}{5}$ m² as 1, what number does $\frac{3}{10}$ m² correspond to?

P Containers A, B, and C contains $\frac{5}{7}$ L, $\frac{15}{28}$ L, and $\frac{45}{56}$ L of water, respectively.

① If you consider the amount of water in A as 1, what number does the amount of water in B correspond to?

② If you consider the amount of water in B as 1, what number does the amount of water in C correspond to?

Page 73
3 →

Q There are small and large swimming pools. The length of the small pool, 15 m, is $\frac{3}{5}$ of the length of the large pool. How long is the large pool?

Q A school is going to organize a marathon. Students in 1st and 2nd grades are going to run a distance that is $\frac{3}{4}$ of the distance students in 3rd and 4th grades are going to run. The students in 3rd and 4th grades are going to run 1,600 m, and this distance is $\frac{4}{5}$ of the distance students in 5th and 6th grades are going to run. Find the following distances:

① The distance students in 1st and 2nd grades are going to run

② The distance students in 5th and 6th grades are going to run

5 Let's Investigate How to Express Rates

Answers→ Page 257

Page 80
2 →

R Find the values of the ratios and identify equivalent ratios.

① 1 : 4 ② 12 : 10 ③ 7 : 14

④ 18 : 15 ⑤ 42 : 24 ⑥ 15 : 30

R Find the values of the ratios and identify equivalent ratios.

① 8 : 12 ② 25 : 10 ③ 24 : 16

④ 27 : 36 ⑤ 30 : 45 ⑥ 18 : 12

Page 82
3 →

S Simplify the following ratios.

① 14 : 21 ② 24 : 27 ③ 10 : 8 ④ 42 : 12

S We are going to mix some water and noodle soup base at ratios from Ⓐ through Ⓓ below to make some noodle soup. Select all the ratios that make noodle soup that will taste the same as the noodle soup made of water and noodle soup base mixed in the ratio of 3 : 2.

Ⓐ 15 : 10 Ⓑ 9 : 3 Ⓒ 12 : 18 Ⓓ 36 : 24

Page 83
5 →

T Simplify the following ratios.

① $0.8 : 1.4$ ② $3.2 : 4$ ③ $\frac{3}{4} : \frac{1}{2}$ ④ $\frac{6}{7} : 2$

T Among Ⓐ through Ⓕ below, select all the ratios that can be simplified into $4 : 5$.

Ⓐ $1.2 : 0.15$ Ⓑ $0.12 : 0.15$ Ⓒ $1 : 1.25$

Ⓓ $2 : \frac{5}{2}$ Ⓔ $\frac{1}{6} : \frac{2}{15}$ Ⓕ $1\frac{1}{4} : 1$

Page 84
2 →

U Find the value of x in each of the following.

① $9 : 24 = x : 8$ ② $30 : x = 5 : 6$

U Find the value of x in each of the following.

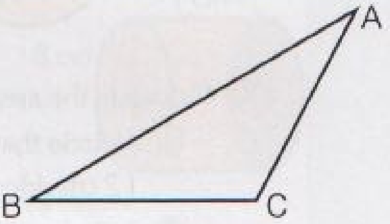
① $x : \frac{4}{3} = 3 : 2$ ② $x : 4 = \frac{7}{5} : 0.8$

6 Let's Investigate Geometric Figures that Have the Same Shape but Different Size

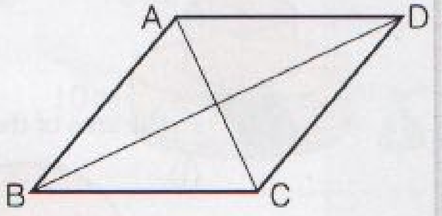
Answers→ Page 257

Page 96
4 →

V Draw a 2 times enlarged drawing and a $\frac{1}{2}$ reduced drawing of triangle ABC on the right.

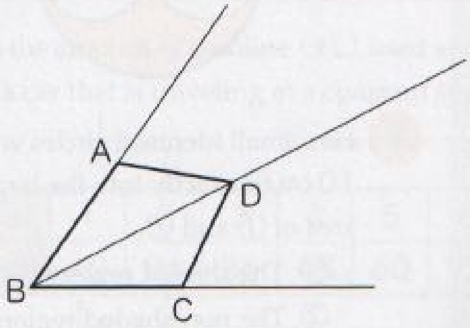


V Draw a 2 times enlarged drawing and a $\frac{1}{2}$ reduced drawing of rhombus ABCD on the right. Compare the enlarged and reduced rhombuses with the original rhombus in the length of diagonals.



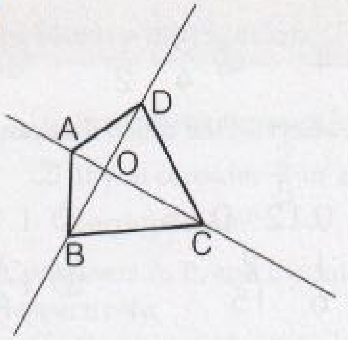
Page 97
5 →

W Draw a 2 times enlarged drawing and a $\frac{1}{2}$ reduced drawing of quadrilateral ABCD on the right.



Page 97
5 →

W Draw a 2 times enlarged drawing of quadrilateral ABCD on the left using point O, the intersection of the diagonals of quadrilateral ABCD, as the center of the enlarged drawing.



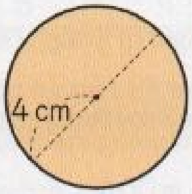
7 Let's Think about How to Calculate the Area of Circles

Answers → Page 257

Page 109
1 →

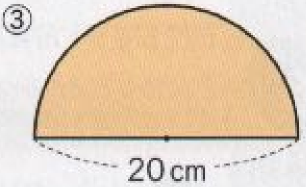
X Find the area of the following figures.

①



②

③



X Calculate the area of the following figures.

① A circle that fits exactly into the square with 12 cm sides as shown on the right

② A circle with the circumference of 62.8 cm

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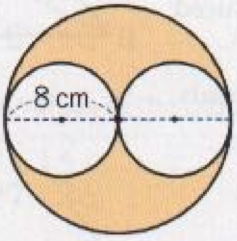
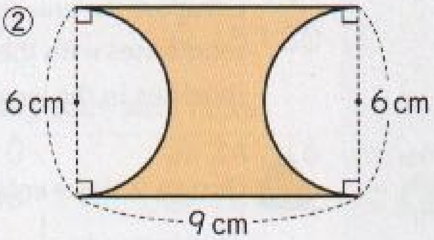
3 →

Y Find the area of the shaded regions.

①



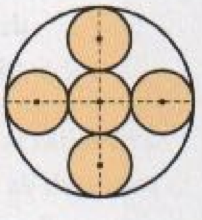
②



Y Five small identical circles with the diameter of 10 cm fit exactly into the large circle. Find the area of ① and ②.

① The shaded regions

② The non-shaded regions

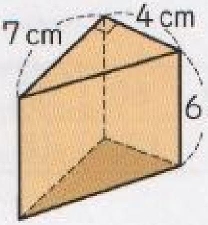


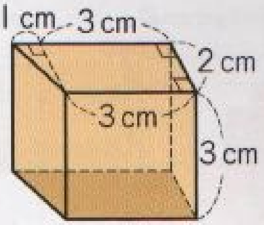
8 Let's Think about How to Calculate the Volume of Prisms and Cylinders

Answers → Page 257

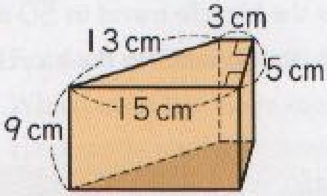
Page 123 1 →

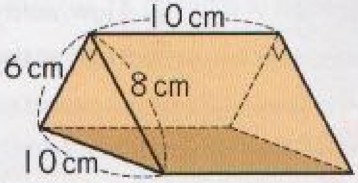
Z Find the volume of the prisms below.

①


②


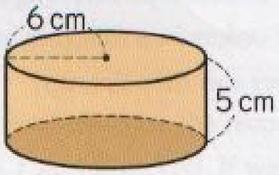
Z Find the volume of the prisms below.

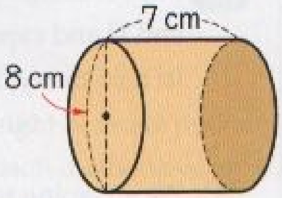
①


②


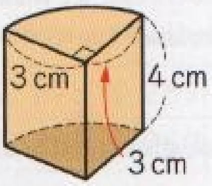
Page 124 2 →

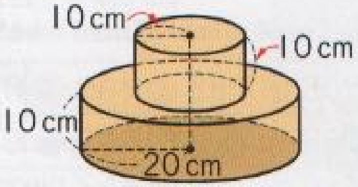
AA Find the volume of the cylinders below.

①


②


AA Find the volume of each of the solid figures below.

①


②


10 Let's Further Investigate Proportional Relationships

Answers → Page 257

Page 140 1 →

AB The table below summarizes the amount of gasoline (x L) used and the distance (y km) traveled by a car that is traveling at a constant speed.

$\frac{1}{2}$ times
ⓑ times

Amount of gasoline x(L)	1	2	3	4	5	6
Distance traveled y(km)	12	24	36	48	60	72

ⓐ times
ⓒ times

① Is the distance traveled proportional to the amount of gasoline used?

② Find the numbers that should go in ⓐ, ⓑ, and ⓒ.

Page 140
1 →

AB The table below summarizes the amount of time (x minutes) a bicycle traveled at a constant speed and the distance (y km) traveled by the bicycle.

Time x (minutes)	20	40	60	80
Distance y (km)	4.5	9	13.5	18

① Is the distance proportional to the time?
② How many km does the bicycle travel in 50 minutes?
③ How many minutes does it take for the bicycle to travel 27 km?

11 Let's Investigate Systematically

Answers→ Page 257

Page 165
1 →

AC Four students are lining up to take a picture. Represent them as A, B, C, and D and express possible orders in a diagram and a table.
In how many different orders can they line up?

