New Mathematics
for Elementary School

5A



TOKYO SHOSEKI

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About the "D" Symbol

＊Available in Japanese only.

- Sections marked with this symbol have additional materials available online.
- Tell your teacher or parent before you use the Internet.

To Study Online

Use the URL below or QR code on the right to access online contents.

https://tsho.jp/02p/m5a/

Notice to Teachers and Parents

Use of the online contents marked with the "D" symbol is free of charge, if you have Internet access.

Prior learning			Later learning
4th Grade Perpendicular and Parallel Lines and Quadrilaterals	6	Congruent Shapes Let's Investigate Figures That Have the Same Shape and Size ··· 72	6th Grade Well-balanced Geometric Figures/ Geometric Figures that Have the Same Shape but Different Size
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Notice to Teachers and Parents

"New Mathematics 5A Plus" is an optional learning material for students who want or need to work on it.
Not all students need to use "New Mathematics 5A Plus."

5B Table of Contents

10 Addition and Subtraction of Fractions
11 Average
12 Per Unit Quantity
13 Area of Quadrilaterals and Triangles
14 Rates
15 Percentage Bar Graphs and Pie Charts
16 Investigating Changes
17 Regular Polygons and Length Around Circles
18 Prisms and Cylinders

- To understand the structures of large whole numbers and decimal numbers, and how to calculate with these numbers, use prior learning.
- Carefully look at lengths of sides, measures of angles, and perpendicular and parallel sides.
- When investigating the relationship between the two quantities that change in relationship to each other...
- To solve a problem, what type of table or graph should you use.

Based on what you learned though fourth grade, further explore the world of mathematics in fifth grade.



Mathlin

Mathlin gives you some hints as you study.

Doorways to Learning

1

We are going to arrange counters in the shape of an equilateral triangle.
How many counters will be needed to make the 10th equilateral triangle?

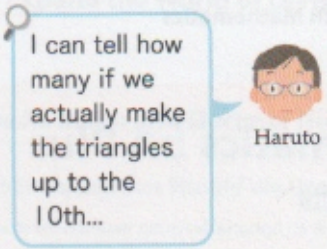


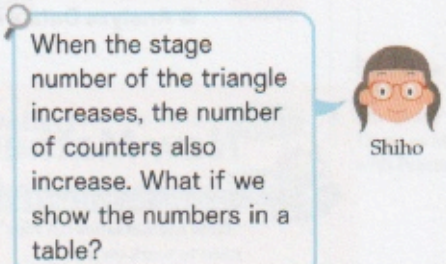
Grasp the problem.

- What problem are we going to work on today?

- What idea may be useful to solve the problem?
- Is there anything you have learned before that you can use to solve this problem?

1 Plan how to find the answer.





Let's think about how to find the number of counters by focusing on the relationship between the stage number and the number of counters at each stage.

Write down your ideas.

- Is your idea clear to others?

2 Put your idea into a diagram, a table, and a math sentence.



There are diagrams you can use on page 155.



Stage number □	1	2	3	4	5	
Number of counters ○ (pieces)	3	6				

Misaki and some other students are explaining their classmates' ideas.

Kota



1st 2nd 3rd 4th ...

For the 10th shape, $10 \times 3 = 30$ Answer 30 pieces

Haruto

Stage number □	1	2	3	4	5	
Number of counters ○ (pieces)	3	6	9	12	15	

$10 \times 3 = 30$ Answer 30 pieces

Kota thinks that the 10th shape will have □ groups of 10 pieces...



Misaki

Learn with your classmates.

3 Look at Kota's drawing and math sentence and explain his idea.

4 Look at Haruto's table and math sentence and explain his idea.



What pattern did he find in the table?

5 Look back and summarize today's lesson.

- Can you understand your classmates' ideas are based on their diagrams, tables and math sentences?
- What is common and what is different about your own idea and your classmates' ideas?
- What are the good points in your classmates' ideas?

Summary

If we find the relationship between the stage number and the number of counters needed at each stage using a diagram or a table, you can find answers through calculation, however large the numbers are.

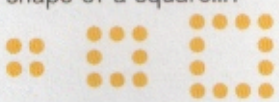
Look back and summarize today's lesson.

- What did you learn from today's investigation?
- Which way of thinking was useful?
- What do you want to investigate next?



Riku

What if we arrange counters in the shape of a square...?



...

The next page shows a photocopy of my notebook page.



Kota

Doorways to Learning

Keep a Math Notebook



When studying mathematics, you use what you learned before. Keep a good record of your learning in your notebook so that you can look back and solve new problems.



I wonder what Kota wrote in his notebook.



Kota

Write down today's date and math problem, and grasp the problem.

Write down your idea.

April 11

<Problem>

We are going to arrange counters in the shape of an equilateral triangle.
How many counters will be needed to make the 10th equilateral triangle?



- Think about how to find the number of counters by focusing on the relationship between the stage number and how many counters there are.

<My idea>



The third triangle has three groups of three, the fourth one has three groups of four, so the tenth one has three groups of ten.

10 × 3 = 30 Answer 30 pieces

Note Taking Tip 1

If he made a mistake, he did not erase it.
He drew a line ═══ through it.

Note Taking Tip 2

In a balloon, he wrote about what he needed to be careful of and what he noticed during the lesson.

<Haruto's Idea>

Stage number □	1 (×3)	2 (×3)	3 (×3)	4 (×3)	5 (×3)	6 (×3)
the number of counters ○(pieces)	3	6	9	12	15	18

Looking at the table vertically, it has a relationship of □ × 3 = ○.

10 × 3 = 30 Answer 30 pieces

1

<summary>

If we find the relationship between the stage number and the number of counters needed at each stage using a diagram or a table, you can find answers through calculation, however large the numbers are.

<My Reflection>

In both figures and tables, by considering several cases, I was able to find the relationship between the stage number and the number of counters.

2

Looking at the table of Haruto, the number of counters increases by three!

Write down your classmates' ideas that you thought were useful.

Look back and summarize today's lesson.

Write down your reflections on today's lesson.

In <My Reflection>, write down ideas like those listed below so that you can check your progress:

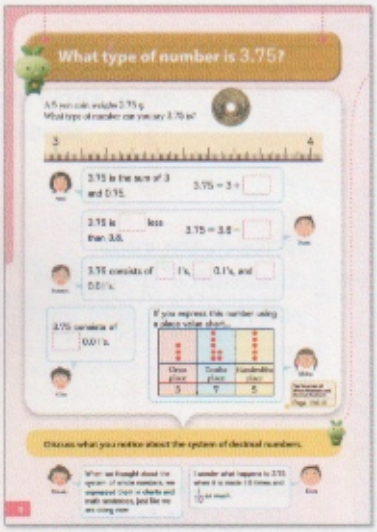
- What you noticed
- What you want to investigate next
- What you have learned to do
- What you thought as you listened to your classmates' ideas

Door ways to Learning

A Guide to Learning with This Textbook

Step to Learning

Entry to Learning



Discuss the following with your classmates and set a goal for what to learn:

- **Math in your daily life**
- **What you have learned so far**

Today's Learning

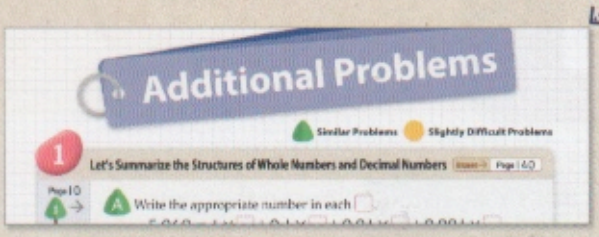
- **Today's Problem**
- **Goal of the Lesson**
- **Hint for Your Thinking**
- **Key Viewpoints and Ways of Thinking**
- **Summary**
- **Practice Problem**

  Key calculation problems are shaded.

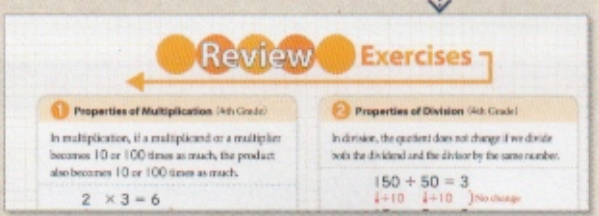
Additional Problems → Page 128 A

Properties of Multiplication Page 150 ①

New Mathematics Plus (Additional Material)



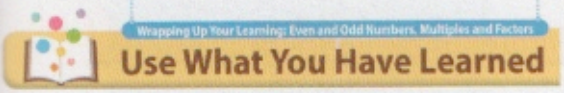
Need more practice? Try these!
The answers are toward the back of the textbook so that you can check them yourself.



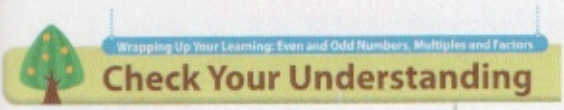
Use this section when you want to review what you learned before.

"New Mathematics Plus" is useful if you want to study more.

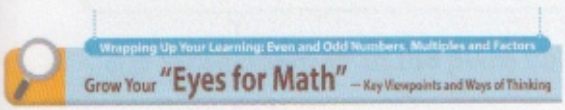
Wrapping Up Your Learning



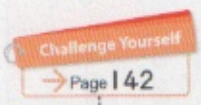
Put what you just learned into use.

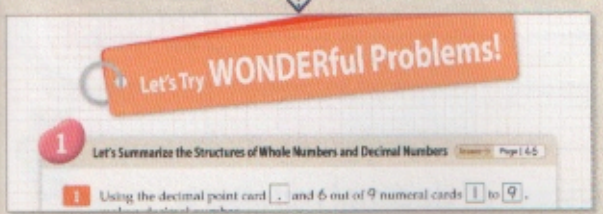


Review what you just learned and practice.



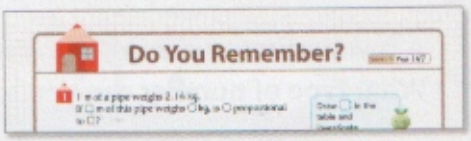
Summarize key viewpoints and ways of thinking.



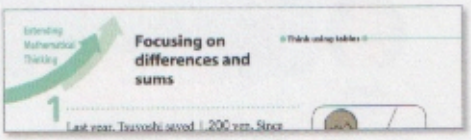


Use this section to explore more and dig deeper. The answers are toward the back of the textbook so that you can check them yourself.

Other Pages



Review what you learned before. The answers are toward the back of the textbook so that you can check them yourself.

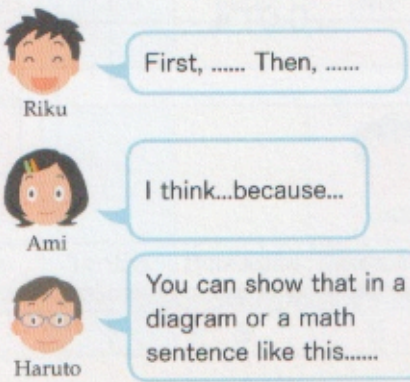


Think using a diagram or a table.



Find information in graphs and tables and use it to solve problems.

Share What You Think!



Notice to Teachers and Parents

- Sections marked with Advanced include topics not covered by the National Course of Study for fifth grade. Not all students need to work on these sections.
- "Do You Remember?" sections are not for studying new topics and not included in the number of class hours. Use these sections for self-study and home learning.

What type of number is 3.75?

A 5 yen coin weighs 3.75 g.
What type of number can you say 3.75 is?






Ami

3.75 is the sum of 3 and 0.75. $3.75 = 3 + \square$

3.75 is $\square$ less than 3.8. $3.75 = 3.8 - \square$


Kota


Haruto

3.75 consists of $\square$ 1's, $\square$ 0.1's, and $\square$ 0.01's.

3.75 consists of $\square$ 0.01's.


Riku

If you express this number using a place value chart...

Ones place	Tenths place	Hundredths place
3	7	5


Shiho

The Structure of Whole Numbers and Decimal Numbers
Page 150 ④

Discuss what you notice about the system of decimal numbers.


Misaki

When we thought about the system of whole numbers, we expressed them in charts and math sentences, just like we are doing now.

I wonder what happens to 3.75 when it is made 10 times and $\frac{1}{10}$ as much.


Kota

1 Let's Summarize the Structures of Whole Numbers and Decimal Numbers

Elevation of Tokugo Pass (in Matsumoto City, Nagano Prefecture, Japan)



Diameter of a hammer throw circle



1 Compare two numbers: 2,135 and 2.135.

1 Write down ● in the place value chart below to express each of these numbers.

2,135

Thousands place	Hundreds place	Tens place	Ones place	Tenths place	Hundredths place	Thousandths place
2	1	Ⓐ 3	5			

2.135

Thousands place	Hundreds place	Tens place	Ones place	Tenths place	Hundredths place	Thousandths place
			2 .	1	Ⓑ 3	5

Let's summarize the structures of whole numbers and decimal numbers.

2 How many groups of what number does the "3" marked Ⓐ indicate there are? What about the "3" marked Ⓑ?

Examine other numbers, too.

3 Fill in the □ with appropriate numbers to describe 2.135.

□ 1's ····· 2

□ 0.1's ····· 0.1

□ 0.01's ····· 0.03

□ 0.001's ····· 0.005

2.135 in total

□ 1,000's ····· 2,000

□ 100's ····· 100

□ 10's ····· 30

□ 1's ····· 5

2,135 in total



Shiho

4 Fill in the □ with appropriate numbers to show the structure of the number 2.135 as a math sentence.

$2.135 = 1 \times \square + 0.1 \times \square + 0.01 \times \square + 0.001 \times \square$



Kota

$2{,}135 = 1{,}000 \times \square + 100 \times \square + 10 \times \square + 1 \times \square$

Summary

In whole numbers and decimal numbers, where a numeral (0 through 9) is written determines which place it is in. Each numeral indicates how many groups of the place value are in the numbers.

Whole numbers and decimal numbers share the same structure.

We can use the numerals 0 through 9 and a decimal point to express whole numbers and decimal numbers of any size.



Whenever we have 10 of any place value, we regroup them with 1 of the next higher place value.

1 Fill in the □ with appropriate numbers.

$7.608 = 1 \times \square + 0.1 \times \square + 0.01 \times \square + 0.001 \times \square$

Additional Problems → Page 128 A

2 Fill in the □ with appropriate inequality signs.

Inequality Signs Page 150 ⑤

① 0.1 □ 0 ② 2.967 □ 3 ③ 3 □ 3.15 − 1.5

Additional Problems → Page 128 B



Riku

We investigated the structure of numbers by focusing on place value and how many groups of a numeral were represented.

2 How many 0.001's are in 2.135?

Let's think about ways to look at numbers as groups of 0.001.

1 How many 0.001's are in 0.005, 0.03, 0.1, and 2?

0.005 ····· ☐ 0.001's

0.03 ····· ☐ 0.001's

0.1 ····· ☐ 0.001's

2 ····· ☐ 0.001's

2.135 has ☐ 0.001's in it.

Ones place	Tenths place	Hundredths place	Thousandths place
2	1	3	5
0	0	0	1


Haruto

If you select a decimal place value carefully, **you can use whole numbers to examine the sizes of decimal numbers.**

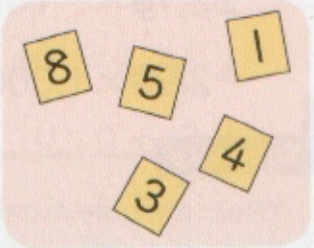

Ami

3 How many 0.001's are in the numbers ① through ④ below?

① 0.003 ② 0.048 ③ 0.999 ④ 6.7

Additional Problems → Page 128 C

3 Place the numeral cards on the right in the ☐'s below to make different numbers, large and small.



☐☐.☐☐☐


Use all the cards.

Let's use the structure of numbers to think about the following questions.