AC We are going to make 4-digit whole numbers using the four cards $\boxed{0}$, $\boxed{1}$, $\boxed{2}$, and $\boxed{3}$. What whole numbers can we make? How many different whole numbers can we make?

Page 171
1 →

AD From 6 pieces of red, blue, yellow, green, white and black origami paper, 2 pieces are selected.
What are some of the possible combinations? Also, how many different combinations are there?

AD You are going to eat at a ramen noodle restaurant. There are two choices of soup: soy sauce-based and miso-based. There are five choices of toppings: slices of roast pork, seasoned bamboo shoots, some corn, sheets of dried sea weed, and hard boiled eggs.
If you choose one type of soup and two different types of toppings, what are some of the possible combinations?
Also, how many different combinations are there?

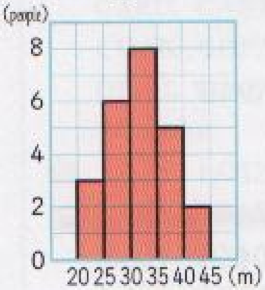
12 Let's Investigate the Characteristics of Data and Make Judgments

Answers→ Page 257

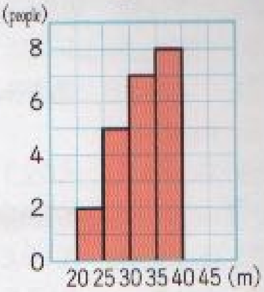
Page 184
4 →

AE The histograms on the right summarize how far the students in teams A and B threw a softball.

Team A's Records of Softball Throwing Distances



Team B's Records of Softball Throwing Distances



① For each of Teams A and B, which class has the largest frequency?

② Which team has more students who threw a softball at least 35 m? How many students in each team threw a softball at least 35 m?

③ Look at each of the histograms above and describe the characteristics of how the values are spread.

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5 →

AF The tables below are the records of the weight of sweet potatoes that groups 1 and 2 picked. If we use comparison methods Ⓐ or Ⓑ below, which group picked heavier sweet potatoes?

Weight of Sweet Potatoes Picked by Group 1 (g)

① 265	② 278	③ 287	④ 309
⑤ 319	⑥ 268	⑦ 269	

Weight of Sweet Potatoes Picked by Group 2 (g)

① 280	② 261	③ 287	④ 302
⑤ 310	⑥ 284	⑦ 275	⑧ 269

Ⓐ The mean of weights

Ⓑ The class that has the largest frequency among the classes divided at intervals of 10 g starting at 260 g

AF A shoe manufacturer is discussing shoes of which size it should manufacture the most next year. The mean of the sizes of shoes it sold this year was 24.5 cm. The median and the mode of the sizes of shoes it sold this year were both 24 cm. The company thinks it should manufacture shoes of the mean size the most.

Do you agree or disagree with this idea? Also, explain your reasoning.

Answers

Additional Problems

Page246-255

1 Let's Investigate Well-balanced Geometric Figures

A ① line JK ② 2 axes

A Perpendicular

B ① side FG ② 2 cm ③ 45°

B Parallel

C Ⓒ, Ⓔ

2 Let's Express Quantities and Their Relationships as Math Sentences

D ① $1.2 \div x$ ② 0.3L

D ① $10 - x$ ② $x \times (10 - x)$

③ 16 cm²

E ① $30 - x = y$ ② $40 \times x = y$

③ $60 \div x = y$ ④ $x + 0.2 = y$

E $(x + 8) \times 3 \div 2 = y$

3 Let's Think about Multiplication of Fractions

F ① $\frac{5}{4}\left(1\frac{1}{4}\right)$ ② $\frac{2}{3}$ ③ 21 ④ 44

F Ⓐ4 Ⓑ3 Ⓒ6 Ⓓ2 Ⓔ9

Ⓐ9 Ⓑ2 Ⓒ6 Ⓓ3 Ⓔ4

G ① $\frac{3}{8}$ ② $\frac{3}{20}$ ③ $\frac{4}{27}$ ④ $\frac{1}{36}$

G ① $\frac{2}{5}$ ② 104 ③ $\frac{10}{3}\left(3\frac{1}{3}\right)$ ④ 30

H ① $\frac{4}{15}$ ② $\frac{7}{48}$ ③ $\frac{15}{28}$ ④ $\frac{35}{18}\left(1\frac{17}{18}\right)$

H ① $\frac{1}{2} \times \frac{3}{10} = \frac{3}{20}$ $0.5 \times 0.3 = 0.15$

$\frac{3}{20} = 3 \div 20 = 0.15$

② $\frac{5}{8} \times \frac{3}{4} = \frac{15}{32}$

$0.625 \times 0.75 = 0.46875$

$\frac{15}{32} = 15 \div 32 = 0.46875$

I ① $\frac{3}{25}$ ② $\frac{2}{3}$ ③ $\frac{22}{27}$ ④ $\frac{5}{8}$

⑤ $\frac{1}{9}$ ⑥ $\frac{3}{5}$ ⑦ $\frac{9}{5}\left(1\frac{4}{5}\right)$ ⑧ $\frac{65}{72}$

I Ⓐ4 Ⓑ3 Ⓒ5 Ⓓ2

(Ⓐ5 Ⓑ2 Ⓒ4 Ⓓ3)

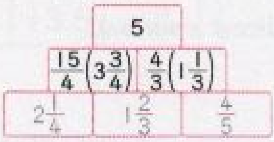
J ① $\frac{21}{4}\left(5\frac{1}{4}\right)$ ② $\frac{25}{4}\left(6\frac{1}{4}\right)$

③ $\frac{8}{9}$ ④ $\frac{11}{3}\left(3\frac{2}{3}\right)$

⑤ $\frac{63}{40}\left(1\frac{23}{40}\right)$ ⑥ $\frac{2}{3}$

⑦ $\frac{20}{7}\left(2\frac{6}{7}\right)$ ⑧ $\frac{11}{4}\left(2\frac{3}{4}\right)$

J



K ① $\frac{8}{11}$ ② 28 ③ 6

K Ⓐ $\frac{2}{3}$ Ⓑ $\frac{3}{10}$ Ⓒ $\frac{2}{3}$ Ⓓ3 Ⓔ2 Ⓕ $\frac{1}{2}$

4 Let's Think about Division of Fractions

L ① $\frac{14}{15}$ ② $\frac{5}{4}\left(1\frac{1}{4}\right)$ ③ $\frac{25}{36}$ ④ $\frac{75}{32}\left(2\frac{11}{32}\right)$

L certain number ⋯ $\frac{4}{15}$ correct answer⋯ $\frac{16}{75}$

M ① $\frac{8}{15}$ ② $\frac{1}{2}$ ③ $\frac{3}{4}$ ④ $\frac{35}{32}\left(1\frac{3}{32}\right)$

⑤ $\frac{6}{7}$ ⑥ $\frac{3}{4}$ ⑦ $\frac{1}{4}$ ⑧ $\frac{8}{3}\left(2\frac{2}{3}\right)$

⑨ $\frac{1}{30}$ ⑩ $\frac{3}{5}$ ⑪ $\frac{1}{20}$

M ① $\frac{3}{14}$ ② $\frac{5}{4}\left(1\frac{1}{4}\right)$

N ① $\frac{28}{3}\left(9\frac{1}{3}\right)$ ② $\frac{9}{2}\left(4\frac{1}{2}\right)$ ③ $\frac{2}{15}$ ④ $\frac{1}{8}$

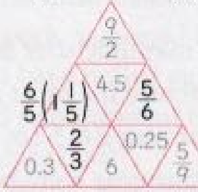
⑤ $\frac{22}{3}\left(7\frac{1}{3}\right)$ ⑥ $\frac{7}{54}$ ⑦ $\frac{7}{6}\left(1\frac{1}{6}\right)$ ⑧ $\frac{5}{8}$

N Ⓐ3 Ⓑ2 Ⓒ6

O ① $\frac{2}{3}$ ② $\frac{9}{4}\left(2\frac{1}{4}\right)$ ③ 6

④ $\frac{2}{7}$ ⑤ $\frac{18}{5}\left(3\frac{3}{5}\right)$ ⑥ $\frac{3}{100}$

O



Times as Much with Fractions

P ① $\frac{7}{6}\left(1\frac{1}{6}\right)$ times ② $\frac{3}{4}$

P ① $\frac{3}{4}$ ② $\frac{3}{2}\left(1\frac{1}{2}\right)$

Q 25 m

Q ① 1,200 m ② 2,000 m

5 Let's Investigate How to Express Rates

R Values of Ratios…① $\frac{1}{4}$ ② $\frac{6}{5}$ ③ $\frac{1}{2}$ ④ $\frac{6}{5}$ ⑤ $\frac{7}{4}$ ⑥ $\frac{1}{2}$ Equivalent Ratios…② and ④, ③ and ⑥

R Values of Ratios…① $\frac{2}{3}$ ② $\frac{5}{2}$ ③ $\frac{3}{2}$ ④ $\frac{3}{4}$ ⑤ $\frac{2}{3}$ ⑥ $\frac{3}{2}$ Equivalent Ratios…① and ⑤, ③ and ⑥

S ① 2 : 3 ② 8 : 9 ③ 5 : 4 ④ 7 : 2

S Ⓐ, Ⓓ

T ① 4 : 7 ② 4 : 5 ③ 3 : 2 ④ 3 : 7

T Ⓑ, Ⓒ, Ⓓ

U ① 3 ② 36

U ① 2 ② 7

6 Let's Investigate Geometric Figures that Have the Same Shape but Different Size

V (Omitted)

V (Figure omitted)

Lengths of diagonals … Double in the enlarged drawing, and $\frac{1}{2}$ in the reduced drawing

W (Omitted)

W (Omitted)