2 Among the numbers you can make, what is the smallest number?

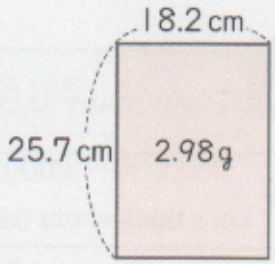
3 Among the numbers you can make, what is the second largest number?

4 Among the numbers you can make, what is the number closest to 50?


Misaki

I wonder what happens if we make other problems by changing the numerals on the cards and the position of the decimal point.

A piece of paper used for this textbook weighs approximately 2.98 g before printing.



4 Write down the numbers that are 10 times, 100 times and 1,000 times as much as 2.98 in the chart below.

Thousands place	Hundreds place	Ten place	Ones place	Tenth place	Hundredths place	Thousandths place
			2	. 9	8	

10 times, 100 times, 10 times, 1,000 times, 10 times

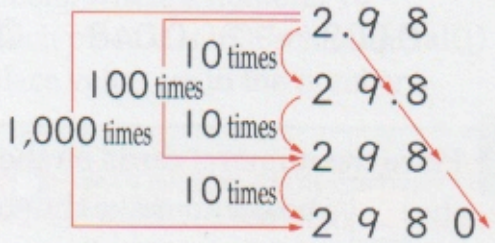
Let's investigate what happens to 2.98 when it is made 10, 100, and 1,000 times as much.

1. When a number is made 10, 100 and 1,000 times as much, what happens to the place values of the digits?
2. Write math sentences that show making 2.98 10, 100 and 1,000 times as much.

$2.98 \times 10 =$ ☐

$2.98 \times 100 =$ ☐

$2.98 \times 1{,}000 =$ ☐



Summary

When a whole number and decimal number is made 10, 100, ... times as much,

- **the digits** move up 1, 2, ... places to the left.
- **the decimal point** moves 1, 2, ... places to the right.

4 How many times as much as 6.19 are 61.9, 619 and 6,190?

5 ① 2.37×10 ② $15.2 \times 1{,}000$ ③ 3.14×100



Kota: Today, we examined numbers by multiplying them by 10, 100... Next...

Tokyo Skytree is 634 m tall.



5 Write the numbers that are $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1,000}$ of 634 in the chart below.

Thousands place	Hundreds place	Ten place	Ones place	Tenth place	Hundredths place	Thousandths place
	6	3	4			

$\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1,000}$

Let's investigate what happens to 634 when it is made $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ as much.

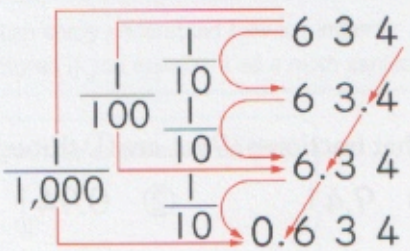
1 When a number is made $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ as much, what happens to the place values of the digits?

2 Write math sentences that show $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ of 634.

$634 \div 10 =$ ☐

$634 \div 100 =$ ☐

$634 \div 1{,}000 =$ ☐



Summary

When whole numbers and decimal numbers are made $\frac{1}{10}$, $\frac{1}{100}$ ... as much,

- **the digits** move down 1, 2, ... places to the right.
- **the decimal point** moves 1, 2, ... places to the left.

6 What fraction of 12.4 are 1.24, 0.124 and 0.0124?

7 ① $35.6 \div 10$ ② $23.85 \div 1{,}000$ ③ $62.5 \div 100$

Check Your Understanding

1 Write the appropriate number in each □.

① $873 = 100 \times \square + 10 \times \square + 1 \times \square$

② $3.05 = 1 \times \square + 0.1 \times \square + 0.01 \times \square$

Can you express the structure of whole numbers and decimal numbers with a math sentence?

Page 9 1

2 Write the appropriate inequality sign in each □.

① $0 \square 0.001$ ② $51 \square 51.2 - 2$

Can you tell which number is greater or less?

Page 9 1

3 How many 0.001's are in 4.823?

Can you tell how many groups of the base amount are in the number?

Page 11 2

4 How many times as much as 0.325 are ① through ④ below?

① 32.5 ② 3,250 ③ 3.25 ④ 325

Can you tell how many times as much one number is as another by looking at the position of the decimal point?

Page 12 4

5 What fraction of 94.1 are ① through ③ below?

① 9.41 ② 0.941 ③ 0.0941

Can you tell what fraction of one number another is by looking at the position of the decimal point?

Page 13 5

6 Calculate.

① 341.9×10 ② 9.81×100

③ $67.5 \times 1{,}000$ ④ $341.9 \div 10$

⑤ $9.81 \div 100$ ⑥ $67.5 \div 1{,}000$

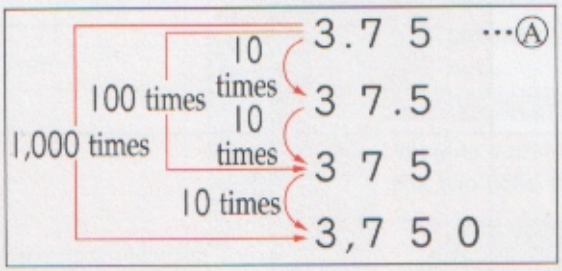
Can you find what 10 times and $\frac{1}{10}$ as much as numbers are?

①~③ Page 12 4
④~⑥ Page 13 5

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the structures of whole numbers and decimal numbers and summarize what they have in common

Riku and Misaki are reviewing what they learned about whole numbers and decimal numbers.
Write the appropriate number or word in each □.




Riku

Whole numbers and decimal numbers share the same structure. In whole numbers and decimal numbers, where a numeral is written determines which place it is in. Each numeral indicates how many groups of the number that represents its place there are.
3.75, marked Ⓐ, can be expressed as the following math sentence:

$3.75 = 1 \times \square + 0.1 \times \square + 0.01 \times \square$


You can easily understand how a number is structured if you express it as a math sentence.


Misaki

10, 100, and 1,000 times of 3.75 can be expressed as the following math sentences:

$3.75 \times 10 = \square$ ···Ⓑ
$3.75 \times 100 = \square$
$3.75 \times 1{,}000 = \square$

Whole numbers and decimal numbers share the same structure. So, you can make 10 times of 3.75 by moving the decimal point 1 place to the □ as shown in Ⓑ.

Look back on what you have learned in "Let's Summarize the Structures of Whole Numbers and Decimal Numbers" and discuss.


Ami

Now I can use the numerals 0 through 9 and a decimal point to express whole numbers and decimal numbers of any size.

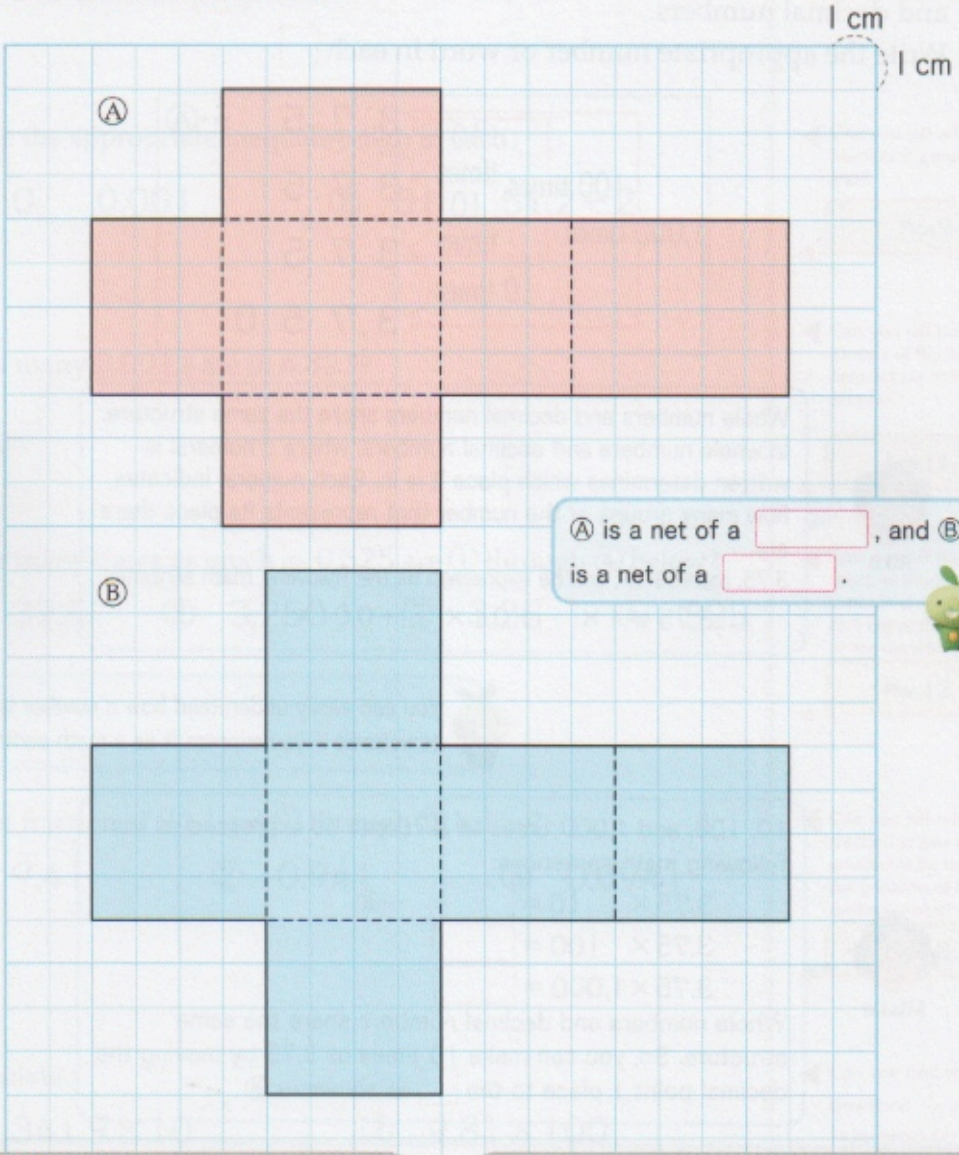

Haruto

Whole numbers and decimal numbers share the same structure, but what about fractions? I want to investigate fractions in detail, too.

Challenge Yourself
→Page 142

How big will these solids be?

Guess which solid figure takes up more space, the one made from net Ⓐ or Ⓑ?



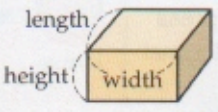
Ⓐ is a net of a _____, and Ⓑ is a net of a _____.

Discuss your guess and why you think so.



Misaki

The size of a cuboid depends on the lengths of three sides: length, width, and height.



I wonder if we can compare the amount of space based on the lengths of three sides.

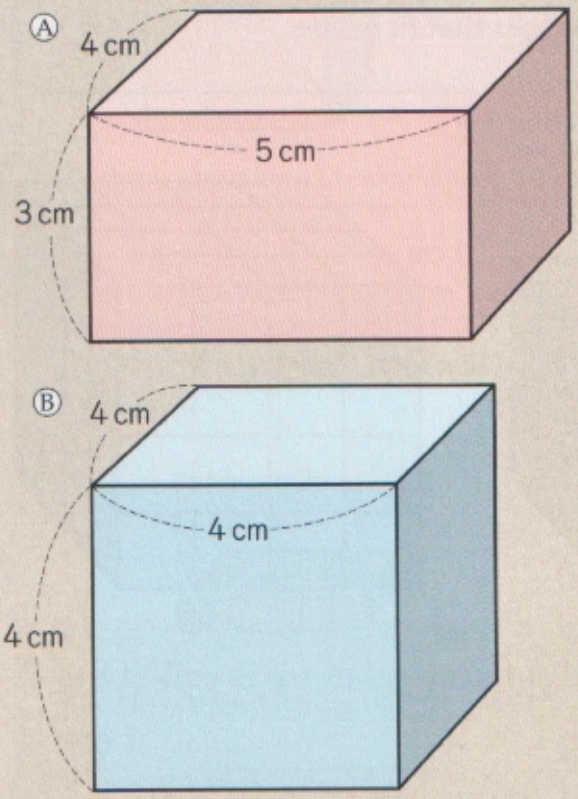


Riku

2 Volume of Cubes and Cuboids

Let's Think about How to Express the Amount of Space Occupied by Cubes and Cuboids

They drew and cut out nets Ⓐ and Ⓑ and made the cube and the cuboid.

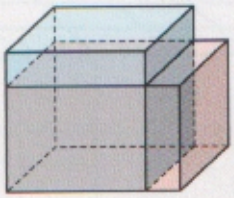


The sum of the length, width, and height is the same between the two, but...



Ami

Imagine superimposing the two solid figures...



Haruto

Both have a part that sticks out of the other.



Kota

I wonder how I can compare Ⓐ and Ⓑ in the amount of space they take up?

1 How to Express the Amount of Space

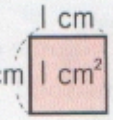
1 Which one takes up more space, cuboid Ⓐ or cube Ⓑ, and by how much? Think about how to compare them.

It looks like we can compare them if we make them using the same sized blocks.



Shiho

We can express a length by telling how many cm fill the length, and an area by telling how many 1 cm^2 squares can be placed inside.

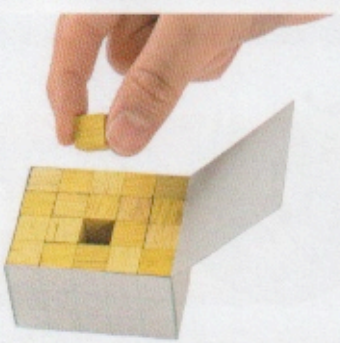




Riku

Let's think about how to express the amount of space that solid figures take up.

How to Express Area
Page 151 ⑪

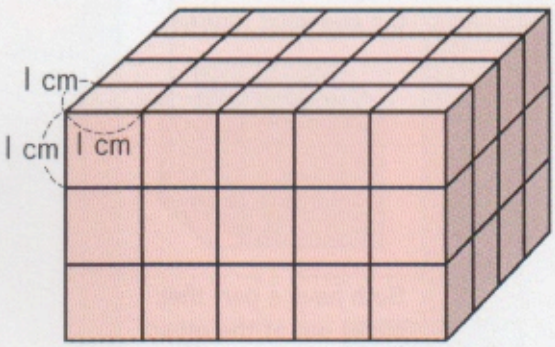


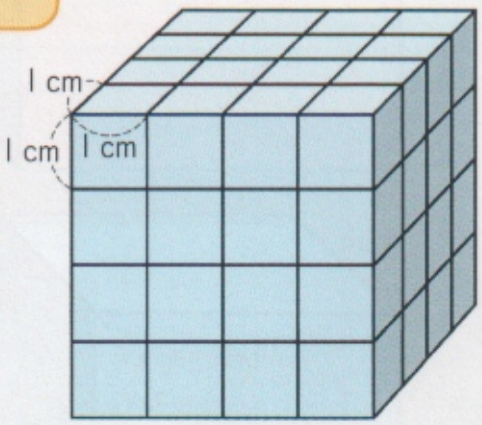
1 How many cubes with 1 cm edges will fill up Ⓐ and Ⓑ on the previous page?
Which solid figure takes up more space? How much more?

Summary

We can express the amount of space cubes and cuboids take up by **the number of cubes with 1 cm edges that fit inside.**

An amount of space is expressed by the number of the base amount that fits inside, just like length and area.

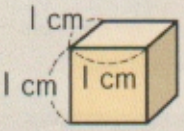




The amount of space a solid figure takes up is called the **volume** of the solid figure.

The volume of a cube with 1 cm edges is **1 cubic centimeter**, and we write it as **1 cm^3.**

1 cm^3

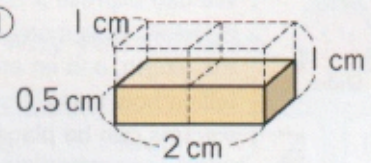


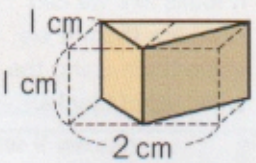
2 What is the volume of each Ⓐ and Ⓑ on the previous page in cm^3?
Which one is bigger, and by how much?

1 Use 24 cube blocks with 1 cm edges to make many different cuboids.

What are the volumes of the cuboids you made in cm^3?

2 Find the volume of each of the solid figures on the right.

① 

② 

English

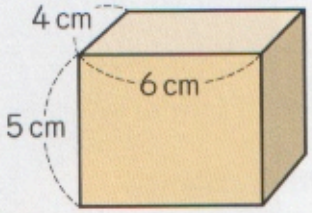
'Taiseki' is called 'volume' in English.

Misaki

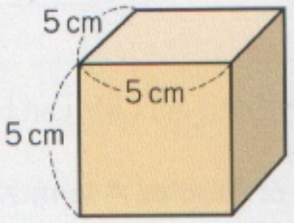
It looks like we can calculate volume just like we calculate area.

2 Find the volumes of the cuboid © and the cube Ⓓ below.

©



Ⓓ



Counting the number of 1 cm^3 cubes is tedious, isn't it?

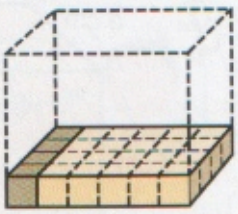


Haruto

Let's think about how to calculate the volume.

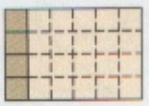
1 Investigate how many 1 cm^3 cubes will fit in ©.

(1) How many 1 cm^3 cubes will be in the bottom layer?



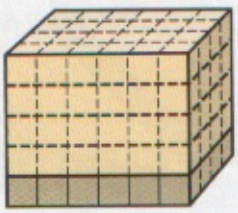


If you look at it from directly above...



□ × □ = □ (pieces)

(2) How many layers will fit inside?



□ (layers)



Kota

The height is □ cm, so...

(3) Calculate the total number of 1 cm^3 cubes.

The number of 1 cm^3 that will fit in © is 120 because:

$4 \times 6 \times 5 = 120$

So the volume of © is 120 cm^3.

The length, width, and height of a cuboid are...



Ami

2 Calculate the volume of Ⓓ.

To calculate the volume of a cuboid or a cube,

1. Measure the length, width, and height.
2. Multiply the three numbers that express the lengths of the three edges.


Shiho

You can find the volume of a cube or cuboid if you know its length, width, and height.

Summary

The volume of cubes and cuboids can be calculated using the following formulas.

Volume of cuboids = length × width × height

Volume of cubes = edge × edge × edge

This is similar to the way we calculated areas of rectangles and squares.


Riku

The length of one edge is all you need to find the volume of a cube.

3 What are the volumes of the cube and cuboids below in cm^3?

① 6 cm, 7 cm, 5 cm

② 8 cm, 8 cm, 8 cm

③ 6 cm, 4 cm, 6 cm, 4 cm

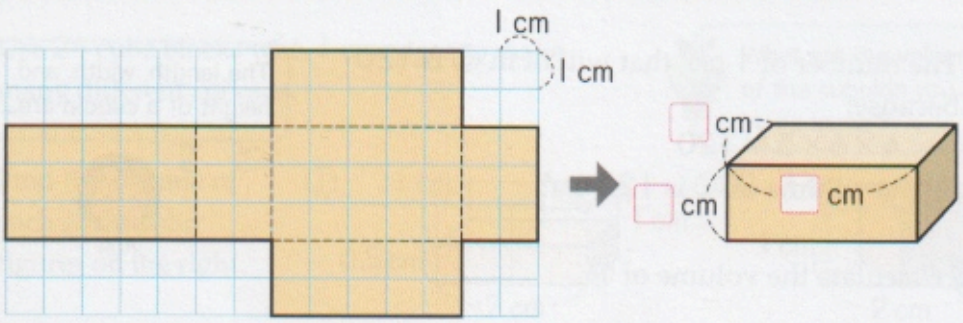
④ 1 m, 10 cm, 40 cm

Units of Length
Page 151 ⑫


Be careful with the units.

Additional Problems
→Page 128 D

4 The picture below is a net of a cuboid. Find the volume of this cuboid.

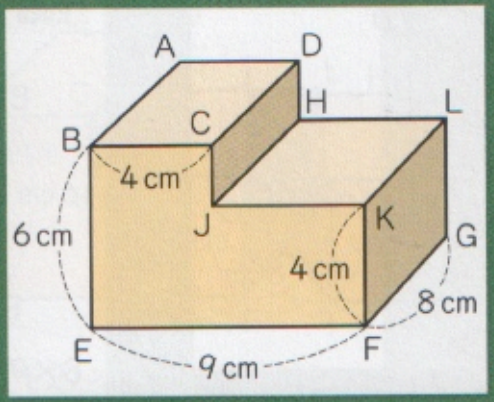



Haruto

If you use a formula, you can calculate volume easily.

How to find volume

3 Find the volume of the solid figure on the right.





Grasp the problem.

- What problem are we going to work on today?

1 Plan how to find the answer.

If you carefully look at the shape of a face...


Kota

When we found the area of a shape like  ...


Ami

Let's think about how to find the volume of solid figures like  .

- What idea may be useful to solve the problem?
- Is there anything you have learned before that you can use to solve this problem?

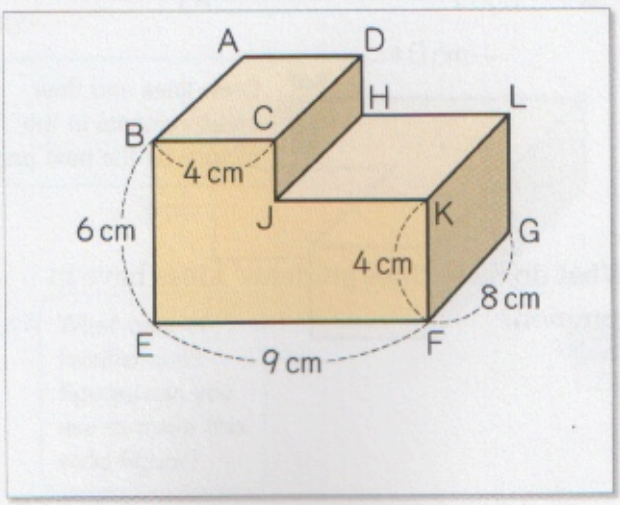
2 Express your idea using pictures, math sentences, and words


There are diagrams you can use on page 155.

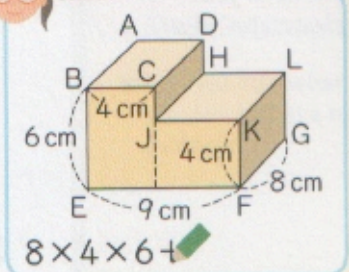

〈Write and Move〉

Write down your ideas.

- Is your idea clear to others?



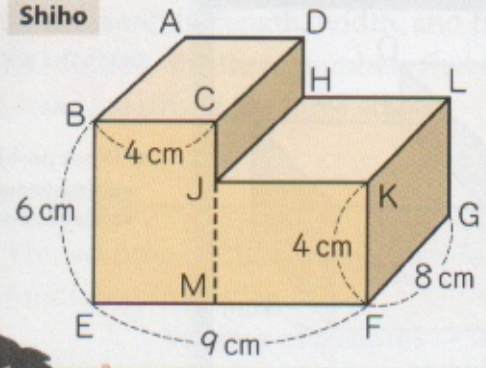

Shiho



- If you figured it out one way, try to think of another way.

Riku and his classmates are explaining their classmates' ideas.

Shiho

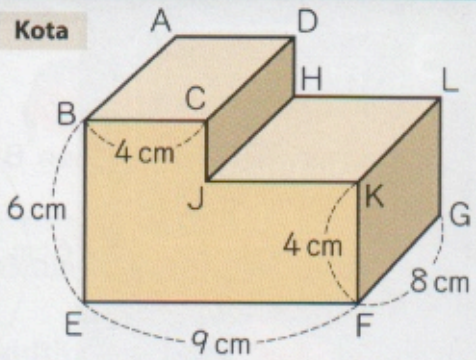




I think Shiho's idea is to use the segment connecting J and M to...

Riku

Kota



$8 \times 9 \times 6 - 8 \times 5 \times 2 = 432 - 80$

$= 352$

Answer $352\ cm^3$

Learn with your classmates.

- Can you understand your classmates' ideas based on their diagrams and math sentences?
- What is common and what is different about your own idea and your classmates' ideas?
- What are the good points in your classmates' ideas?

3 Look at the picture Shiho drew, and write math sentences to express her idea.

4 Look at Kota's math sentence, and explain his idea using pictures.



Draw lines and their measurements in the picture above.

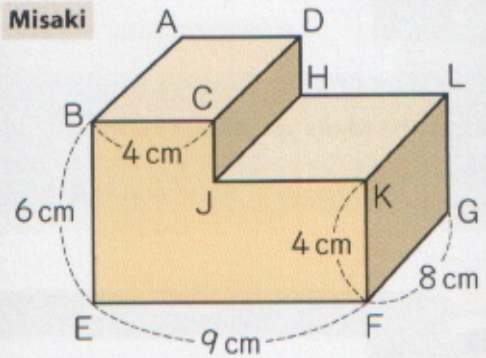
5 Look at Misaki's math sentence shown on the next page, and explain her idea using pictures.



Draw lines and their measurements in the picture on the next page.

6 What do these three students' ideas have in common?

Misaki



$8 \times (9+2) \times 4 = 8 \times 11 \times 4$

$= 352$

Answer 352 cm³

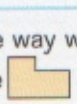
Ami

Haruto

7 Look back and summarize today's lesson.

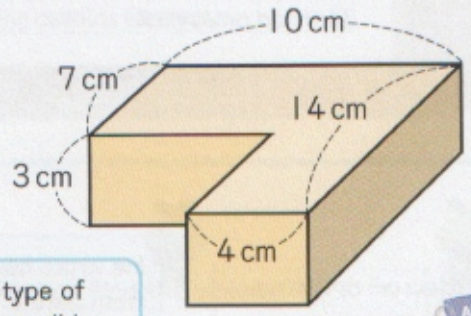
Summary

You can calculate the volume of solid figures like  **by thinking about how cubes and cuboids make up the figures.**

This approach is similar to the way we thought about the areas of shapes like  by using rectangles and squares.

Riku

5 Calculate the volume of the solid figure below in different ways.



Misaki

What type of familiar solid figures can you use to make this solid figure?

Additional Problems → Page 129 E

Look back and summarize today's lesson.

- What did you learn from today's investigation?
- Which way of thinking was useful?
- What do you want to investigate next?

Put it into use.

- Can you use what you learned in a new ploblem?



Look back at the ideas you used to solve problems.

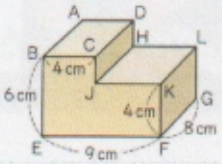


Shiho

April 22

<Problem>

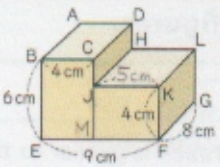
Find the volume of the solid figure on the right.



- Let's think about how to find the volume of solid figures like  .

She broke the solid figure into cubes or cuboids because she knew how to calculate their volumes.

<My idea>



I thought about the figure as made up of two cuboids.

$8 \times 4 \times 6 + 8 \times 5 \times 4 = 192 + 160$

$= 352$

1 The formula for determining the volume of a cuboid was learned on April 21.

Answer 352 cm^3

When you work on problems, not only write math sentences and answers but also consider using:

- **Diagrams**
- **Tables**
- **Graphs**

Classmates' reflections.



Riku

I was able to use the idea of splitting the figure like we did to find the area of  in Grade 4.



He wrote how his learning from before was useful.

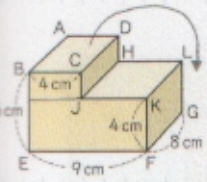
Note Taking Tip 1

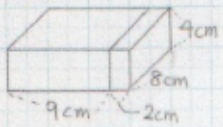
Today's lesson was based on what she had learned. So, she added the date of the lesson so that she can go back to the page of her notebook.

Note Taking Tip 2

She drew a diagram to show how she rearranged parts of the original solid figure in addition to a math sentence so that her classmate's idea was clearly represented.

<Misaki's idea> ②





The figure was split and moved into one cuboid.

$? \times (9 + 2) \times 4 = 8 \times 11 \times 4$

$= 352$

Answer $352\ cm^3$

<Summary>

You can calculate the volume of solid figures like [figure] by thinking about how cubes and cuboids make up the figures.

<My reflection>

I thought of dividing the figure into two cuboids. Some thought the given figure could be made by removing a cuboid from a bigger cuboid, and others thought about rearranging the parts of the figure to make a new cuboid. Either way, we were all using cuboids somehow.

Misaki carefully looked at the characteristics of the original shape and rearranged it into a cuboid, because she knew how to calculate the volume of a cuboid.

She found it a good idea to find cubes or cuboids, because she knew how to calculate their volumes.



Ami

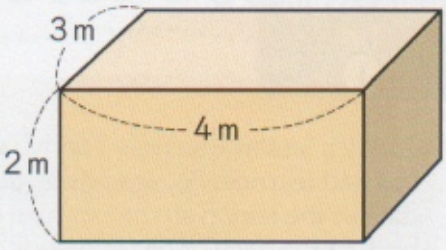
I think we can find the volume of a figure as long as it can be split into cubes and cuboids.



She wrote about further ideas she would like to investigate.

2 Various Units of Volume

1 Think about how to express the volume of the cuboid shown on the right.



Let's think about how to express the volume of large objects.


Misaki

1 m = 100 cm, so the volume can be calculated as...

When you calculated large areas...

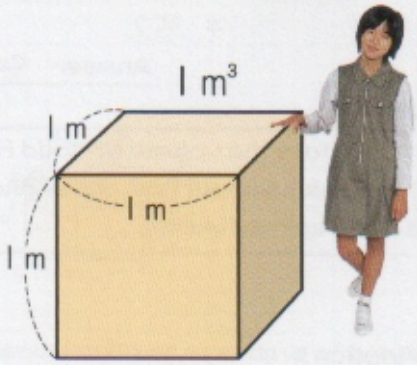

Haruto

Summary

When we express the volume of large objects, we use cubes with 1 m edges as the unit.

You can change the unit to use.

The volume of a cube with 1 m edges is 1 **cubic meter**, and it is written as 1 m^3.

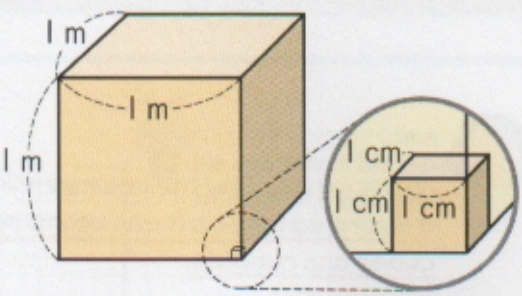


1 How many m^3 is the volume of the cuboid above?


Riku

If you fill the cuboid with 1 m^3 cubes, there will be ☐ cubes along the length, ☐ cubes along the width, and ☐ cubes along the height, so...

2 How many 1 cm^3 cubes will be along the width, length, and height of a 1 m^3 cube?



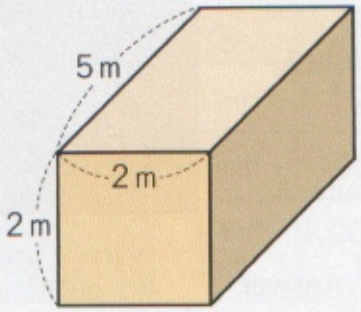
3 How many 1 cm^3 cubes will fit inside a 1 m^3 cube?