7 Let's Think about How to Calculate the Area of Circles

X ① 50.24 cm² ② 3.14 cm² ③ 157 cm²

X ① 113.04 cm² ② 314 cm²

Y ① 100.48 cm² ② 25.74 cm²

Y ① 392.5 cm² ② 314 cm²

8 Let's Think about How to Calculate the Volume of Prisms and Cylinders

Z ① 84 cm³ ② 21 cm³

Z ① 405 cm³ ② 240 cm³

AA ① 565.2 cm³ ② 351.68 cm³

AA ① 28.26 cm³ ② 15,700 cm³

10 Let's Further Investigate Proportional Relationships

AB ① Yes

② Ⓐ $\frac{1}{2}$ Ⓑ $\frac{1}{3}$ Ⓒ $\frac{1}{3}$

AB ① Yes

② 11.25 $\left(\frac{45}{4}\right)$ km ③ 120 minutes

11 Let's Investigate Systematically

AC ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA

24 orders

AC 1,023, 1,032, 1,203, 1,230, 1,302, 1,320, 2,013, 2,031, 2,103, 2,130, 2,301, 2,310, 3,012, 3,021, 3,102, 3,120, 3,201, 3,210 18 numbers

AD (Red, Blue), (Red, Yellow), (Red, Green), (Red, White), (Red, Black), (Blue, Yellow), (Blue, Green), (Blue, White), (Blue, Black), (Yellow, Green), (Yellow, White), (Yellow, Black), (Green, White), (Green, Black), (White, Black)

15 combinations

AD (Example) Represent soy sauce-based soup as Ⓢ, miso-based soup as Ⓜ, pork as P, bamboo shoots as B, corn as C, sea weed as S, and egg as E. Possible combinations are as follows:

(Ⓢ, P, B), (Ⓢ, P, C), (Ⓢ, P, S), (Ⓢ, P, E), (Ⓢ, B, C), (Ⓢ, B, S), (Ⓢ, B, E), (Ⓢ, C, S), (Ⓢ, C, E), (Ⓢ, S, E), (Ⓜ, P, B), (Ⓜ, P, C), (Ⓜ, P, S), (Ⓜ, P, E), (Ⓜ, B, C), (Ⓜ, B, S), (Ⓜ, B, E), (Ⓜ, C, S), (Ⓜ, C, E), (Ⓜ, S, E)

20 combinations

12 Let's Investigate the Characteristics of Data and Make Judgments

AE ① A…30 m or more and less than 35 m

B…35 m or more and less than 40 m

② Team B A…7 students, B…8 students

③ (Example) The histogram for Team A has the highest bar in the middle and looks almost symmetric. Team B has higher bars on the right side.

AF Ⓐ Group 1 (group 1…285 g, group 2…283.5 g)

Ⓑ Group 2 (group 1…260 g or more and less than 270 g, group 2…280 g or more and less than 290 g)

AF (Example) Disagree

Reason…The company should focus on the size of shoes that sold the most (the mode).

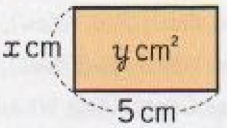
Let's Try WONDERful Problems!

2 Let's Express Quantities and Their Relationships as Math Sentences

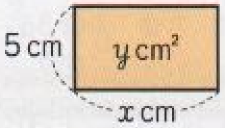
Answers → Page 265

1 Think about the area of rectangles.

Ⓐ The area of a rectangle with the width of x cm and the length of 5 cm is y cm^2.



Ⓑ The area of a rectangle with the width of 5 cm and the length of x cm is y cm^2.



① In each of the situations Ⓐ and Ⓑ above, express the relationship between x and y as a math sentence.
If the value of x is given, is the value of y determined?

② Misaki says as follows. Is what she says correct or incorrect?

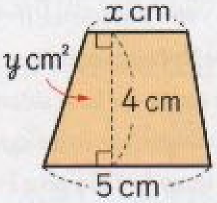
In both Ⓐ and Ⓑ, y is proportional to x.


Misaki

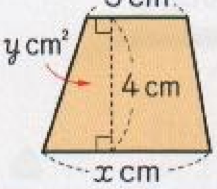

Find it out with tables.

Next, think about the area of trapezoids.

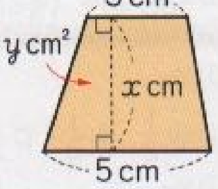
Ⓒ The area of a trapezoid with the top base of x cm, the bottom base of 5 cm, and the height of 4 cm is y cm^2.



Ⓓ The area of a trapezoid with the top base of 3 cm, the bottom base of x cm, and the height of 4 cm is y cm^2.



Ⓔ The area of a trapezoid with the top base of 3 cm, the bottom base of 5 cm, and the height of x cm is y cm^2.



③ In each of the situations Ⓒ, Ⓓ, and Ⓔ above, express the relationship between x and y as a math sentence.
If the value of x is given, is the value of y determined?

④ In which situation(s) among Ⓒ, Ⓓ, and Ⓔ is y proportional to x? Find it out with tables.

3 Let's Think about Multiplication of Fractions

Answers→ Page 265

1 Fill in the blanks Ⓐ through Ⓙ so that the multiplication math sentences work both horizontally and vertically.

①

$\frac{3}{5}$	×	Ⓐ	=	$\frac{21}{40}$
×		×		
Ⓑ	×	$\frac{5}{9}$	=	$\frac{5}{63}$
=		=		
Ⓒ	×	Ⓓ	=	Ⓔ

Kota

$\frac{3}{5}\times\boxed{Ⓐ}=\frac{21}{40}$

$\frac{3\times b}{5\times a}=\frac{21}{40}$

Simplify the fractions to make their denominators as small as possible.
In ②, use proper fractions, mixed numbers, and whole numbers to fill the blanks.

②

Ⓕ	×	$2\frac{2}{3}$	=	$3\frac{1}{3}$
×		×		
$1\frac{3}{5}$	×	Ⓖ	=	$1\frac{13}{15}$
=		=		
Ⓗ	×	Ⓘ	=	Ⓙ

Shiho

$\boxed{Ⓕ}\times\frac{8}{3}=\frac{10}{3}$

A fraction that is equivalent to $\frac{10}{3}$ and has a numerator that can be divided by 8 is $\frac{40}{12}$, so...

2 Use each of the numbers from 3 through 7 once to fill the $\boxed{}$ in the math sentence below. What math sentence has the largest product?

$$\boxed{}\frac{\boxed{}}{\boxed{}}\times\frac{\boxed{}}{\boxed{}}$$

In the mixed number, make sure that the numerator is smaller than the denominator.

4 Let's Think about Division of Fractions

Answers → Page 265

Mathematics Junior High School

1 The quotient can be expressed as a fraction, as shown on the right. So far, we have only studied fractions with numerators and denominators that are whole numbers. Now, we are going to expand our thinking by using fractions and decimal numbers. Complete the following calculations and write the quotients as fractions.

$$a \div b = \frac{a}{b}$$

① $0.4 \div 0.7 = \dfrac{0.4}{0.7}$

$= \dfrac{0.4 \times 10}{0.7 \times 10}$

② $3 \div 1.25 = \dfrac{3}{1.25}$

$= \dfrac{3 \times 100}{1.25 \times 100}$

③ $\dfrac{2}{3} \div \dfrac{8}{9} = \dfrac{\frac{2}{3}}{\frac{8}{9}}$

$= \dfrac{\frac{2}{3} \times \frac{9}{8}}{\frac{8}{9} \times \frac{9}{8}}$

They are using the fact that the size of a fraction does not change even if the numerator and the denominator are multiplied by the same number, aren't they?

④ $1.5 \div 2.25$

⑤ $\dfrac{24}{7} \div \dfrac{48}{35}$

2 Write +, −, ×, or ÷ in each ☐ to complete the math sentences.

① $\frac{1}{2}\ \square\ \frac{1}{3}\ \square\ \frac{1}{6} = 1$

② $\frac{1}{6}\ \square\ \frac{1}{3}\ \square\ \frac{1}{4} = 2$

③ $\frac{1}{4}\ \square\ \frac{1}{2}\ \square\ \frac{1}{6} = 3$

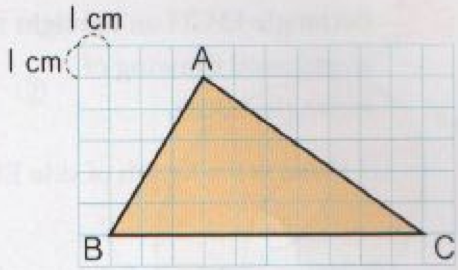
④ $\frac{1}{3}\ \square\ \frac{1}{2}\ \square\ \frac{1}{6} = 4$

If you want to calculate something first, you may use ( ). Is there only one answer for each?

5 Let's Investigate How to Express Rates

Answers→ Page 265

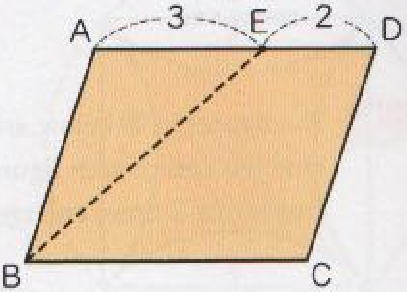
1 The area of triangle ABC on the right is 25 cm².



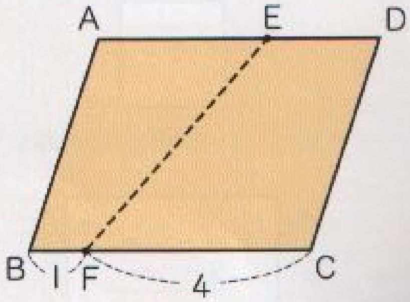
① Draw point D so that BD : DC=2 : 3.

② Calculate the area of triangle ABD and triangle ADC.