□ × □ × □ = □

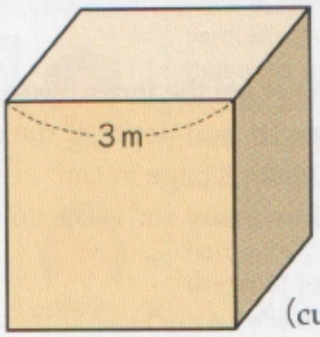
1 m^3 = 1,000,000 cm^3

1 What are the volumes of the cuboid and cube below in m^3?

①



②



(cube)

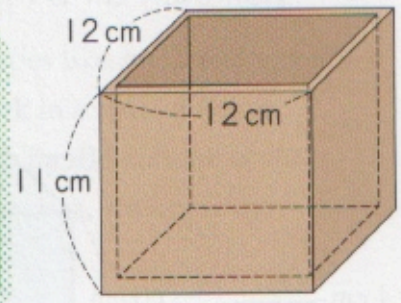
2 Using metersticks, tape, and wooden sticks, make a cube with a volume of 1 m^3.





Ami: Even when edges are in m, you can still use the formula for volume.

2 We used 1-cm thick boards to make a cuboid-shaped container as shown on the right.
How many cm^3 is the volume of water in the container?

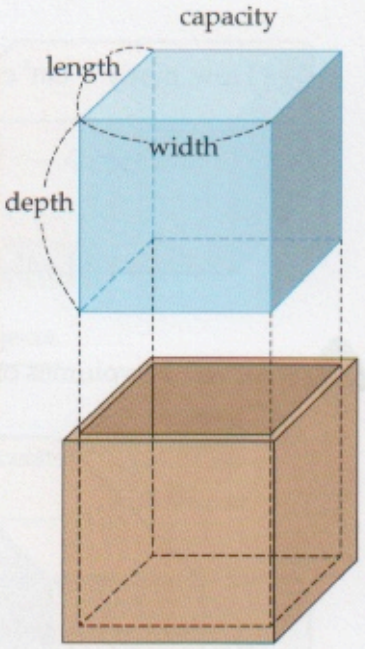


1 What are the lengths of this container you need to know to calculate the volume of water it can hold?



Kota: The walls of this container are thick.

The inside measurements of a container are called its **inner measurements.**
The amount of water or other things a container can hold is called its **capacity.**



2 What are the inner measurements of the width, length, and depth of the container on the previous page in cm?
How many cm^3 is the volume of the container?

□ × □ × □ = □

Answer □ cm^3

If the inner measurements of the width, length, and depth of a container are all 10 cm, the container can hold 1 L of water.

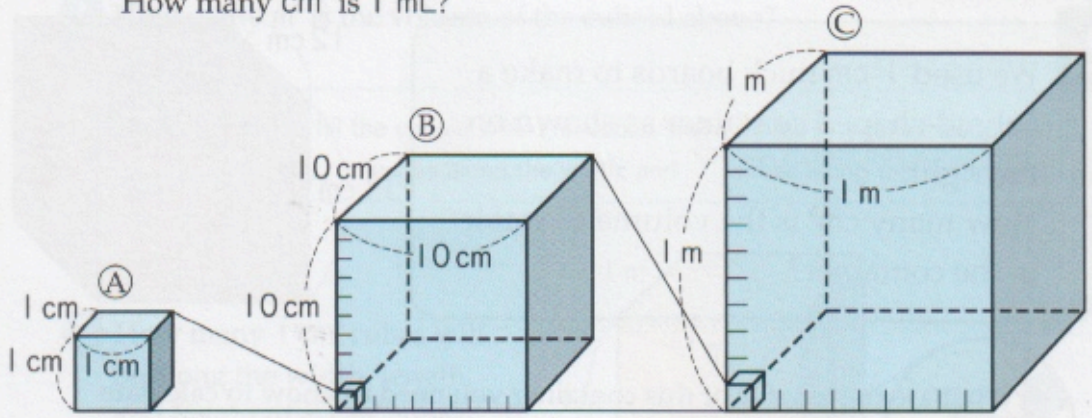
1 L is equal to 1,000 cm^3.

1 L = 1,000 cm^3

Let's investigate the equality relationships between the units you have studied.

3 1 L is equal to 1,000 mL.
How many cm^3 is 1 mL?

1 mL = 1 cm^3



1 cm^3
1 mL

1,000 cm^3
1 L (1,000 mL)

1 m^3
□ L

4 How many L is 1 m^3?

How many cubes with 10 cm edges will be along the width, length, and height of a 1 m^3 cube?

From the relationship, 1 L = 1,000 cm^3, you can tell the relationships among liter-based units and units such as cm^3 and m^3.

Kota

5 The table below summarizes relationships between the units of length, area, and volume that you have studied.

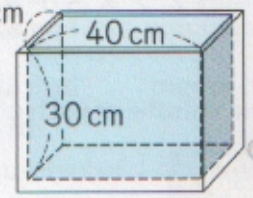
	Ⓐ	Ⓑ	Ⓒ
Length of each edge	1 cm	10 cm	1 m
Area of a square	1 cm^2	100 cm^2	1 m^2
Volume of a cube	1 cm^3 1 mL	1,000 cm^3 1 L	1 m^3 1 kL

Riku: Look at the table vertically. You can find that units of area and volume are based on units of length.

Shiho: Look at the table horizontally. When the length of each edge of a cube is made 10 times as much, its volume becomes...

Ami: 1,000 times as much as 1 L is 1 kL.

3 What is the capacity of the water tank on the right in cm^3? What is it in L?

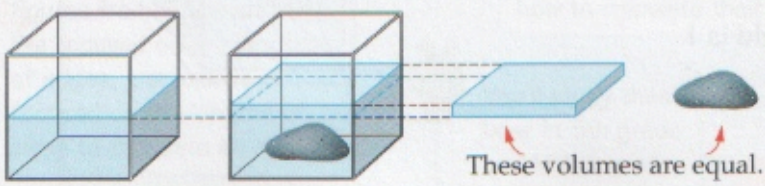


Additional Problems
→Page 129 F

Mathlin Newsletter

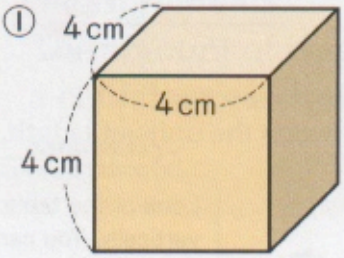
How to Find the Volume of a Rock

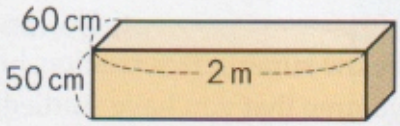
How can you determine the volume of irregular shapes like rocks and eggs? One way to do this is to use water. Submerge the rock in a tank of water. When the rock is submerged, the water level goes up by the volume of the rock. Therefore, if we calculate the amount the water rises, we know the volume of the rock.



Check Your Understanding

1 What are the volumes of the cube and cuboid below in cm^3?

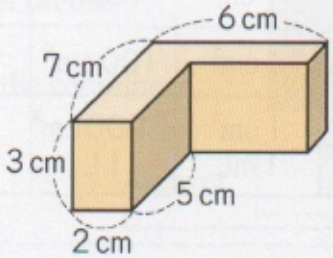
① 

② 

Can you find the volumes of cubes and cuboids?

Page 19 2

2 Someone calculated the volume of the solid figure on the right using the math sentence below.
Explain this person's thinking by drawing lines in the picture.

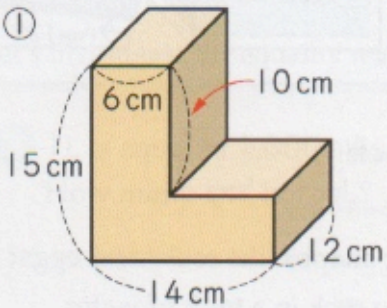


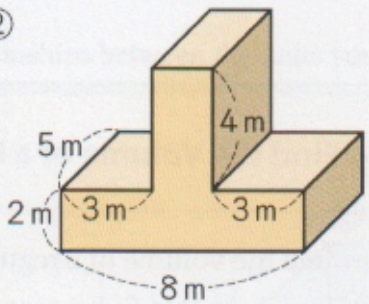
$5 \times 2 \times 3 + 2 \times 6 \times 3$

Can you figure out how volume was calculated from a math sentence?

Page 21 3

3 Calculate the volume of each of the following solid figures.

① 

② 

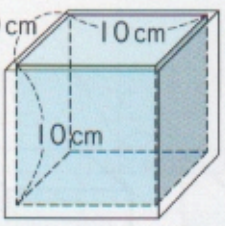
Can you find the volume of a solid figure shaped like  ?

Page 21 3
Page 26 1

4 Write the appropriate unit in each ☐.

① The volume of a cube with 1 m edges is 1 ☐.

② The capacity of the container on the right is 1 ☐.



Do you understand the units of volume and capacity?

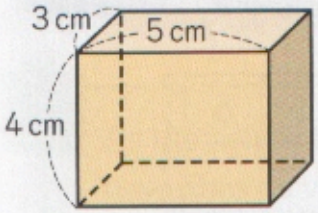
① Page 26 1
② Page 27 2

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the characteristics of solid figures and think about how to calculate their volumes

Let's review the reason why the volume of a cuboid can be calculated by the formula, length × width × height, while comparing how the area of a rectangle is calculated.
Fill in the ☐ with appropriate numbers.

Cuboid



① Since we can put ☐ 1 cm^3 cubes along the length of the cuboid and ☐ 1 cm^3 cubes along the width, there will be ☐ 1 cm^3 cubes in each layer.
Since the height is ☐ cm, there are ☐ layers.

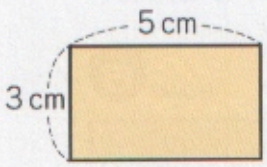
② Therefore, the total number of 1 cm^3 cubes will be

(length) ☐ × (width) ☐ × (height) ☐ = ☐, and the volume is ☐ cm^3.


Kota

Just like we [...] the lengths of the sides of a rectangle, we [...] the lengths of the edges of a cuboid.

Rectangle



① We can put ☐ pieces of 1 cm^2 squares along the length of the rectangle and ☐ pieces of 1 cm^2 squares along the width.

② Therefore, the total number of 1 cm^2 squares will be

(length) ☐ × (width) ☐ = ☐, and the area is ☐ cm^2.


Misaki

Both the volume of a cuboid and the area of a rectangle are expressed in terms of how many unit shapes fit inside the figure.

Look back on what you have learned in "Let's Think about How to Express the Amount of Space That Cubes and Cuboids" and discuss.


Riku

Now I can calculate the volumes of cubes and cuboids and look at solid figures from a new angle. We focused on the lengths of edges, just like we focused on the lengths of sides to calculate an area.


Ami

Some of the solid figures around us are not cubes or cuboids. I want to find out about their characteristics and how to calculate their volumes, too.



You'll study them later in 5th grade and in 6th grade.

Challenge Yourself → Page 143

How do quantities change?

As one quantity increases, how does the other quantity change?

① There is a book that has 80 pages in total.
As the number of pages read increases, the number of pages to read will...



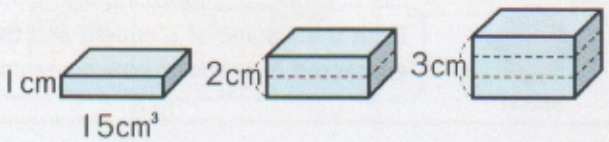
Number of pages read	1	2	3	4	5	6
Number of pages to read	79	78				

② A girl and her brother have the same birthday, and he is younger by 3. As the brother's age increases, the sister's age will ...



Brother's age	1	2	3	4	5	6
Sister's age	4	5				

③ There is a cuboid with a height of 1 cm and a volume of 15 cm^3.
As the height increases, the [ ] will...



As the height of a cuboid changes from 1 cm to 2 cm, 3 cm..., what quantity changes and how does it change?



Misaki: The cuboid gets bigger, and its volume increases.

I want to investigate it using a table, like we did in ① and ②.



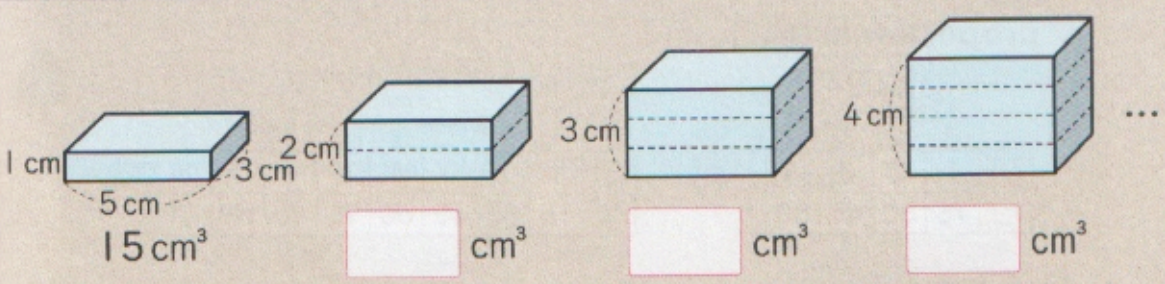
Kota

OJO

Este me gusta para volumen en crecimiento exponencial

Proportional Relationship

3 Let's Investigate How Quantities Change (1)



1 As the height of a cuboid changes from 1 cm to 2 cm, 3 cm..., how does its volume change?

1 When the height, □ cm, becomes 2 cm, 3 cm, ..., how many cm³ will the volume ○ be? Fill in the table below.

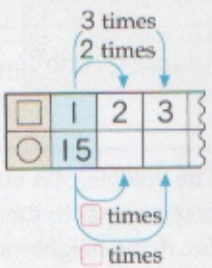
3×5×□=○
15 ×□=○

〈Move〉

Height □ (cm)	1	2	3	4	5	6	7	8
Volume ○ (cm³)	15							

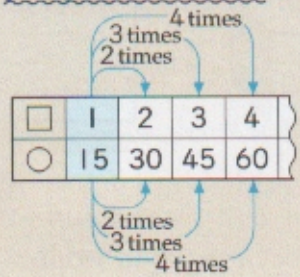
Let's investigate the relationship between the height □ (cm) of a cuboid and its volume ○ (cm³).

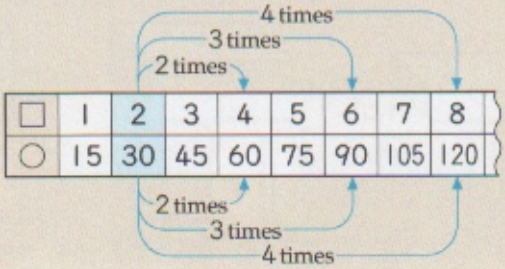
2 Suppose □ (the height) is 1. When □ increases to 2 times, how does ○ (the volume) change? When □ increases to 3 times or 4 times, how does ○ change? In the table above, draw arrows like the ones on the right and find out about the changes.



3 Suppose □ is 2. Investigate how quantities change like you did in 2.

Suppose there are a pair of quantities, □ and ○, and when □ becomes 2 times, 3 times..., ○ also becomes 2 times, 3 times... In this case, you can say that ○ is **proportional to** □.





In the case of the cuboid in 1, **its volume is proportional to its height.** — Haruto



Ami: When we investigated changes in 4th grade, we focused on the increase by 15 cm³. We are looking at the table horizontally like we did before, but today, we are focusing on how many times as much the quantities are, like 2 times as much, 3 times as much, ...