2 Parallelogram ABCD is split into two shapes by a line.

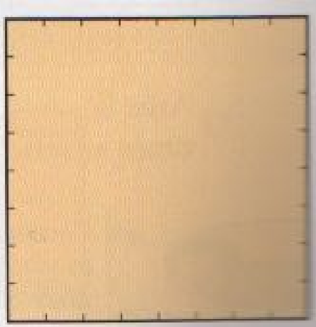


① Point E is drawn so that AE : ED =3 : 2. Find the ratio of the area of triangle ABE and the area of quadrilateral EBCD.



② Next, point F is drawn so that BF : FC =1 : 4. Find the ratio of the area of quadrilateral ABFE and the area of quadrilateral EFCD.

3 Split the square into 3 shapes using 2 lines so that the ratio of the areas will be 1 : 3 : 4.

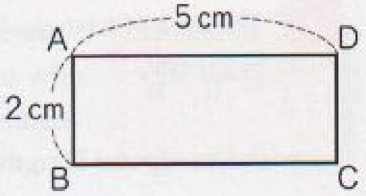


There are lots of different answers.

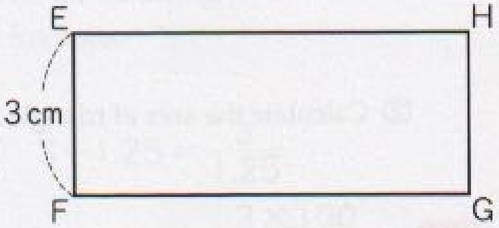
6 Let's Investigate Geometric Figures that Have the Same Shape but Different Size

Answers → Page 265

1 Rectangle EFGH on the right is an enlarged drawing of rectangle ABCD.



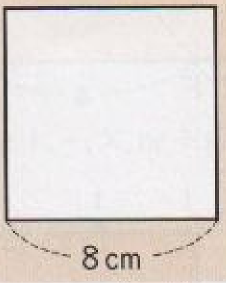
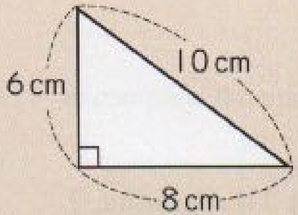
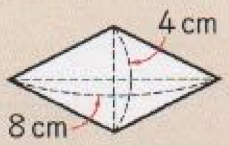
① What is the length of side EH?



② We are going to draw an enlarged drawing of rectangle EFGH.
If we make side EF longer by 5 cm, side EH must be extended by how many cm?

Mathematics 3rd Grade of Junior High

2 The figures in Ⓑ below are 2 times enlarged drawings of the figures in Ⓐ. Find the area of each figure and determine how many times as much the area is of a 2 times enlarged drawing as the area of the original figure.

	Square	Right Triangle	Rhombus
Ⓐ	 	 	
Ⓑ	 	 	
	Area is ☐ times as much	Area is ☐ times as much	Area is ☐ times as much

I wonder how many times as much the area will be when the length of the sides are made 3 times as long. What if we made the length of sides 4 times as long...

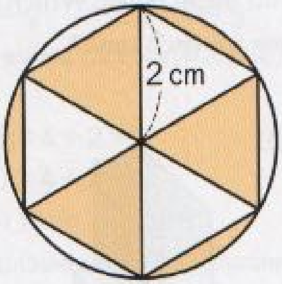
When the length of the sides are made 3 times as long, the area...

7 Let's Think about How to Calculate the Area of Circles

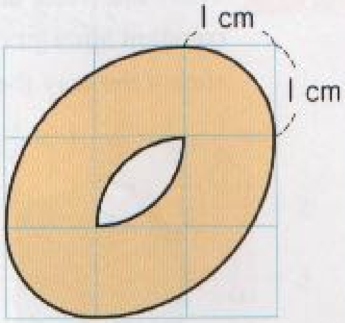
Answers→ Page 265

1 Calculate the area of each of the shaded regions.

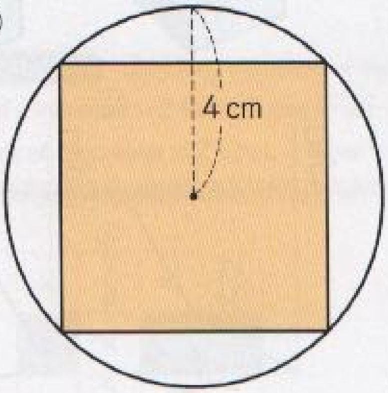
①



②

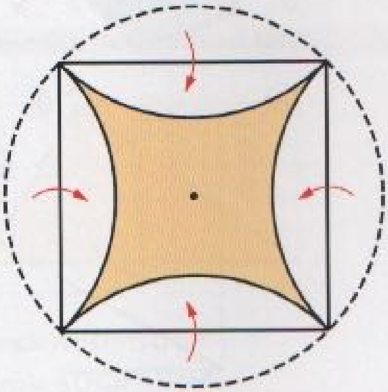


③



Fold along the sides of the square.

④

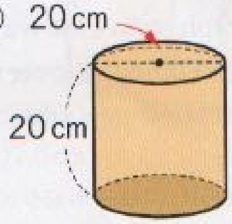


8 Let's Think about How to Calculate the Volume of Prisms and Cylinders

Answers→ Page 265

1 How many times as much is the volume of cylinder Ⓐ as the volume of cylinder Ⓑ?

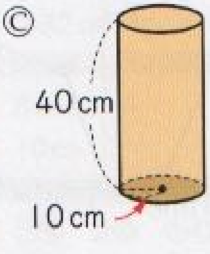
Ⓐ



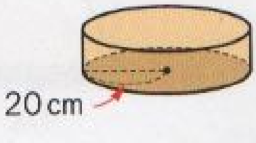
Ⓑ

2 If cylinders Ⓒ and Ⓓ are of equal volume, find the height of Ⓓ.

Ⓒ



Ⓓ



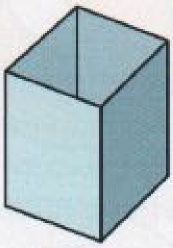
10 Let's Further Investigate Proportional Relationships

Answers→ Page 265

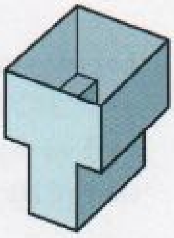
Mathematics 2nd Grade of Junior High

1 There are water tanks in the shapes shown below. We are going to pour a constant amount of water every minute into these tanks. Which graph shows the way the depth changes over time for each tank?

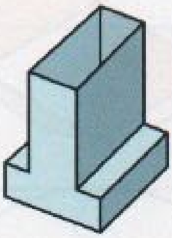
①



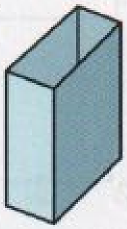
②



③

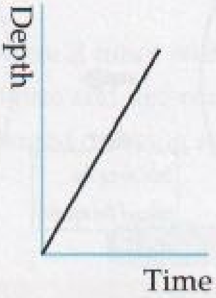


④

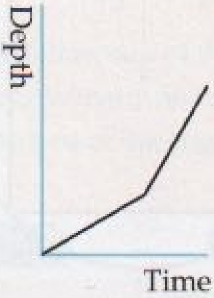


Ⓐ

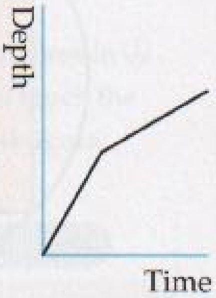
Ⓑ



Ⓒ

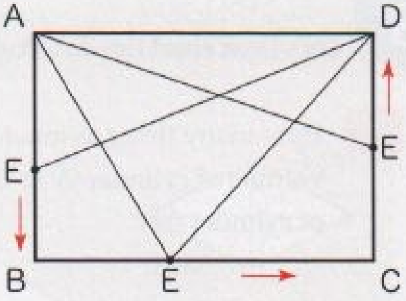


Ⓓ

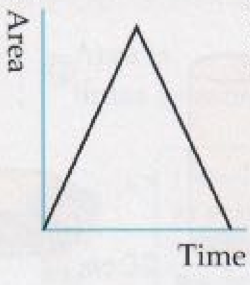


Mathematics 2nd Grade of Junior High

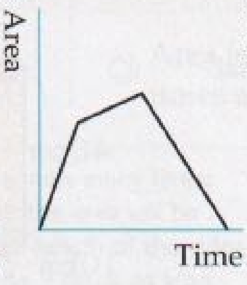
2 Point E is moving along the sides of rectangle ABCD at a constant speed from A → B → C → D.
Which graph shows the way the area of triangle AED changes over time?



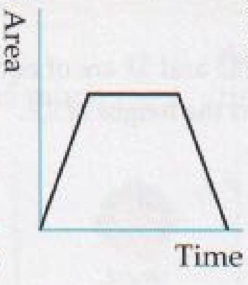
Ⓕ



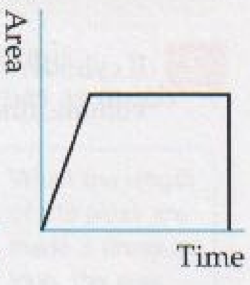
Ⓖ



Ⓗ



Ⓘ



② Let's Express Quantities and Their Relationships as Math Sentences

1 ①Ⓐ $x \times 5 = y$ Ⓑ $5 \times x = y$

It is determined (in both Ⓐ and Ⓑ).

② Correct

③Ⓒ $(x+5) \times 4 \div 2 = y$

Ⓓ $(3+x) \times 4 \div 2 = y$

Ⓔ $(3+5) \times x \div 2 = y$

It is determined (in all the situations Ⓒ through Ⓔ).