2 Calculate the volume of the cuboid in 1 with a height of 30 cm.



You could use the formula of calculating the volume of a cuboid, but...

If we use the fact that the volume of a cuboid is proportional to its height... — Ami



Let's use the proportional relationship to solve.

Height □ (cm)	1	2	3	4	5	6	7	8	9	10	30
Volume ○(cm³)	15	30	45	60	75	90	105	120	135	150	

The volume of a cuboid is proportional to its height. So, if the height increases to 30 times, from 1 cm to 30 cm, the volume also... — Riku



If the height increases to 3 times, from 10 cm to 30 cm... — Shiho



1 If the height is 30 cm, what is the volume in cm^3?

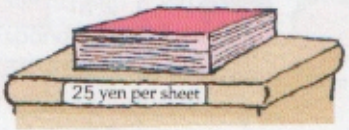
If you use the proportional relationship, you can calculate **a missing value of the volume in the table**, like the value for a height of 30 cm.

Misaki

1 Consider the relationship between the two quantities shown in the table below.
Is ○ proportional to □ ?
If they are in a proportional relationship, find the value of ○ when □ is 10.

Draw □ and ○ in the table and investigate.

① The cost ○ (yen) of buying □ pieces of colored paper, which is 25 yen per piece



Number of pieces □	1	2	3	4	5	6	7	8
Cost ○ (yen)	25	50	75	100	125	150	175	200

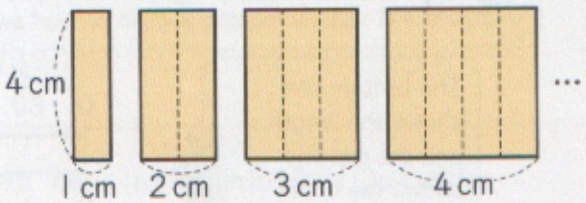
② The total cost ○ (yen) of buying □ pieces of colored paper, which is 25 yen per piece, and a 50-yen eraser

Haruto

Both the cost in ① and the total cost in ② increases by 25 yen, but...

Number of pieces □	1	2	3	4	5	6	7	8
Total cost ○ (yen)	75	100	125	150	175	200	225	250

③ The width □ (cm) of a rectangle with a length of 4 cm and the area ○ (cm^2) of the rectangle



Width □(cm)	1	2	3	4	5	6	7	8
Area ○(cm^2)	4	8	12	16	20	24	28	32

Additional Problems
→Page 130 G

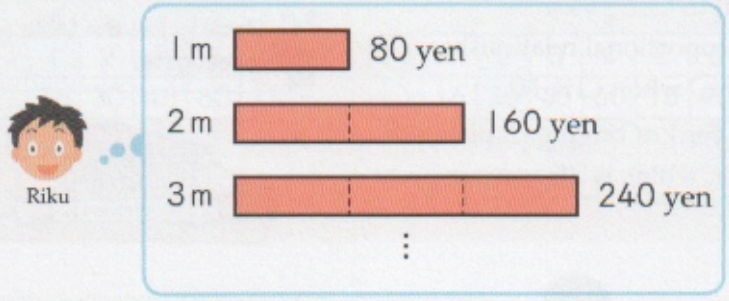
Kota

You can find many proportional relationships around you, like the number of pieces of colored paper and their cost.

Gento is at a store to buy □ m of ribbon for the party.

3

There is a roll of ribbon that costs 80 yen per m.
If the length of the ribbon he buys changes from 1 m to 2 m, 3 m..., how will the cost of the ribbon change?



1 Is the cost of ribbon, ○yen, proportional to the length, □ m?

Draw □ in the table and investigate.

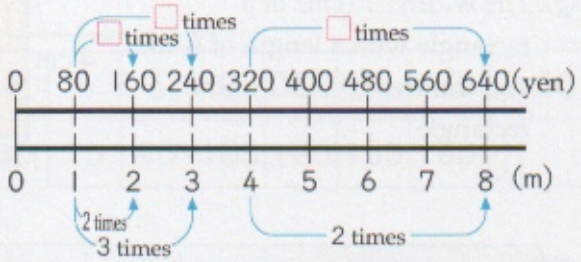
Length □(m)	1	2	3	4	5	6	7	8
Cost ○ (yen)	80	160	240	320	400	480	560	640

The cost is proportional to the length.

Shiho: When doing multiplication, I often came across number line diagrams.

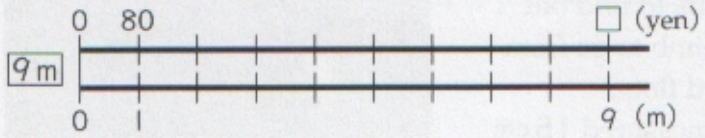
2 Express the table in **1** as a double number line diagram.

The bottom line shows the length, and the top line shows the cost.



3 Use the double number line diagram to find the costs of 9 m ribbon and 15 m ribbon.

Let's use a double number line diagram to solve the problem.

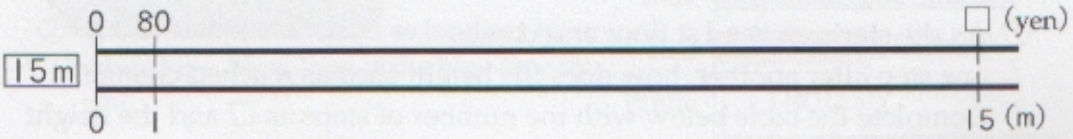


The cost of the ribbon is proportional to its length. So, if the length increases to 9 times from 1 m to 9 m...

Misaki

The cost to find is what corresponds to 9 if we consider 80 yen as 1. So, a math sentence to find it out will be...

Kota



Many tick marks are omitted, such as 160 yen for 2 m and 240 yen for 3 m.

Using a double number line diagram, **you can make a math sentence and find the answer.**

Ami

Page 148 shows how to draw a double number line diagram.

Look back on what you have learned "Let's Investigate How Quantities Change (1)" and discuss.

Haruto

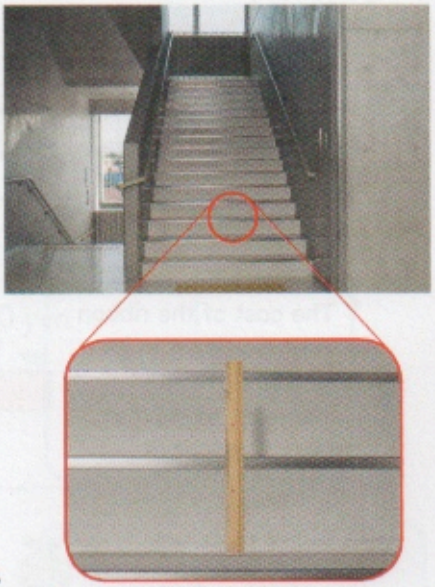
Now I can find out the relationship of quantities that increase to 2 times, 3 times...by looking at a table horizontally.
I learned that the volume of a cuboid in 1 is proportional to its height.

Shiho

Now I'm familiar with the structure of a double number line diagram.
I want to use a diagram to solve many different problems.

Use What You Have Learned

Aya's school is a 3-story building. She is going to use stairs to find out how high she has to climb to go from the first floor to the 3rd floor. One step of the stairs measured 15 cm high.



Assume that all the steps of the stairs are 15 cm high.

① As she starts on the 1st floor and climbs one step after another, how does the height she has reached change? Complete the table below with the number of steps as □ and the height from the 1st floor as ○ cm.

Number of steps □	1	2	3	4	5	6	7
Height from the 1st floor ○ (cm)	15						

② Is the height from the 1st floor, ○ cm, proportional to the number of steps that she climbs, □? Express the relationship between □ and ○ as a math sentence.

③ She climbed 48 steps to reach the 3rd floor from the 1st floor. How high did she climb when she went from the 1st floor to the 3rd floor in cm? What about in m?

④ Pick a building near you. Like Aya did, measure the height of one step of the stairs, count the number of steps □ to climb, and calculate how high you climb, ○ cm, to reach the 2nd and the 3rd floor from the 1st floor.

Assume that the height from the 1st floor level is proportional to the number of steps you climb.

Do You Remember?

Answers → Page 147

1 1 m of a pipe weighs 2.14 kg.
If □ m of this pipe weighs ○ kg, is ○ proportional to □?

Draw □ in the table and investigate.

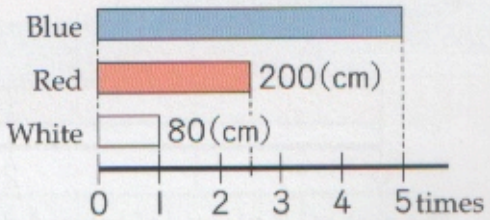
Pipe length □ (m)	1	2	3	4	5	6
Pipe weight ○ (kg)	2.14	4.28	6.42	8.56	10.7	12.84

2 There are white, red, and blue strips of tape. The white tape is 80 cm long, and the red tape is 200 cm long.

① How many times as long is the red tape as the white tape?

② The blue tape is 5 times as long as the white tape. How long is the blue tape in cm?

Times As Much Page 151 ⑩



Warm-up

3 Find the product of each of the following based on $7 \times 4 = 28$.

Properties of Multiplication Page 150 ①

① 7×12

$7 \times 4 = 28$
$\times\square \quad \times\square$
$7 \times 12 = \square$

② 70×40

$7 \times 4 = 28$
$\times\square \quad \times\square \quad \times\square$
$70 \times 40 = \square$

③ 7×40

④ 7×400

Playing with Numbers and Calculations

Multiplication and Division Puzzle

Fill in the blanks with numbers so that the product of the three numbers in any column, row, or along a diagonal is equal to the number in ____.

① 216

Ⓐ	4.5	Ⓑ
Ⓒ	6	Ⓓ
9	Ⓔ	Ⓕ

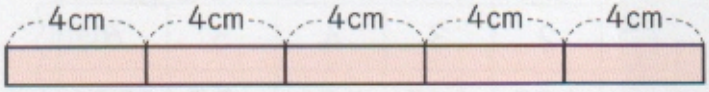
② 1,000

Ⓚ	Ⓛ	Ⓜ
Ⓝ	10	Ⓞ
Ⓟ	12.5	20

What kinds of multiplication have we studied?

$4 \times 5 = 20$

The length of 5 pieces of 4 cm connected side by side



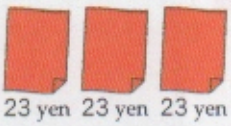
You do not need to think about the tapes overlapping where they are connected.

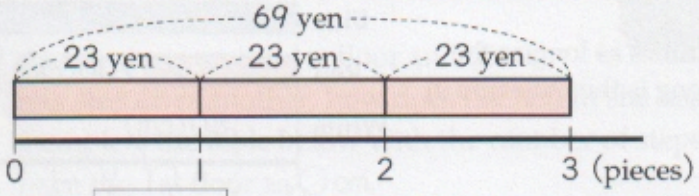
$4+4+4+4+4$

The whole length is [5 times] as long as [4cm] = [20cm].

$23 \times 3 = 69$

The cost of 3 pieces of colored drawing paper, which is 23 yen for each





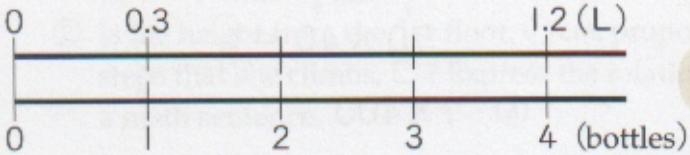
$23+23+23$

The cost of [3 pieces] of colored drawing paper, which is [23 yen for each], is [69 yen].

$0.3 \times 4 = 1.2$

0.3 L bottles of beverage
Total quantity of 4 bottles of beverage





$0.3+0.3+0.3+0.3$

Let's discuss what we studied about multiplication.


Haruto

The number of objects in each group × the number of groups = the total number of objects.


Shiho

We also studied how to do multiplication using the algorithm.

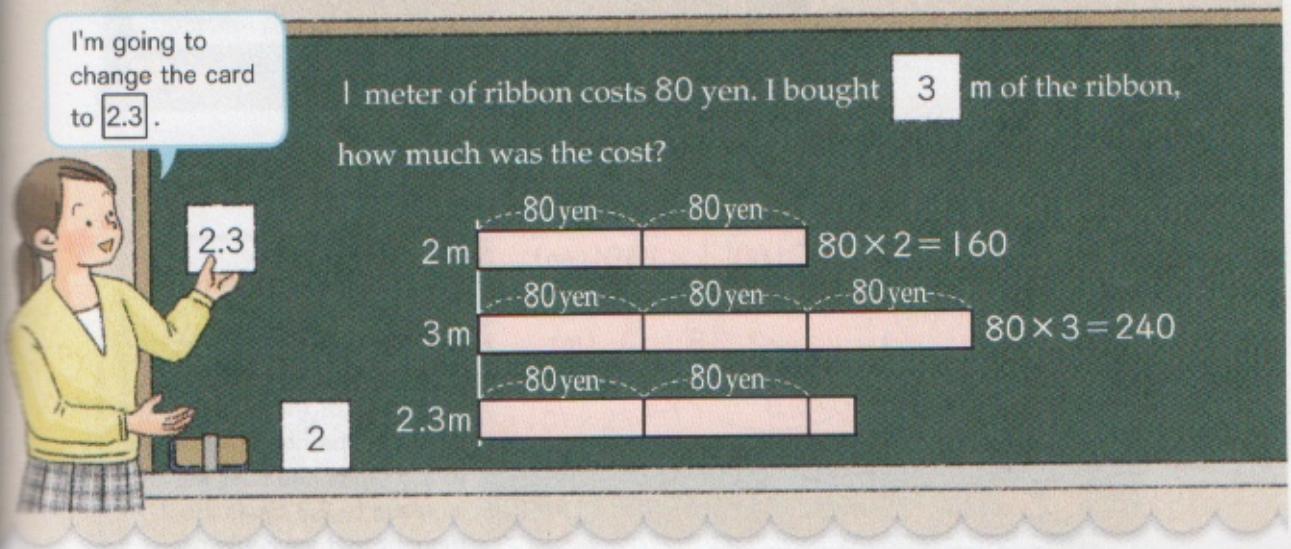

Misaki

We studied multiplication of whole numbers and multiplication of decimal numbers by whole numbers.


Riku

I wonder if there is any type of multiplication that we haven't studied.

4 Let's Expand the World of Multiplication



1 1 meter of ribbon costs 80 yen. I bought 2.3 m of the ribbon. How much was the cost?

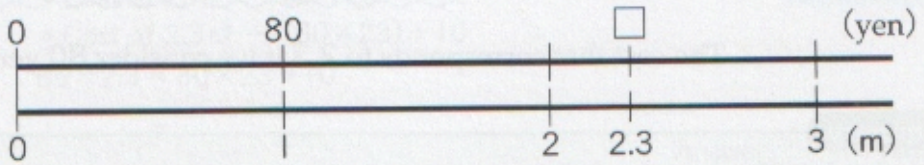
Let's think about what math sentence we should write.


Ami

If it were 3 m, we could think of it as 3 times the cost of 1 m of ribbon, or 80 yen, but...


Kota

It's 2.3 m, so we cannot think of it as groups of 80 yen, the cost of 1 m of ribbon.