④Ⓔ

Ways of Thinking ②④ Find out whether the value of y increases to 2 times, 3 times… as the value of x increases to 2 times, 3 times…

③ Let's Think about Multiplication of Fractions

1 ①

			Ⓐ		
	$\frac{3}{5}$	×	$\frac{7}{8}$	=	$\frac{21}{40}$
	×	▨	×	▨	▨
Ⓑ	$\frac{1}{7}$	×	$\frac{5}{9}$	=	$\frac{5}{63}$
	=	▨	=	▨	▨
	$\frac{3}{35}$	×	$\frac{35}{72}$	=	$\frac{1}{24}$
	Ⓒ		Ⓓ		Ⓔ

②

	Ⓕ				
②	$1\frac{1}{4}$	×	$2\frac{2}{3}$	=	$3\frac{1}{3}$
	×	▨	×	▨	▨
Ⓖ	$1\frac{3}{5}$	×	$1\frac{1}{6}$	=	$1\frac{13}{15}$
	=	▨	=	▨	▨
	2	×	$3\frac{1}{9}$	=	$6\frac{2}{9}$
	Ⓗ		Ⓘ		Ⓙ

2 $6\frac{4}{5} \times \frac{7}{3}$

④ Let's Think about Division of Fractions

1 ① $\frac{4}{7}$ ② $\frac{12}{5}\left(2\frac{2}{5}\right)$ ③ $\frac{3}{4}$

④ $\frac{2}{3}$ ⑤ $\frac{5}{2}\left(2\frac{1}{2}\right)$

2 (Example)

① $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$, $\frac{1}{2}\times\frac{1}{3}\div\frac{1}{6}=1$

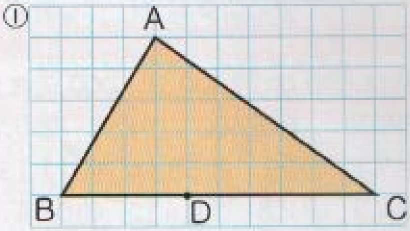
② $\frac{1}{6}\div\frac{1}{3}\div\frac{1}{4}=2$, $\left(\frac{1}{6}+\frac{1}{3}\right)\div\frac{1}{4}=2$

③ $\frac{1}{4}\div\frac{1}{2}\div\frac{1}{6}=3$, $\frac{1}{4}\div\left(\frac{1}{2}\times\frac{1}{6}\right)=3$

④ $\frac{1}{3}\div\frac{1}{2}\div\frac{1}{6}=4$, $\frac{1}{3}\div\left(\frac{1}{2}\times\frac{1}{6}\right)=4$

⑤ Let's Investigate How to Express Rates

1 ①

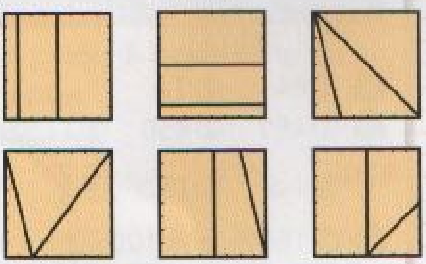


② Triangle ABD…10 cm^2

Triangle ADC…15 cm^2

2 ①3 : 7 ②2 : 3

3 (Example)



⑥ Let's Investigate Geometric Figures that Have the Same Shape but Different Size

1 ①7.5 cm ②12.5 cm

2 (in this order) 4, 4, 4

⑦ Let's Think about How to Calculate the Area of Circles

1 ①6.28 cm^2 ②6.28 cm^2

③32 cm^2 ④13.76 cm^2

⑧ Let's Think about How to Calculate the Volume of Prisms and Cylinders

1 8 times

2 10 cm

⑩ Let's Further Investigate Proportional Relationships

1 ①Ⓐ ②Ⓓ ③Ⓒ ④Ⓑ

2 Ⓗ

13 Answers Finishing Touches

1 Numbers and Calculations

1 How to Express and the Structure of Numbers Page 199

1 ①208,050,000 ②357
③600 million ④4,700

2 ①42.195 ②12.9 ③1,230
④0.0098 ⑤$\frac{2}{3}$ ⑥20

3 ①0.125 ②1.8 ③$\frac{7}{10}$ ④$2\frac{3}{100}\left(\frac{203}{100}\right)$

4 ①0.2 ②1.4 ③2.5 ④3
⑤3.8 ⑥$\frac{2}{3}$ ⑦$\frac{5}{3}\left(1\frac{2}{3}\right)$ ⑧$\frac{11}{3}\left(3\frac{2}{3}\right)$

2 Addition and Subtraction Pages 200 and 201

5 ①11,400 ②39,500 ③1,000 ④8,103
⑤700 ⑥5,600 ⑦687 ⑧7,746

6 ①7.1 ②20 ③58.14 ④11.13
⑤2.3 ⑥2.8 ⑦0.432 ⑧0.64

7 ①$\frac{9}{7}\left(1\frac{2}{7}\right)$ ②$\frac{17}{15}\left(1\frac{2}{15}\right)$
③$3\frac{11}{12}\left(\frac{47}{12}\right)$ ④$\frac{3}{4}$ ⑤$\frac{13}{35}$
⑥$1\frac{7}{18}\left(\frac{25}{18}\right)$

8 ①84 ②7.6

9 ①497 ②850 ③11.75 ④20.1
⑤1.6 ⑥13.3 ⑦$\frac{2}{3}$ ⑧2

10 ①$1500-x=700$ ②$x+150=y$
③$x+y=800$

3 Multiplication and Division Page 203

11 ①24,702 ②136.268 ③34
④8 ⑤20.8 ⑥6.12
⑦36.608 ⑧8.33 ⑨1.2
⑩3.5 ⑪3.2 ⑫5
⑬4 ⑭$\frac{2}{3}$ ⑮1
⑯$\frac{9}{4}\left(2\frac{1}{4}\right)$ ⑰$\frac{2}{7}$ ⑱$\frac{13}{18}$

12 ①87 ②20 ③$\frac{1}{15}$ ④$\frac{47}{24}\left(1\frac{23}{24}\right)$

13 ①700 ②72 ③4.8 ④180
⑤2,994 ⑥23.23

14 ①$150\times x=750$ ②$x\times 1.5=y$

4 Properties and Processing of Numbers Page 205

15 ①even ②odd ③odd ④even

16 ①8 ②15 ③40

17 ①7 ②16 ③6

18 ①24,000 ②100,000
③400,000 ④900,000

19 greater than or equal to 47,500, and less than 48,500

20 Ami…Ⓓ(Ⓒ) Kota…Ⓒ

2 Geometric Figures

1 The Properties of Geometric Figures Pages 207 and 208

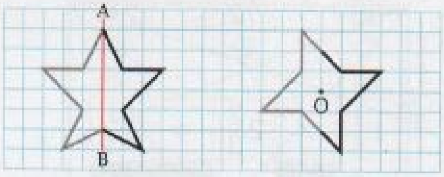
1

Name / Characteristic	Quadrilateral	Trapezoid	Parallelogram	Rhombus	Rectangle	Square
Two pairs of opposite sides are parallel			○	○	○	○
All four sides have equal lengths				○		○
All four angles have equal measures					○	○
The lengths of the two diagonals are equal					○	○
The two diagonals are perpendicular to each other				○		○

2 Line Symmetric…Ⓐ, Ⓒ, Ⓔ
Point Symmetric…Ⓑ, Ⓒ, Ⓓ

3 (Omitted)

4



5 ⓐ 50° ⓑ 110° ⓒ 60° ⓓ 120°

2 Area and Volume Pages 209 and 210

6 ①10 cm^2 ②17.64 cm^2 ③12.5 cm^2
④5 cm^2 ⑤15 cm^2 ⑥60 cm^2

7 ①530 m^2 ②530 m^2

8 ①Area…12.56 cm^2
Length around…12.56 cm
②Area…25.12 cm^2
Length around…33.12 cm

9 ①204 cm^3 ②512 m^3 ③320 cm^3

10 ①22.5 cm^3 ②706.5 cm^3 ③32 cm^3

3 Measurement

1 Comparing Quantities and Units Pages 212 and 213

1 ①(in this order) shaku, 3, 90.9

②(in this order) sun, 5, 15.15

2 ① cm ② m ③ km²

④ cm² ⑤ g ⑥ kg

3 ① 10 ② 100 ③ 10,000

④ 100 ⑤ 100 ⑥ 1,000,000

⑦ 1,000 ⑧ 1,000,000

⑨ 1,000 ⑩ 1,000

4 Changes and Relationships

1 Changes and Proportional and Inverse Proportional Relationships Pages 214 and 215

1

Width (cm)	1	2	3	4	5	6
Length around (cm)	12	14	16	18	20	22

2 ①

Number of triangles x	1	2	3	4	5	6
Number of sticks y	3	5	7	9	11	13

② 21 sticks

3 ① Proportional Relationships…Ⓑ

Inverse Proportional Relationships…Ⓐ

② Ⓐ $y = 20 \div x$ Ⓑ $y = 40 \times x$

③ Ⓖ

4 Ⓐ(Example)

The length of each side x(cm)	1	2	3	4
Area y(cm²)	1	4	9	16

Ⓑ(Example)

Height x(cm)	1	2	3	4	5
Area y(cm²)	2	4	6	8	10

Ⓒ(Example)

Speed per minute x(km/min)	1	2	4	5	10
Time y(min)	20	10	5	4	2

Proportional Relationships…Ⓑ

Inverse Proportional Relationships…Ⓒ

2 Speed and Per Unit Quantity Page 217

5 ① 400L (2,800 ÷ 7 = 400)

② 146,000L (400 × 365 = 146,000)

6 Room B

7 ① 125m per minute (500 ÷ 4 = 125)

② 14m per second (420 ÷ 30 = 14)

③ 6,000m (200 × 30 = 6,000)

④ 50km

$$\left(\begin{array}{l}1\text{ hours }15\text{ minutes} = 1\frac{15}{60}\text{hours} = 1\frac{1}{4}\text{hours}\\ 40 \times 1\frac{1}{4} = 50\end{array}\right)$$

⑤ 30 minutes

$$\left(\begin{array}{l}1.8\text{km} = 1{,}800\text{m}\\ 1{,}800 \div 60 = 30\end{array}\right)$$

8 ① 228km per hour

$$\left(\begin{array}{l}2\text{ hours }25\text{ minutes} = 2\frac{25}{60}\text{hours} = 2\frac{5}{12}\text{hours}\\ 552 \div 2\frac{5}{12} = \frac{6{,}624}{29} = 228.4\cdots\end{array}\right)$$

② 63m per second

$$\left(\begin{array}{l}2\text{ hours }25\text{ minutes} = 145\text{ minutes} = 8{,}700\text{ seconds}\\ 552\text{km} = 552{,}000\text{m}\\ 552{,}000 \div 8{,}700 = 63.4\cdots\end{array}\right)$$

3 Rates Page 219

9 ① 1% ② 10% ③ 105% ④ 150%

10 ① 2.1 ② 40 ③ 6 ④ 1,600

11 ① 120% ② 675 people ③ 3,000 people

12 ① $\frac{2}{7}$ ② 5 : 3 ③ 5

13 400mL

5 Utilization of Data Pages 222 and 223

1 ① Ⓐ Bar graph Ⓑ Broken line graph

Ⓒ Pie chart

Ⓓ Histogram

② (1) Ⓑ (2) Ⓐ (3) Ⓒ (4) Ⓓ

③ (1)

④ Not correct

(Reason) (Example) At East School, 360 × 0.25 = 90 students liked fried bread, while at West School, 180 × 0.25 = 45 students liked fried bread. So, East School has more students who liked bread.