Math Sentence

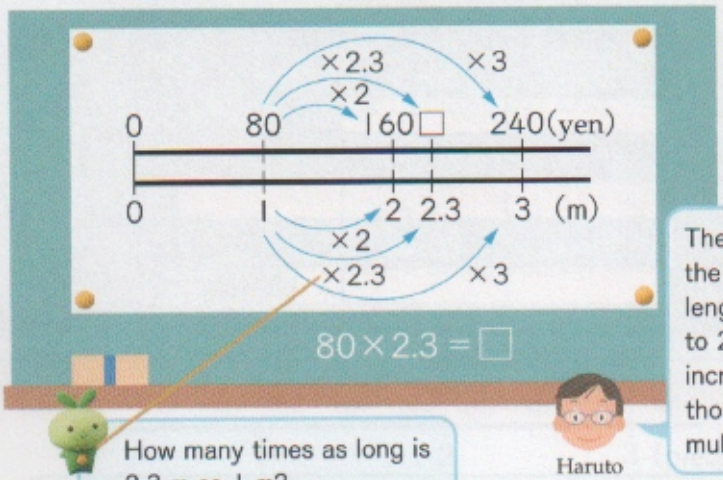
1 Explain why the math sentence is written in this way.

I thought we could think of the lengths as if they were whole numbers.

Ami

2 m·····	80	×	2	= 160
3 m·····	80	×	3	= 240
2.3 m···	80	×	2.3	= □

Price for 1 m × Length bought = Cost



How many times as long is 2.3 m as 1 m?
2.3 ÷ 1 = 2.3(times), so it's 2.3 times.

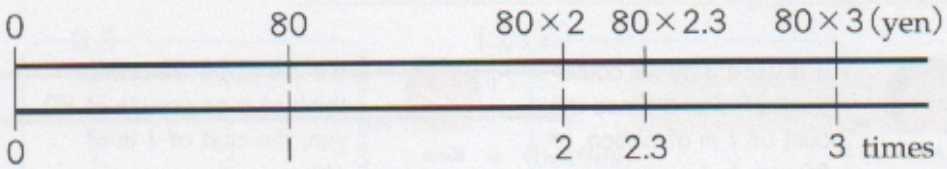
Haruto

The cost is proportional to the length of the ribbon. If the length of the ribbon increases to 2.3 times, the cost will increase to 2.3 times. So, I thought that we could use multiplication.

Times as Much
Page 151 ⑩

80×2 ··· Calculation to find 2 times (2 groups of) 80 yen

The cost that corresponds to 2 if we consider 80 yen as 1



80×2.3 ··· Calculation to find 2.3 times the cost of 80 yen ribbon

The cost that corresponds to 2.3 if we consider 80 yen as 1

Summary

Even when the length of ribbon is a decimal number, we can use a multiplication sentence to find the total cost, **just like we did when the lengths were whole numbers.**

80×2.3

Shiho

I wonder how I can calculate 80×2.3.

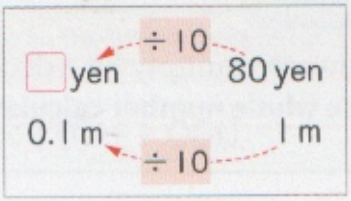
Let's think about how to multiply a whole number by a decimal number.

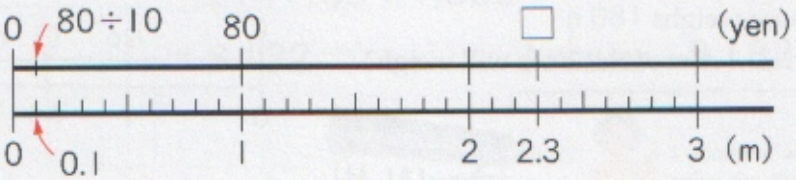

Kota

About how much will it be?
I think it's greater than 80×2 and [...] than 80×3.


Riku

2.3 m is made up of 23 0.1 m pieces. So, we can find the price for 0.1 m, and then find what 23 times that price is.





- Price of 0.1 m·····80÷10
- Cost of 2.3 m·····(80÷10)×23

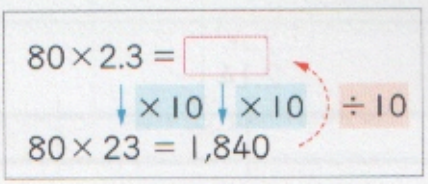
$80 \times 2.3 = 80 \div 10 \times 23$

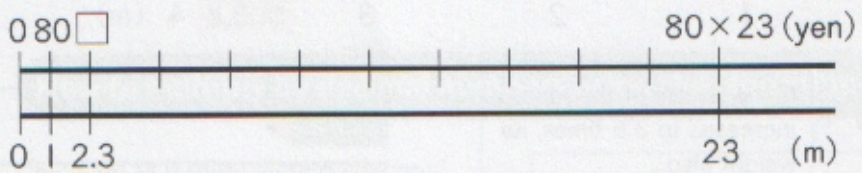
$= \square$

Answer □ yen


Misaki

If the length of the ribbon becomes 10 times as long, the cost will also be 10 times as much.





- Cost of 23 m ·····80×23
- Cost of 2.3 m·····(80×23)÷10

$80 \times 2.3 = 80 \times 23 \div 10$

$= \square$

Answer □ yen

2 Explain Riku's and Misaki's methods.

3 What do these two students' methods have in common?

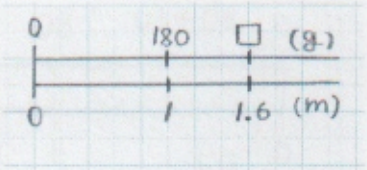
Both of them make multiplication between ☐ only.


Ami

Summary

You can multiply by a decimal number **by thinking about how to use whole number calculations.**

1 1 meter of hose weighs 180 g.
How much will 1.6 m of this hose will weigh?



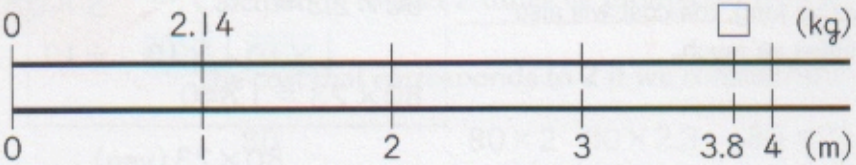
Additional Problems
→Page 131 H


Riku

Thinking about how we make use of whole number multiplication is the same as how we thought about calculations with decimal numbers before.

2 There is a 1 m pipe that weighs 2.14 kg.
How much will 3.8 m of this pipe weigh?






Shiho

If the length of the pipe increases to 3.8 times, its weight also...

Math Sentence ▶

I wonder if we can make both 2.14 and 3.8 into ☐.


Ami

Let's think about how to calculate.

1 Explain the calculation method on the right.

$2.14 \times 3.8 = \square$

$\times \square \quad \times \square \quad \times \square$ ÷ 1,000

$214 \times 38 = 8,132$

Summary

To calculate 2.14×3.8, first multiply 2.14 by 100 and 3.8 by 10 to make them 214 and 38.

We used a property of multiplication so that we can make use of whole number multiplication.

$$2.14 \times 3.8 = (2.14 \times 100) \times (3.8 \times 10) \div 1{,}000$$
$$= 214 \times 38 \div 1{,}000$$
$$= 8.132$$

Answer 8.132 kg



Misaki: I wonder if we can calculate using the algorithm.

Let's think about how the multiplication algorithm was used in the calculation.



Kota: Do the whole number calculation 214×38 first, and then...

Position of the decimal point

2.14	— 100 times →	214	····· Move 2 places to the right
× 3.8	— 10 times →	× 38	····· Move 1 place to the right
1712		1712	
642		642	
8.132	— 1,000 times →	8132	····· Move 3 places to the left.

2+1

1/1,000

How to multiply by decimal numbers using the algorithm

1. Calculate as if there were no decimal points.
2. To determine the location of the decimal point of the product, add the number of places that are to the right of the decimal points of the multiplicand and the multiplier. Then move the decimal point of the product from right to left the same number of places.

2.14 →	2	places to the right
× 3.8 →	1	place to the right
1712	2+1	
642		
8.132 ←	3	places to the left

Even when the multiplier is a decimal number, we can calculate just like we did with whole numbers, can't we?

2 Find the product of each of the following based on $176 \times 54 = 9{,}504$.

① 17.6×54 ② 176×5.4 ③ 1.76×5.4

Additional Problems → Page 131 I

3 Place the decimal points in the products.

① $1.7 \times 2.3 = 391$ ② $76.5 \times 8.3 = 63495$

Additional Problems → Page 131 J

4 Estimate the products first, and then calculate using the multiplication algorithm.

① 4.37×5.6 ② 3.81×7.4 ③ 3.9×2.1

④ 19.6×3.02 ⑤ 54×6.8 ⑥ 816×2.3

Additional Problems → Page 132 K

(Practice)



Ami: When placing the decimal point of the product, be careful not to simply slide it down, just like you do when doing addition or subtraction.

3 Explain how the multiplication algorithm was used in the calculations on the right.

(1) $4.92 \times 7.5 = 36.900$ (2) $0.18 \times 3.4 = 0.612$



Riku: How are these different from the multiplication algorithm you have studied?

Let's think about the size of the products.

1 What fraction of 36,900 is product (1)? What fraction of 612 is product (2)?



Misaki: **Depending on the size of the product**, you can delete or place 0.

5 ① 2.35×5.6 ② 3.6×9.5 ③ 875×1.2

④ 0.17×1.2 ⑤ 0.23×3.1 ⑥ 0.6×1.5

Additional Problems → Page 132 L



Haruto: We studied how to make $\frac{1}{10}$ or $\frac{1}{100}$ of numbers before, and that helps here.

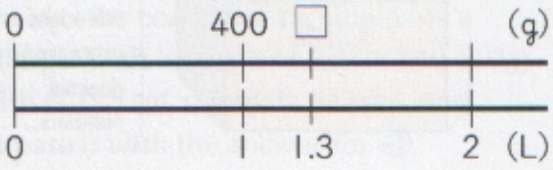
4

There is soil that weighs 400 g for each L. How much does 1.3 L of this soil weigh in g? How much does 0.6 L of this soil weigh in g?



1 Write math sentences, then find the answer.

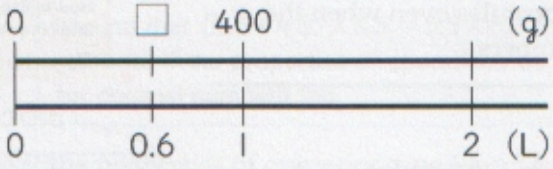
Ⓐ 1.3 L



Math Sentence

Answer □ g

Ⓑ 0.6 L



Math Sentence

Answer □ g

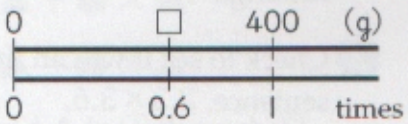


Shiho: This is multiplication, but in the case of 0.6 L, the product is [...] than 400.

Let's investigate the relationship between the size of the multiplier and the size of the product.

2 If we consider 400 g as 1, math sentences Ⓐ and Ⓑ are calculating the weights corresponding to what numbers?

Ⓑ



Summary

When you multiply by a number less than 1, the product will be less than the multiplicand.



$400 \times 0.1 = 40$
$400 \times 1 = 400$
$400 \times 1.01 = 404$

Even when we calculate the size that corresponds to a number less than 1, we can use multiplication.

6 Which of the following will have a product that is less than 6?

Ⓐ 6×0.9 Ⓑ 6×1.4 Ⓒ 6×2.08 Ⓓ 6×0.85

7 ① 8.3×0.7 ② 29.3×0.4 ③ 0.9×0.6
④ 0.2×0.03 ⑤ 0.5×0.8 ⑥ 1.25×0.4

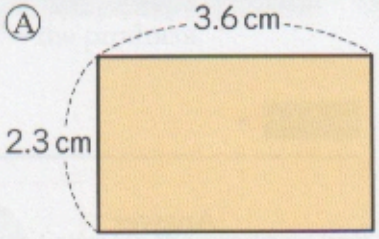
Additional Problems → Page 133 M

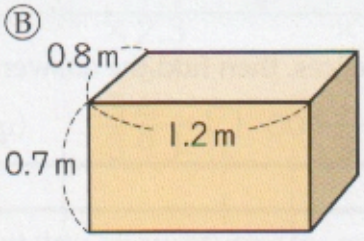


Kota: Now I have an even clearer picture of multiplication.

5

Find the area of rectangle Ⓐ and the volume of cuboid Ⓑ.





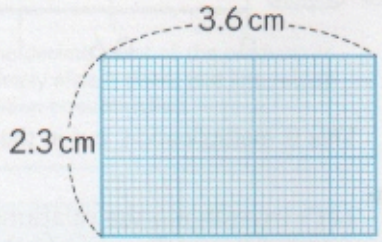
Lengths of the edges are expressed in decimal numbers...



Kota

Let's see if we can use the formulas even when the lengths become decimal numbers.

How to Express Area and Formulas for Area
Page 151 ⑪



1. How many squares with 1 mm sides are inside rectangle Ⓐ.

2. How many cm^2 is the area of the rectangle Ⓐ?



100 squares with 1 mm sides will equal 1 cm^2, won't it?

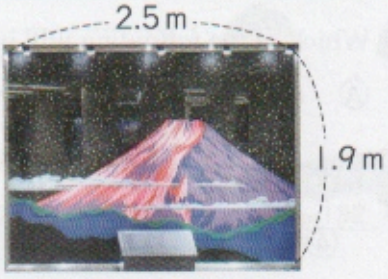
3. Check to see if we can find the area of the rectangle Ⓐ with the math sentence, 2.3×3.6.

4. Calculate volume of Ⓑ using both centimeters and meters as units for the lengths of the edges, then compare the results.

Summary

We can use the formulas to calculate area and volume even when the lengths of the sides or edges are decimal numbers, **just like when they are whole numbers.**

8. How many cm^2 is the area of the rectangular wall painting on the right?



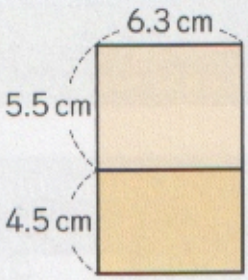
Shining, a painting on a piece of Ogatsu Slate from Ishinomaki, Miyagi Prefecture, Japan, placed on a wall in Tokyo Station building



Shiho

Is there any other way of thinking that is shared with whole number calculation?

6 How many cm² are the areas of the rectangles on the right?



1 How many cm² are the areas of the top and bottom rectangles? How many cm² are the areas altogether?

2 Consider the combined rectangles as a rectangle with a length of 10 cm and a width of 6.3 cm, calculate its area, and compare it with the answer for 1.

As you can see, 5.5 + 4.5 = 10.

I found that $(5.5+4.5)\times 6.3 = 5.5\times 6.3+4.5\times 6.3$.
I wonder if the properties of operations we have found work for decimal numbers too.

Ami

Let's see if the properties of operations we found for whole number calculations also work for decimal numbers.

Ⓐ $\blacksquare\times\bullet=\bullet\times\blacksquare$

Ⓑ $(\blacksquare\times\bullet)\times\blacktriangle=\blacksquare\times(\bullet\times\blacktriangle)$

Ⓒ $(\blacksquare+\bullet)\times\blacktriangle=\blacksquare\times\blacktriangle+\bullet\times\blacktriangle$

Ⓓ $(\blacksquare-\bullet)\times\blacktriangle=\blacksquare\times\blacktriangle-\bullet\times\blacktriangle$

3 Choose decimal numbers to put in ■, ● and ▲ in Ⓐ through Ⓓ. Then, check to see if the left side of the equal sign is equal to the right side.

Equal sign
Page 150 ⑤

Haruto

Suppose ■ = 3.6 and ● = 0.4. In Ⓐ, compare the product of 3.6 × 0.4 with the product of 0.4 × 3.6.

Summary

The properties of operations we found for whole number calculations also work for decimal numbers **just like they work for whole numbers.**

9 Think about making the calculation simpler by using properties of operations Ⓐ through Ⓓ above.

① $1.7\times 4\times 2.5$ ② $2.4\times 1.8+2.6\times 1.8$

③ 15.3×4 ④ 9.8×15

Additional Problems
→Page 133 N

15.3 = 15 + 0.3, so...