2 ① Classroom A…34 m Classroom B…33 m

② No

③ Classroom A…34 m Classroom B…36 m

④ Classroom A…34 m Classroom B…34.5 m

⑤ Classroom A…greater than or equal to 30 m, and less than 35 m

Classroom B…greater than or equal to 35 m, and less than 40 m

Answers Do You Remember?

Page 33

1 ① 10 ② 0.91 ③ 6.13
④ 49.64 ⑤ 6.144 ⑥ 2
⑦ 2.05 ⑧ 2.5 ⑨ 38
⑩ 19.8

2 ① 3 ② 5 ③ 7 ④ 8

3 ① C
② Math sentence… $300 \times 0.6 (= 180)$
Answer… 180 g

Playing with Numbers and Calculations

① Ⓐ 756 Ⓑ 756
② Ⓐ 1,472 Ⓑ 1,472
③ Ⓐ 20.16 Ⓑ 20.16

Page 53

1 ① greater than or equal to 935,000, and less than 945,000
② greater than or equal to 34,500, and less than 35,500

2 ① 28 ② 24 ③ 30

3 ① 9 ② 6 ③ 12

4 ① $64 - x = y$ ② $x \times 4 = y$
③ $2 \div x = y$

5 ① 2.5 kg ② 0.4 m

Playing with Numbers and Calculations

① $a = 20$
② $b = 2,\ c = 30$ | $b = 10,\ c = 6$
$b = 3,\ c = 20$ | $b = 12,\ c = 5$
$b = 4,\ c = 15$ | $b = 15,\ c = 4$
$b = 5,\ c = 12$ | $b = 20,\ c = 3$
$b = 6,\ c = 10$ | $b = 30,\ c = 2$
③ $d = 2,\ e = 3,\ f = 5$
$d = 2,\ e = 5,\ f = 3$

Page 75

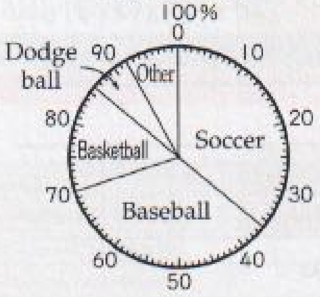
1 ① $\frac{1}{8}$ ② $\frac{14}{5}\left(2\frac{4}{5}\right)$ ③ $\frac{3}{4}$ ④ 20
⑤ $\frac{2}{5}$ ⑥ $\frac{7}{9}$ ⑦ $\frac{3}{14}$

2

Favorite Sports

Sports	Students	Rate (%)
Soccer	108	36
Baseball	102	34
Basketball	48	16
Dodge ball	18	6
Other	24	8
Total	300	100

Favorite Sports



3 ① $\frac{3}{2}\left(1\frac{1}{2}\right)$ times ② $\frac{8}{9}$ L ③ $\frac{4}{7}$

Playing with Numbers and Calculations

① 5 ② 9 ③ 4 ④ 7 ⑤ 8 ⑥ 6

Page 89

1 ① $\frac{3}{4}$ hours ② $\frac{5}{12}$ minutes ③ $\frac{4}{3}$ or $1\frac{1}{3}$ hours

2 ① $\frac{13}{3}$ or $4\frac{1}{3}$ kg ② $\frac{3}{13}$ m

3 The car that travels 250 km on 20 L of gasoline

4 60 km per hour, 1 km per minute

5 ① 28.26 cm ② 2 m

6 (omitted)

Playing with Numbers and Calculations

① Ⓐ $\frac{1}{6}$ Ⓑ $\frac{1}{6}$ ② Ⓐ $\frac{4}{15}$ Ⓑ $\frac{4}{15}$
③ Ⓐ $\frac{9}{40}$ Ⓑ $\frac{9}{40}$

Page 119

1 Ⓒ, Ⓓ

2 8 notebooks for 1,000 yen

3 8.4 people

4 ① 14 cm^2 ② 13.5 cm^2 ③ 50 cm^2

5 ① 125 cm^3 ② 126 cm^3

Playing with Numbers and Calculations

①Ⓐ $\frac{3}{2}\left(1\frac{1}{2}\right)$ Ⓑ $\frac{1}{10}$ Ⓒ $\frac{19}{10}\left(1\frac{9}{10}\right)$

Ⓓ $\frac{7}{5}\left(1\frac{2}{5}\right)$ Ⓔ $\frac{1}{2}$

②Ⓕ $\frac{5}{6}$ Ⓖ $\frac{11}{12}$ Ⓗ $\frac{2}{3}$

Ⓘ $\frac{13}{12}\left(1\frac{1}{12}\right)$ Ⓙ $\frac{5}{12}$

Page 133

1 ① 60° ② 120° ③ 31.4cm

2 ① 60% ② 120 ③ 90 cm^3

3 ① $x = 36$ ② $x = 30$

4 ① $x = 4.5$, $y = 2$

②Ⓑ… $\frac{3}{2}$ or 1.5 times enlarged drawing

Ⓒ… $\frac{1}{2}$ reduced drawing

Playing with Numbers and Calculations

① +, +

② ÷, ÷

③	+, −, −	−, ×, ÷
	−, +, −	−, ÷, ×
	×, −, ×	×, ÷, −
	÷, −, ÷	÷, ×, −
④	+, +, +	+, ÷, −
	×, ÷, ÷	−, +, ÷
	÷, ×, ÷	÷, +, −
	÷, ÷, ×	÷, −, +

Page 163

1 ① $\frac{23}{24}$ ② $\frac{1}{18}$ ③ $\frac{1}{8}$ ④ $\frac{5}{6}$

⑤ 7 ⑥ 4 ⑦ $\frac{2}{3}$

2 ① $80 \times x + 120$ (yen)

② When she bought 3 pencils…360yen

When she bought 8 pencils…760yen

3 ① 432 cm^3 ② 45 cm^3 ③ 197.82 cm^3

4 ① 631 ② 613 ③ 2

Playing with Numbers and Calculations

①Ⓐ 5 Ⓑ 7 Ⓒ 3

②Ⓕ 9 Ⓖ 5 Ⓗ 7 Ⓘ 4

Let's Think Using a Double Number Line Diagram

Given below is the situation of math problem **1** on page 35.

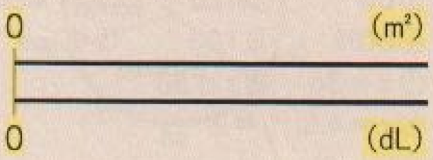
With 1 dL of paint, we can paint $\frac{3}{7}$ m^2 of boards.
How many m^2 of boards can we paint with 2 dL of this paint?

- With 1 dL of paint, we can paint $\frac{3}{7}$ m^2 of boards.
- Find how many m^2 we can paint with 2 dL of this paint.

● Express the multiplication problem above as a number line diagram.

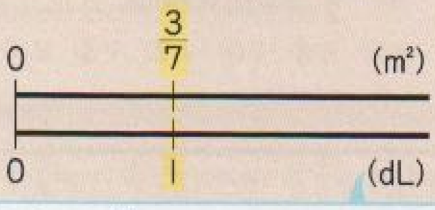
1

At the left end of the diagram, draw a tick mark and write "0."
Draw 2 parallel lines.
At the right end of the bottom line, write "(dL)."
At the right end of the top line, write "(m^2)."



2

Since it is "$\frac{3}{7}$ m^2 with 1 dL," on the bottom line, draw a tick mark that expresses the base amount (1 dL) and write "1." On the top line, draw a tick mark and write "$\frac{3}{7}$."

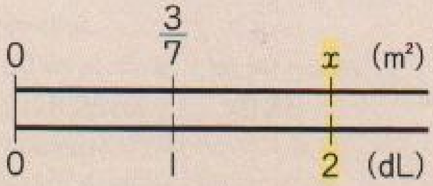


Since you wrote "1" as the base amount at the bottom numbe line, you wrote "(dL)" at the bottom number line as well.

3

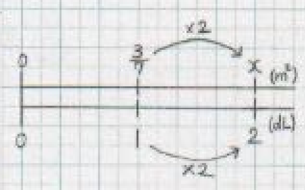
Express the unknown number as x.

Meaning "x m^2 with 2 dL," draw a tick mark that expresses how many times as much 2 dL is as the base amount and write "2" on the bottom line. On the top line, draw a tick mark and "x."



Ami

The area we can paint with 1 dL of this paint is $\frac{3}{7}$ m^2, and we have 2 dL of this paint. So, the answer can be found with the math sentence $\frac{3}{7} \times 2$. Drawing a number line diagram helps you come up with a math sentence.