Check Your Understanding

1 Calculate.



Estimate first, and then calculate.

① 8×1.7 ② 14×3.9 ③ 7.8×2.9

④ 21.3×3.5 ⑤ 4.2×5.34 ⑥ 10.3×3.14

⑦ 5.5×4.4 ⑧ 4.26×6.5 ⑨ 315×4.6

⑩ 0.34×2.5 ⑪ 0.62×1.3 ⑫ 0.47×1.9

Can you multiply by decimal numbers using the algorithm?

①~⑥ Page 44 2
⑦~⑫ Page 46 3

2 There is a 1 m wire that weighs 18.5 g.
How many g will 3.6 m of this wire weigh?

Can you make math sentences out of situations and find the answers?

Page 47 4

3 Which of the math sentences in the ( ) will have a product that is less than the multiplicand?

① $(4 \times 1.2 \quad 4 \times 0.8)$

② $(1.6 \times 0.7 \quad 1.6 \times 1.1)$

③ $(0.3 \times 0.9 \quad 0.3 \times 1.4)$



When you multiply by a number less than 1, the product will be...

Do you understand the relationship between the size of the multiplier and the size of the product?

Page 47 4

4 Calculate.

① 24×0.8 ② 0.69×0.37 ③ 0.4×0.5

Can you multiply by a number less than 1?

Page 47 4

5 Find the area of a rectangle with a length of 2.7 m and a width of 4.35 m.

Can you find the area of a rectangle with sides expressed in decimal numbers?

Page 48 5

6 Think about easier ways to calculate the following using properties of operations, and then find the answers.

① $4 \times 7.63 \times 2.5$ ② $6.4 \times 2.3 + 3.6 \times 2.3$

Can you calculate using the properties of operations?

Page 49 6

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

1 Focus on properties of multiplication and think about how to multiply using the algorithm

When Shiho calculated 3.14×2.6 using the algorithm, she first found the product of 314×26.
Then, she thought about where to place the decimal point of the product, and placed it between 8 and 1.
Shiho is explaining why she did so. Complete her explanation.

$$\begin{array}{r} 3.14 \\ \times\ 2.6 \\ \hline 1884 \\ 628 \\ \hline 8.164 \end{array}$$



Shiho

First, we find the product of 314×26 Since the product, 8,164, is obtained by making 3.14 100 times as much and 2.6 10 times as much...

2 Find easier ways to calculate based on properties of operations

Haruto calculated 3.99×25 as shown on the right.
Which property of operations did he use?



Haruto

$$\begin{aligned} 3.99 \times 25 &= (4 - 0.01) \times 25 \\ &= 4 \times 25 - 0.01 \times 25 \\ &= 100 - 0.25 \\ &= 99.75 \end{aligned}$$

Ⓐ $(■ \times ●) \times ▲ = ■ \times (● \times ▲)$
Ⓑ $(■ + ●) \times ▲ = ■ \times ▲ + ● \times ▲$
Ⓒ $(■ - ●) \times ▲ = ■ \times ▲ - ● \times ▲$



Ami

If you use properties of operations, you can sometimes make calculations easier.

Look back on what you have learned in "Let's Expand the World of Multiplication" and discuss.



Riku

Now I can multiply a whole number by a decimal number.
I found the properties of multiplication very useful.



Misaki

Now I understand that multiplication is a type of calculation that finds the size that corresponds to the multiplier when the multiplicand is considered as 1.

Challenge Yourself → Page 144

What kinds of division have we studied?

72 ÷ 8

The quotient of 72 ÷ 8 is ☐.

We used multiplication facts to think.



Misaki

84 ÷ 21

$$21\overline{)84}$$

☐

☐

0

If I think of 21 as 20...



Kota

7,200 ÷ 800

72 ÷ 8 = ☐ (×100, ×100)

7,200 ÷ 800 = ☐ (Equal)

The quotient does not change if we multiply both the dividend and the divisor by the same number.



Haruto

Properties of Division Page 150 ②

9.4 ÷ 4

$$4\overline{)9.40}$$

2.35

8

14

12

20

20

0

Think as if 9.4 was ☐ and continue with the calculation.



Ami

Let's discuss what we studied about division.



Riku

We studied the division of a whole number by a whole number and a decimal number by a whole number.



Shiho

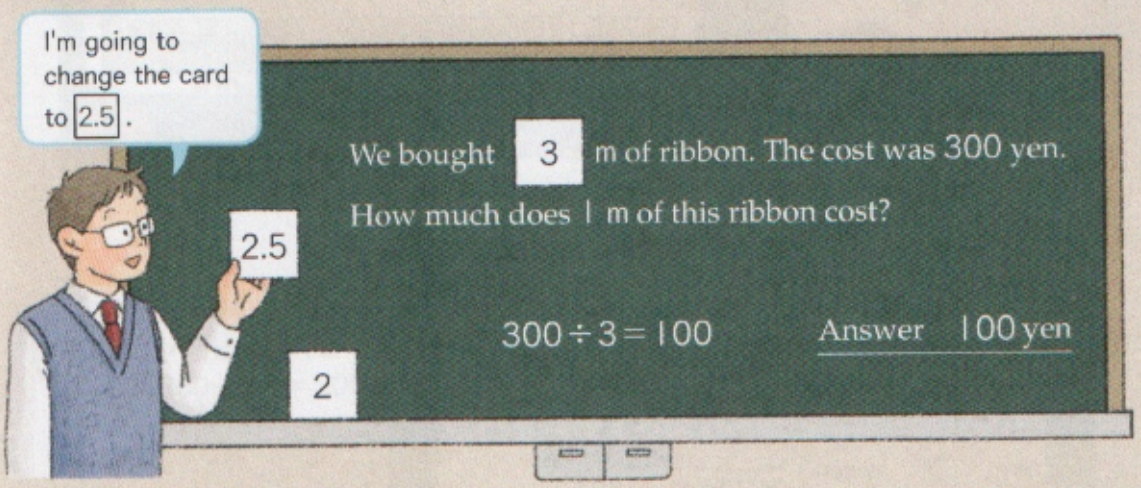
We studied how to do division using the algorithm too.



Misaki

We studied how to multiply a whole number by a decimal number. I wonder if we can also divide a whole number by a decimal number.

5 Let's Expand the World of Division



1 We bought 2.5 m of ribbon. The cost was 300 yen. How much does 1 m of this ribbon cost?

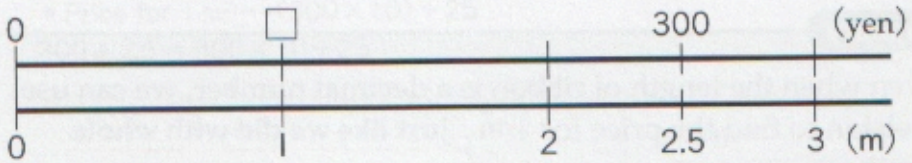
Let's think about what math sentence we should write.



Ami: If it were 3 m, then, we could think of it as 3 pieces of 1 m and divide 300 yen into 3 equal pieces. But, ...



Riku: With 2.5 m we can't think of dividing 300 yen into 2.5 equal pieces.



Math Sentence

1 Explain why the math sentence is written in this way.

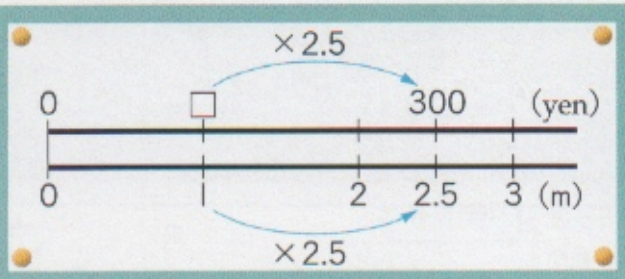
If the length we bought is a whole number, it will be division, ...



Haruto

2 m ···300 ÷ 2 = 150
3 m ···300 ÷ 3 = 100
2.5 m···300 ÷ 2.5 = □

Cost ÷ Length we bought (m) = Price for 1 m



□ × 2.5 = 300

□ = 300 ÷ 2.5

I replaced the unknown price for 1 m with □. If the length we bought increased to 2.5 times, I thought the cost would increase to 2.5 times, too.
So I wrote □ × 2.5 = 300, and to find the number that goes in □.....



Shiho

Times As Much
Page 151 ⑩

□ × 2.5 = 300 means that if we consider □ as 1, 300 corresponds to 2.5.
So, the math sentence to find the number we consider as 1 is a division sentence.



Misaki

Summary

Even when the length of ribbon is a decimal number, we can use division to find the price for 1 m **just like we did with whole numbers.**

300 ÷ 2.5



Riku

I wonder how I can calculate 300 ÷ 2.5.

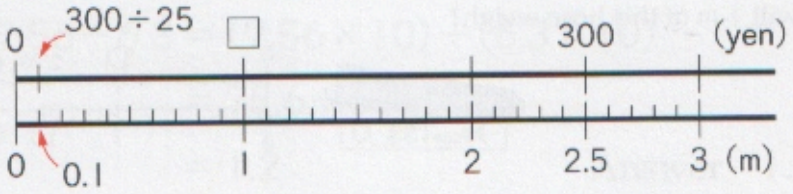
Let's think about how to divide a whole number by a decimal number.


Riku

About how much will it be?
Since
300 ÷ 2 = 150 and
300 ÷ 3 = 100, ...


Ami

2.5 m is made up of 25 pieces of 0.1 m.



- Price for 0.1 m……300 ÷ 25
- Price for 1 m……(300 ÷ 25) × 10

300 ÷ 2.5 = 300 ÷ 25 × 10

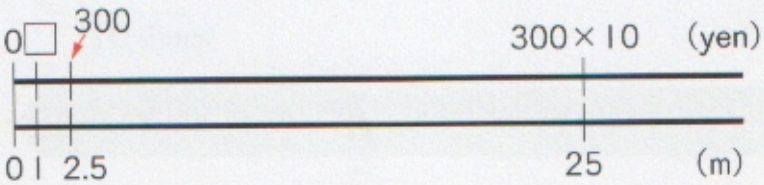
= □

Answer □ yen


Kota

If the length of the ribbon becomes 10 times as long, the cost will be 10 times as much, but the price for 1 m does not change.

300 ÷ 2.5 = □
↓ × 10 ↓ × 10
3,000 ÷ 25 = 120
equal



- Cost of 25 m……300 × 10
- Price for 1 m……(300 × 10) ÷ 25

300 ÷ 2.5 = 300 × 10 ÷ 25

= □

Answer □ yen

2 Explain Ami's and Kota's methods.

3 What do these two students' methods have in common?

Both of them made use of division of □.

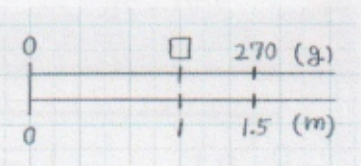

Shiho

Summary

You can divide by a decimal number by **thinking about how to use whole number calculations.**

1 1.5 m of hose weighs 270 g.
How much will 1 m of this hose weigh?

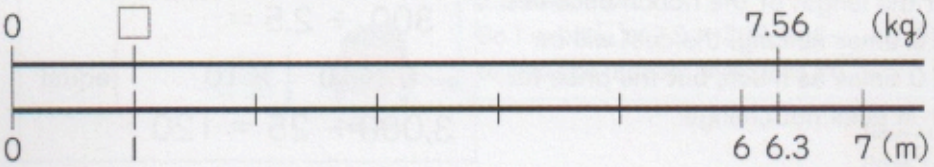
Additional Problems → Page 133 O




Misaki

Thinking about how we make use of whole number division is the same as how we thought about calculations with decimal numbers before.

2 There is a 6.3 m iron pipe that weighs 7.56 kg.
How much will 1 m of this iron pipe weigh?



Math Sentence

Let's think about how to calculate.

I wonder if we can make 6.3 into a □.


Haruto

1 Explain how the calculation on the right was done.

$$7.56 \div 6.3 = \square$$
$$\downarrow \times \square \quad \downarrow \times \square \quad \text{equal}$$
$$75.6 \div 63 = 1.2$$

We should use a property of division...


Shiho

Summary

You can find the quotient of $7.56 \div 6.3$ based on the fact that this quotient is equal to the quotient we obtain when we make both 7.56 and 6.3 10 times as much.

We used a property of division so that we can make use of whole number division.

$$7.56 \div 6.3 = (7.56 \times 10) \div (6.3 \times 10)$$
$$= 75.6 \div 63$$
$$= 1.2$$

Answer 1.2 kg



Kota: I wonder if we can calculate using the algorithm.

Let's think about how the division algorithm is used in the calculation.



Ami: We should move the decimal point to make the math sentence 75.6 ÷ 63...

10 times

$6.3\overline{)7.56}$ → $6.3\overline{)7.5.6}$

10 times

$6.3\overline{)7.56}$

How to divide by decimal numbers using the algorithm

1. Move the decimal point of the divisor to the right to make it a whole number.
2. Move the decimal point of the dividend to the right the same number of places.
3. Divide by the whole number divisor. The decimal point of the quotient will be in the same position as the decimal point of the dividend after it was moved to the right.

$$\begin{array}{r} 1.2 \\ 6.3\overline{)7.5.6} \\ 63 \\ \hline 126 \\ 126 \\ \hline 0 \end{array}$$

Even when the divisor is a decimal number, we can calculate just like we did with whole numbers, can't we?

2 Find the following quotients based on $221 \div 65 = 3.4$.

① $22.1 \div 6.5$ ② $2.21 \div 0.65$ ③ $0.0221 \div 0.065$

Additional Problems → Page 133 P

3 Estimate first, and then calculate.

① $2.38 \div 1.7$ ② $8.96 \div 2.8$ ③ $38.7 \div 8.6$

④ $7.8 \div 6.5$ ⑤ $4.71 \div 3.14$ ⑥ $58.4 \div 7.3$

⑦ $25.8 \div 4.3$ ⑧ $65.6 \div 1.6$ ⑨ $47.7 \div 1.59$

〈Practice〉

Additional Problems → Page 134 Q



Ami: When placing the decimal point of the quotient, make sure that it is in the same position as the decimal point of the dividend after it was moved.

3 Explain how the division algorithm is used in the calculations below.



Riku: I wonder how these are different from the calculations we did before.

(1) $2.34 \div 3.9$

$$\begin{array}{r} 0.6 \\ 3.9\overline{)2.3.4} \\ 234 \\ \hline 0 \end{array}$$

(2) $1.8 \div 2.4$

$$\begin{array}{r} 0.75 \\ 2.4\overline{)1.8.0} \\ 168 \\ \hline 120 \\ 120 \\ \hline 0 \end{array}$$

(3) $8 \div 2.5$

$$\begin{array}{r} 3.2 \\ 2.5\overline{)8.0} \\ 75 \\ \hline 50 \\ 50 \\ \hline 0 \end{array}$$

Let's think about how the division algorithm is used in the calculations.



Explain the reason why.

About (1), 23 < 39, so you write 0 in the ones place of the quotient, place the decimal point, and...



Misaki

About (2), after writing 0 and placing the decimal point, **you consider 18 as 18.0** and...



Kota

4 ① $5.04 \div 8.4$ ② $3.92 \div 5.6$ ③ $2.1 \div 2.5$

④ $1.17 \div 3.6$ ⑤ $6 \div 2.4$ ⑥ $42 \div 5.6$

Additional Problems → Page 134 R



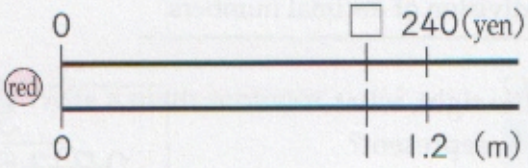
Haruto: After moving the decimal point to change the divisor into a whole number, you can do the same as division of a decimal number by a whole number, which we studied in 4th grade.