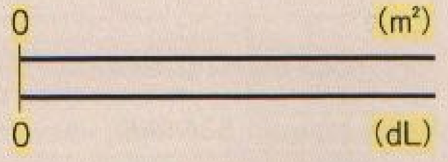


Given below is the situation for math problem **3** on page 38.

With 2 dL of paint, we can paint $\frac{4}{5}$ m² of boards.
How many m² of boards can we paint with 1 dL of this paint?

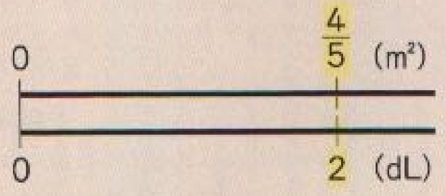
- With 2 dL of paint, we can paint $\frac{4}{5}$ m² of boards.
- Find how many m² we can paint with 1 dL of this paint.

● Express the division problem above as a double number line diagram.



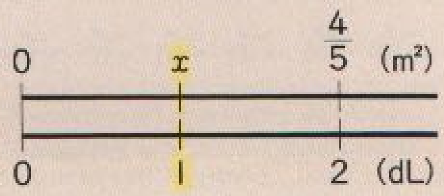
1

At the left end of the diagram, draw a tick mark and write "0."
Draw 2 parallel lines.
At the right end of the bottom line, write "(dL)."
At the right end of the top line, write "(m²)."



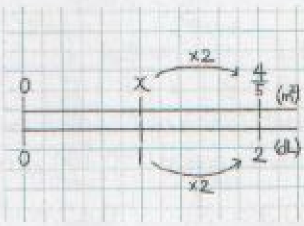
2

Since "$\frac{4}{5}$ m² with 2 dL," at the bottom line, draw a tick mark that expresses the whole amount (2 dL) and write "2." On the top line, draw a tick mark and write "$\frac{4}{5}$."



3 Express the unknown number as x.

Meaning "x m² with 1 dL," draw a tick mark that expresses the base amount (1 dL) and write "1" on the bottom line. On the top line, draw a tick mark and "x."



This can be expressed as the following multiplication sentence:

$x \times 2 = \frac{4}{5}$

So, the math sentence to find the value of x will be:

$x = \frac{4}{5} \div 2$

Riku

Review Exercises

① Properties of Multiplication (4th Grade)

In multiplication, if a multiplicand or a multiplier is made 10 or 100 times as much, the product is also made 10 or 100 times as much.

$$\begin{array}{l} 0.2 \times 34 = 6.8 \\ \downarrow\times 10 \qquad\quad)\times 10 \\ 2 \times 34 = 68 \end{array}$$

$$\begin{array}{l} 2 \times 0.34 = 0.68 \\ \quad\ \downarrow\times 100 \quad)\times 100 \\ 2 \times 34 = 68 \end{array}$$

$$\begin{array}{l} 0.2 \times 0.34 = 0.068 \\ \downarrow\times 10 \ \ \downarrow\times 100 \qquad)\times 1{,}000 \\ 2 \times 34 = 68 \end{array}$$

② Properties of Division (4th Grade)

In division, the quotient does not change if we divide both the dividend and the divisor by the same number.

$$\begin{array}{l} 60 \div 30 = 2 \\ \downarrow\div 10 \ \ \downarrow\div 10 \quad)\text{No change} \\ 6 \div 3 = 2 \end{array}$$

In division, the quotient does not change if we multiply both the dividend and the divisor by the same number.

$$\begin{array}{l} 0.6 \div 0.3 = 2 \\ \downarrow\times 10 \ \ \downarrow\times 10 \quad)\text{No change} \\ 6 \div 3 = 2 \end{array}$$

③ Order and Properties of Operations (4th Grade)

◆ Order of Operations
- Calculate from left to right.
- Calculate what is inside the ( ).
- × and ÷ must be calculated before + and −.

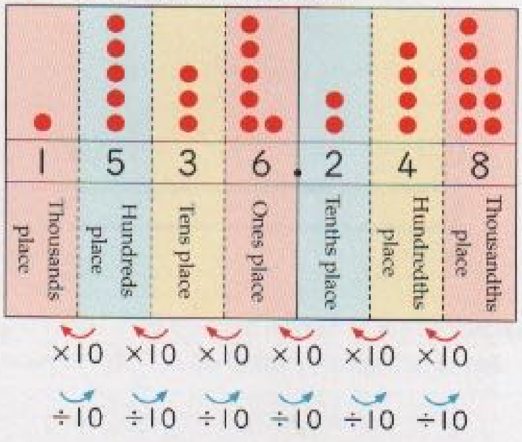
◆ Properties of Operations

(■ + ●) × ▲ = ■ × ▲ + ● × ▲

(■ − ●) × ▲ = ■ × ▲ − ● × ▲

④ Structure of Whole Numbers and Decimal Numbers (4th and 5th Grades)

◆ How to express 1,536.248



⑤ Approximate Numbers (4th Grade)

◆ **Rounding**

If the digit in the place is (0, 1, 2, 3, 4) → round down.

If the digit in the place (5, 6, 7, 8, 9) → round up.

To round to the nearest thousand → round based on the digit in the hundreds place, one place below the thousands	To round to the highest place → round the digit in the second highest place
(thousands) (Hundreds) 13,648 ↓Rounded up 14,000	①② 13,648 ↓Rounded down 10,000
(thousands) (Hundreds) 17,392 ↓Rounded down 17,000	①② 17,392 ↓Rounded up 20,000

⑥ Relationships between Fractions and Decimal / Whole Numbers (5th Grade)

◆ Fractions can be expressed as decimal numbers found by dividing numerators by denominators.

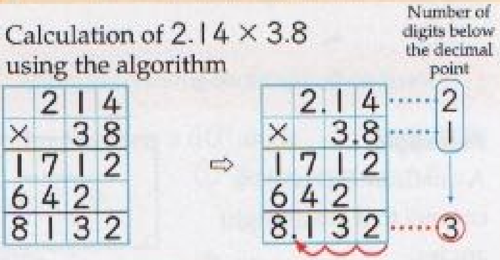
$$\frac{1}{3} = 1 \div 3 = 0.333... \qquad \frac{2}{5} = 2 \div 5 = 0.4$$

◆ Whole and decimal numbers can be expressed as fractions with denominators of 1, 10, 100, and so on.

$$2 = \frac{2}{1} \qquad 0.3 = \frac{3}{10} \qquad 0.47 = \frac{47}{100}$$

⑦ Multiplication Algorithm of Decimal Numbers (5th Grade)

◆ Calculation of 2.14 × 3.8 using the algorithm



Calculate as if there were no decimal points.

Place the decimal point in the product.

⑧ Simplifying Fractions (5th Grade)

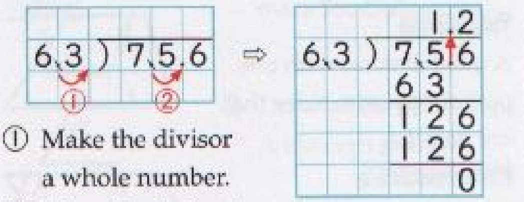
Simplifying Fractions

Refers to dividing both the denominator and the numerator by a common factor.

$\frac{2}{4}=\frac{1}{2}$ $\frac{18}{24}=\frac{3}{4}$ $1\frac{6}{8}=1\frac{3}{4}$

⑨ Division Algorithm of Decimal Numbers (5th Grade)

◆ Calculation of 7.56 ÷ 6.3 using the algorithm



① Make the divisor a whole number.

② Move the decimal point of the dividend.

Place the decimal point in the quotient.

⑩ Finding a Common Denominator (5th Grade)

Finding a common denominator

Refers to changing fractions that have different denominators into equal size fractions with the same denominator.

The common denominator is a common multiple of the original denominators.

$\left(\frac{1}{2},\frac{1}{3}\right) \rightarrow \left(\frac{3}{6},\frac{2}{6}\right)$

$\left(\frac{3}{4},\frac{3}{10}\right) \rightarrow \left(\frac{15}{20},\frac{6}{20}\right)$

⑪ Per Unit Quantity and Speed (5th Grade)

For example, crowdedness can be compared in **per unit quantity**, such as the number of people in 1 m^2 on average.

$\boxed{\text{Crowdedness}} = \boxed{\text{Total number of people}} \div \boxed{\text{Area}}$

Speed is expressed using the distance traveled in a unit amount of time.

Speed per hour ... the speed expressed as the distance traveled in 1 hour

Speed per minute ... the speed expressed as the distance traveled in 1 minute

Speed per second ··· the speed expressed as the distance traveled in 1 second

$\boxed{\text{Speed}} = \boxed{\text{Distance}} \div \boxed{\text{Time}}$

$\rightarrow \boxed{\text{Distance}} = \boxed{\text{Speed}} \times \boxed{\text{Time}}$

$\rightarrow \boxed{\text{Time}} = \boxed{\text{Distance}} \div \boxed{\text{Speed}}$

⑫ Average (5th Grade)

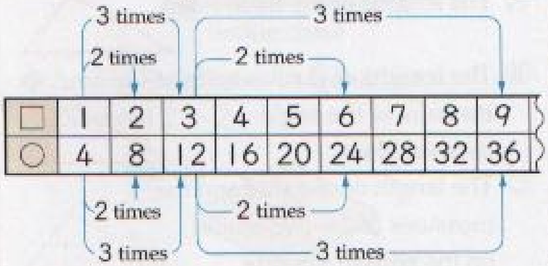
Average

Refers to a number of quantities evened out so that they are all equal.

$\boxed{\text{Average}} = \boxed{\text{Total}} \div \boxed{\text{Number of quantities}}$

⑬ Proportional Relationship (5th Grade)

Suppose there are a pair of quantities, □ and ○, and when □ becomes 2 times, 3 times..., ○ also becomes 2 times, 3 times... In this case, you can say that ○ is **proportional** to □.



□	1	2	3	4	5	6	7	8	9
○	4	8	12	16	20	24	28	32	36

⑭ Rates (5th Grade)

If we consider the base quantity as 1, the number that corresponds to the quantity being compared is called the **rate**.

$\boxed{\text{Rate}} = \boxed{\text{Quantity being compared}} \div \boxed{\text{Base quantity}}$

$\rightarrow \boxed{\text{Quantity being compared}} = \boxed{\text{Base quantity}} \times \boxed{\text{Rate}}$

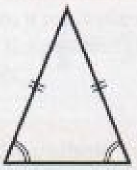
$\rightarrow \boxed{\text{Base quantity}} = \boxed{\text{Quantity being compared}} \div \boxed{\text{Rate}}$

⑮ Different Kinds of Triangles (2nd and 3rd Grades)

Isosceles triangle

A triangle with 2 sides of equal length.

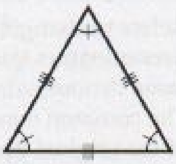
In an isosceles triangle, two angles have the same size.



Equilateral Triangle

A triangle with 3 sides of equal length.

In an equilateral triangle, all three angles have the same size.



Isosceles right triangle

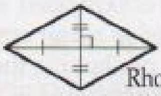
An isosceles triangle that has a right angle.

⑰ Diagonal (4th Grade)

Diagonal

Segments that connect the opposite vertices of quadrilaterals.

Parallelogram



Rhombus

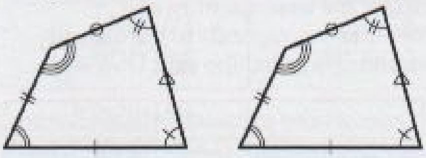
Rectangle

Square

⑱ Congruent Figures (5th Grade)

When two figures can fit on top of each other perfectly, we say that they are **congruent**.

Congruent figures have the same shape and size.



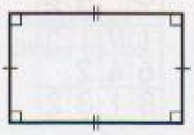
In congruent figures, the lengths of the corresponding sides are equal. Also, the measures of the corresponding angles are equal.

⑯ Various Quadrilaterals (2nd-4th Grades)

Rectangle

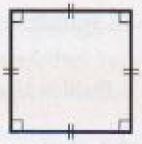
A quadrilateral with 4 corners that are all right angles.

The opposite sides of a rectangle are the same length.



Square

A quadrilateral with 4 corners that are all right angles and all 4 sides are the same length.



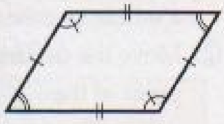
Trapezoid

A quadrilateral with one pair of opposite sides that are parallel.

Parallelogram

A quadrilateral with two pairs of opposite sides that are parallel.

The lengths of opposite sides and the sizes of opposite angles of a parallelogram are equal.

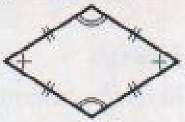


Rhombus

A quadrilateral whose sides are equal in length.

Opposite sides of a rhombus are parallel.

The sizes of opposite angles of a rhombus are equal.



⑲ How to Draw Congruent Triangles (5th Grade)

To draw a congruent triangle, measure one of the following sets of quantities:

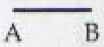
Ⓐ The lengths of the three sides

Ⓑ The lengths of 2 sides and the measure of the angle between them

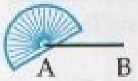
Ⓒ The length of one side and the measures of the two angles on the ends of the side

20 How to Draw an Angle (4th Grade)

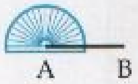
◆ How to draw a 60° angle



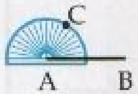
① Draw segment AB.



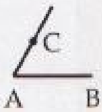
② Line up the center of the protractor with point A.



③ Line up the 0° line with segment AB.



④ Mark a point at the tick mark for 60°.



⑤ Draw a line from point A through point C.

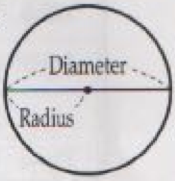
21 Sum of the Angles of a Triangle or a Quadrilateral (5th Grade)

◆ The sum of the 3 angles of any triangle is 180°.
ⓐ+ⓑ+ⓒ = 180°

◆ The sum of the 4 angles of any quadrilateral is 360°.
ⓕ+ⓖ+ⓗ+ⓘ = 360°

22 Length Around Circles (5th Grade)

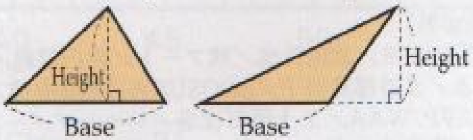
The circumference of any circle is about 3.14 (Pi) times as long as the diameter.
Circumference = Diameter × 3.14
= Radius × 2 × 3.14



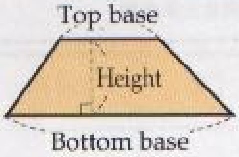
23 The Formulas for Calculating Area of Triangles and Quadrilaterals (5th Grade)

◆ Area of parallelogram = Base × Height

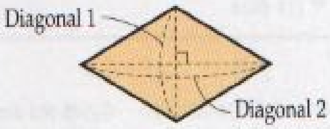
◆ Area of triangle = Base × Height ÷ 2



◆ Area of Trapezoid = (Top Base + Bottom Base) × Height ÷ 2

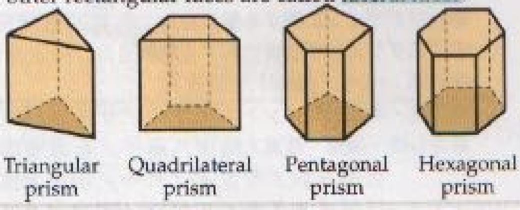


◆ Area of Rhombus = Diagonal 1 × Diagonal 2 ÷ 2



24 Prisms (5th Grade)

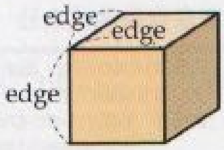
Solid figures like the ones below are called **prisms**. In a prism, the top and the bottom faces that are opposite of each other are called **bases**, and the other rectangular faces are called **lateral faces**.



25 The Formulas for Calculating Volume of Cubes and Cuboids (5th Grade)

◆ Volume of cuboids = length × width × height

◆ Volume of cubes = edge × edge × edge



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Numbers of Jumps Classroom A Made

Day 1	① 61
Day 2	② 60
Day 3	③ 57
Day 4	④ 62
Day 5	⑤ 55
Day 6	⑥ 56
Day 7	⑦ 64
Day 8	⑧ 63
Day 9	⑨ 67
Day 10	⑩ 62
Day 11	⑪ 68
Day 12	⑫ 63
Day 13	⑬ 70
Day 14	⑭ 62
Day 15	⑮ 66

Numbers of Jumps Classroom B Made

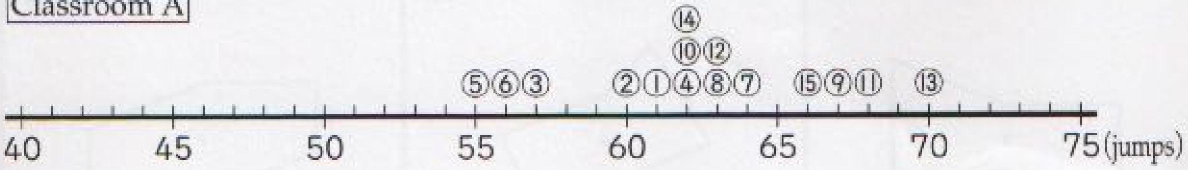
① 54
② 55
③ 53
④ 56
⑤ 65
⑥ 65
⑦ 70
⑧ 67
⑨ 68
⑩ 70
⑪ 56
⑫ 56
⑬ 71
⑭ 67

Numbers of Jumps Classroom C Made

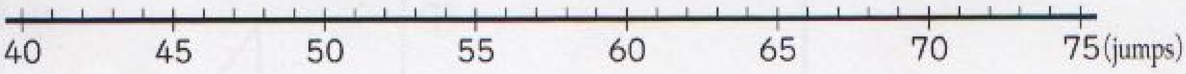
① 56
② 60
③ 60
④ 55
⑤ 59
⑥ 58
⑦ 56
⑧ 57
⑨ 63
⑩ 40
⑪ 67
⑫ 70
⑬ 65
⑭ 73
⑮ 70
⑯ 61
⑰ 70

To be used when you study "How to Analyze Data" on page 176

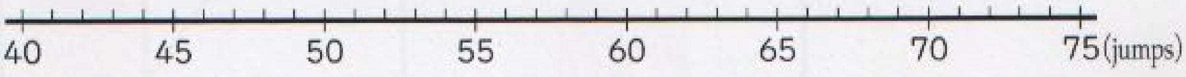
Classroom A



Classroom B



Classroom C



To be used on page 180

To be used on page 10

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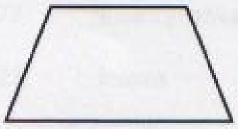
Ⓑ

Ⓒ

Ⓓ

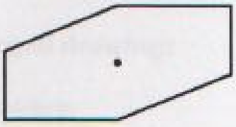
Ⓔ



Fold along the perforation and then cut the pieces out.

To be used on page 14

Ⓕ



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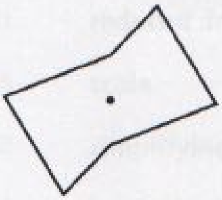

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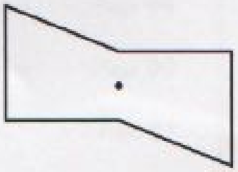

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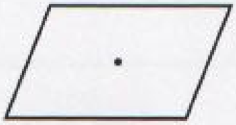

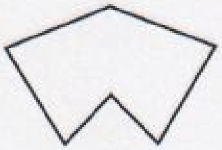

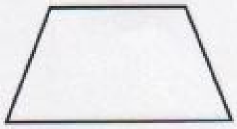

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If you use these terms and symbols correctly, you will find it easier to think and explain your ideas to others.

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