4

A red ribbon costs 240 yen for 1.2 m.
A blue ribbon also costs 240 yen for 0.8 m.
How much does 1 m of each ribbon cost?

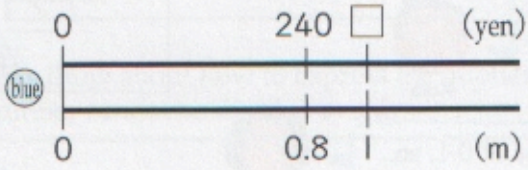


1 Write math sentences, then find the answers.



Math Sentence ▸

Answer □ yen



Math Sentence ▸

Answer □ yen

2 In which case is the quotient greater than the dividend, 240?

Let's investigate the relationship between the size of the divisor and the size of the quotient.

3 In which case is the divisor less than 1?

Summary

When dividing by a decimal number less than 1, the quotient will be greater than the dividend.

If you look at the number line diagram for (blue), you can see that □ is greater than 240. □ is considered as 1, and □ × 0.8 = 240.

5

Which divisions will have a quotient that is greater than 8?

Ⓐ 8 ÷ 1.5 Ⓑ 8 ÷ 0.02 Ⓒ 8 ÷ 0.64 Ⓓ 8 ÷ 5

6

① 19.8 ÷ 0.3 ② 3.9 ÷ 0.6 ③ 7.4 ÷ 0.4
④ 3.75 ÷ 0.6 ⑤ 0.51 ÷ 0.4 ⑥ 6 ÷ 0.5

Additional Problems
→Page 134 S



Shiho: Now I understand that answers for division are not always less than the dividends.

5

0.7 m of ribbon will be given to each student. If there is 2.5 m of ribbon, how many people can get the ribbon? How many m will be left over?



Math Sentence ▸

Let's think about the remainder of division of decimal numbers.

1 In the division algorithm on the right, what amount does the remainder, 4, represent?

$$\begin{array}{r} 3 \\ 0.7\overline{)2.5} \\ 2\,1 \\ \hline 4 \end{array}$$



I wonder if the remainder is 4.

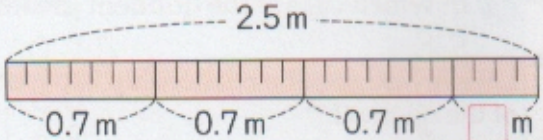
Kota



It means that there are 4 pieces of 0.1, so...

Ami

2 Use a diagram or the calculation to check the answer for division to see that the remainder is 0.4.



$0.7 \times 3 + \square = \square$

divisor quotient remainder dividend

Summary

In division of decimal numbers, the decimal point of the remainder will be in the same place that the decimal point of the dividend was in before it was moved.

$$\begin{array}{r} 3 \\ 0.7\overline{)2.5} \\ 2\,1 \\ \hline 0.4 \end{array}$$

You think about the amount of the remainder based on the amount of the dividend before the decimal point was moved.

7 Find the whole number quotient. Also find the remainder.

① $4.9 \div 2.3$ ② $17.5 \div 9.6$ ③ $340 \div 7.2$



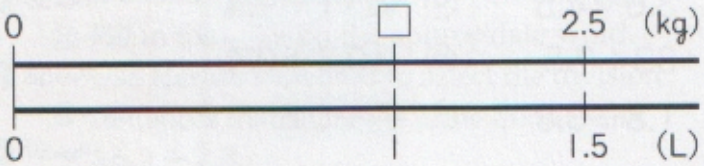
Riku

2.5 cannot be divided evenly by 0.7. I wonder if we have no choice but to show the answer with the remainder.

6

1.5 L of sand weighs 2.5 kg.
How much will 1 L of this sand weigh?





1 Write a math sentence.

Math Sentence

The quotient is 1.666... 6 goes on forever. How can we express the quotient?


Misaki

Let's think about how to express the quotient without the remainder if a number cannot be divided evenly.

2 Round the quotient to the second highest place.

To round to the second highest place, we have to look at the numeral in which place?

$$2.5 \div 1.5 = 1.\overset{\overset{7}{\curvearrowright}}{6}\overset{\curvearrowleft}{6}\cdots$$

Answer approximately 1.7 kg

Approximate Numbers
Page 150 ③

Summary

If a number cannot be divided evenly or a quotient has so many digits, **the quotient is sometimes expressed as an approximate number.**

3 About how many kg is 5 L of this sand? Calculate using the fact that 1 L of this sand weighs about 1.7 kg.

8 There is an iron plate that is 1.8 m^2 and weighs 4.8 kg. How much does 1 m^2 of this plate weigh?
Round the answer to the second highest place.

Check Your Understanding

1 Divide completely.

① $36.1 \div 3.8$ ② $7.44 \div 6.2$ ③ $37.4 \div 8.5$

④ $12.3 \div 4.1$ ⑤ $45.6 \div 3.8$ ⑥ $5.36 \div 6.7$

⑦ $2.24 \div 3.2$ ⑧ $3.6 \div 4.5$ ⑨ $6.11 \div 9.4$

⑩ $6 \div 2.5$ ⑪ $33 \div 7.5$ ⑫ $10.8 \div 0.4$

⑬ $1.96 \div 0.5$ ⑭ $1.8 \div 0.8$ ⑮ $9 \div 0.6$

Can you divide by decimal numbers using the algorithm?

①~⑤ Page 56 2
⑥~⑪ Page 58 3
⑫~⑮ Page 59 4

2 We are going to find the quotients by putting the 6 numbers shown below in the □.
When will the quotient be the largest? When will it be the smallest? Answer without actually calculating.

$2.4 \div \square$

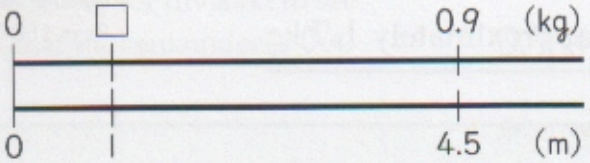
Ⓐ 0.8 Ⓑ 1 Ⓒ 1.25

Ⓓ 0.09 Ⓔ 2.4 Ⓕ 0.1

Do you understand the relationship between the size of the divisor and the size of the quotient?

Page 59 4

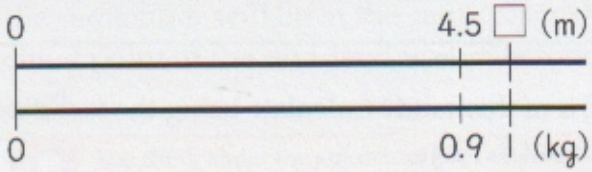
3 4.5 m of hose weighs 0.9 kg.
Write a math sentence to find how much 1 m of this hose weighs.



Can you make a math sentence correctly?

Page 59 4

4 Write a math sentence to find how long 1 kg of the hose in **3** is.



Can you make a math sentence correctly?

Page 59 4

5 Round the quotient to the second highest place.

① $8.3 \div 2.9$ ② $6.13 \div 4.7$ ③ $24.2 \div 8.9$

Can you express quotients as approximate numbers?

Page 61 6

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on properties of division and think about how to calculate

① Haruto is selecting division sentences that share the same quotient as $46.2 \div 2.8$ from the following options: Ⓐ, Ⓑ, Ⓒ, and Ⓓ.
Fill in the ☐ with the appropriate word.
Use Haruto's method to select the division sentences that share the same quotient as $46.2 \div 2.8$.

In division, the ☐ does not change if we multiply or divide both the dividend and the divisor by the same number.
In Ⓐ, only 2.8 is divided by 10, so...



Haruto

Ⓐ $46.2 \div 0.28$	Ⓑ $462 \div 28$
Ⓒ $4.62 \div 0.28$	Ⓓ $4.62 \div 2.8$

② The division $752 \div 1.6$ on the right is done incorrectly.
Explain the reason why and calculate correctly.

$$
\begin{array}{r}
47 \\
1.6\overline{)752} \\
64 \\
\hline
112 \\
112 \\
\hline
0
\end{array}
$$



Kota

The decimal point of 1.6 is moved to the right because 1.6 is multiplied by 10. To keep the quotient from changing, the decimal point of 752 needs to be moved one place to the right...



Ami

When we did multiplication of decimal numbers, we used properties of multiplication to... think, didn't we?

Look back on what you have learned in "Let's Expand the World of Division" and discuss.



Misaki

Now I can divide by decimal numbers. My world of division has expanded.



Riku

I wonder if we can also multiply or divide something by fractions.



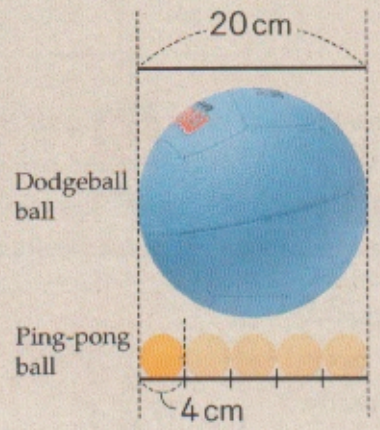
You'll study it in 6th grade.

Challenge Yourself
→Page 144

Times As Much and Decimal Numbers

In our previous study, we compared many different objects in size and expressed how much the difference is and how many times as big one object is as the other object in size.

Comparison of a dodgeball ball and a ping-pong ball in diameter



Difference □ − □ = □

□ cm

times

How many times as long is the diameter of the dodgeball ball as the diameter of the ping-pong ball?

□ ÷ □ = □

□ times

Times as Much Page 151 ⑩


Kota

To find the difference, we subtracted the smaller number from the larger number.


Shiho

To find how many times as long one is as the other, we considered the base amount as 1. In this case, the base amount is 4 cm, the diameter of the ping-pong ball.

If we consider 4 cm as 1, 20 cm corresponds to □.


Riku

1 The lengths of 4 ribbons are shown in the table on the right.
Choose one of them as the base amount and compare the lengths of the ribbons.

Lengths of Ribbons

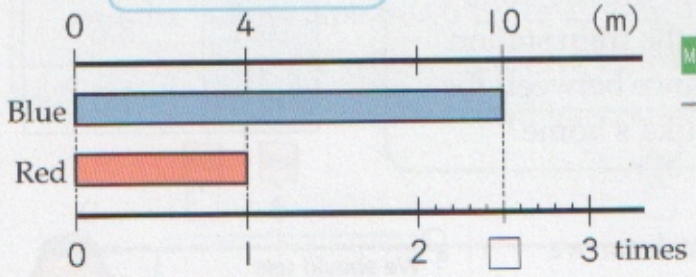
	Length (m)
Red	4
Blue	10
Yellow	5

Let's investigate the relationships between the lengths of the ribbons by finding out how many times as long one ribbon is as another.

1 How many times as long is the blue ribbon as the red ribbon?

The length of the red ribbon is used as the base amount. — Misaki

$4 \times \square = 10$, so... — Haruto



Math Sentence: $\square \div \square = \square$

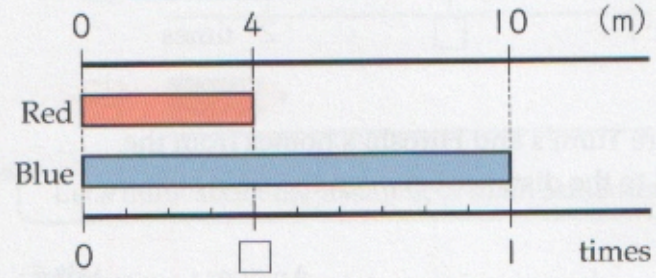
Answer $\square$ times

"2.5 times" means that if we consider 4 m as 1, 10 m corresponds to 2.5. — Ami

$4 \times 2.5 = 10$

2 How many times as long is the red ribbon as the blue ribbon?

The length of the $\square$ ribbon is used as the base amount. — Kota



Math Sentence: $\square \div \square = \square$

Answer $\square$ times

We sometimes used a decimal number less than 1 when we expressed times as many/much.

Summary

When you compare two quantities, **the number of times as much one quantity is as the other quantity depends on which quantity is used as the base amount.**

"0.4 times" means that if we consider 10 m as 1, 4 m corresponds to 0.4.

1 About 1 on the previous page, how many times as long is the yellow ribbon as the red ribbon? How many times as long is the red ribbon as the yellow ribbon?

Additional Problems → Page 134 T

Shiho: About the balls on the previous page, if you consider the diameter of the dodgeball ball, 20 cm, as 1, the diameter of the ping-pong ball, 4 cm, corresponds to 0.2.

2

The table on the right shows the distance between the train station and each of the 4 people's homes.
How many times as far are the other people's homes from the train station compared to the distance between the train station and Haruka's home?

Distance between Home and Station

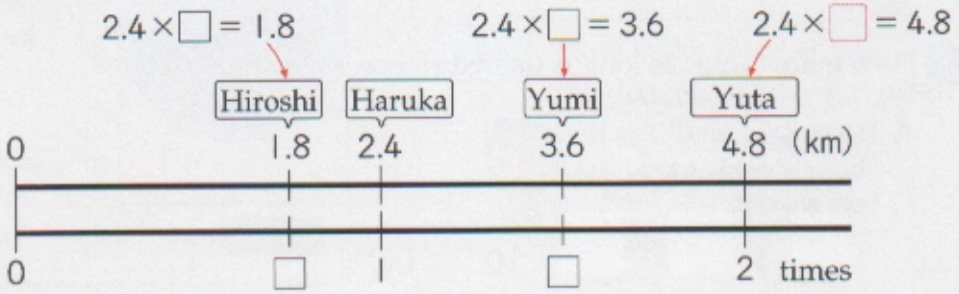
Name	Distance (km)
Haruka	2.4
Yuta	4.8
Yumi	3.6
Hiroshi	1.8

Let's think about which calculation we should use.

We should use 2.4 × □ to calculate...



Shiho



1 How many times as far are Yumi's and Hiroshi's homes from the train station as compared to the distance between the train station and Haruka's home?

Yumi Math Sentence ▸ _____ Answer □ times

Hiroshi Math Sentence ▸ _____ Answer □ times

Summary

Even when we have decimal numbers, we can use division to find how many times one quantity is compared to a base amount.
"0.75 times as far" means that if we consider 2.4 km as 1, 1.8 km will be considered as 0.75.

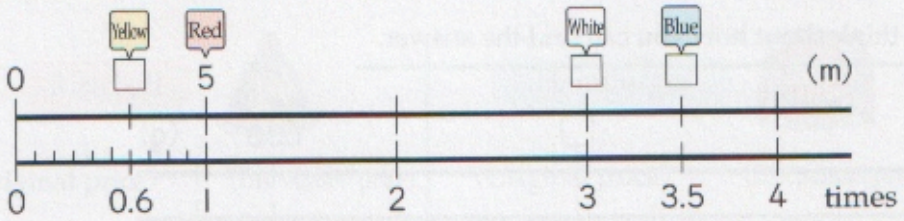


Kota

By using a number line diagram, we were able to think just like we did when the lengths were whole numbers.

3

There are four colored tapes, red, white, blue, and yellow. The red tape is 5 m long. The white tape is 3 times as long as the red tape, the blue tape is 3.5 times as long as the red tape, and the yellow tape is 0.6 times as long as the red tape. How long are the white, blue, and yellow tapes?



1 Write math sentences, and then find the answers.

White Math Sentence _____ Answer ☐ m

Blue Math Sentence _____ Answer ☐ m

Yellow Math Sentence _____ Answer ☐ m

Let's think about the meaning of math sentences.

Look at the number line diagram above to think about the question.

2 Write the appropriate number in each ☐.

White The math sentence, $5 \times 3 = 15$, expresses the relationship that if we consider 5 m as 1, 15 m corresponds to 3.

Blue The math sentence, $5 \times 3.5 = 17.5$, expresses the relationship that if we consider 5 m as 1, 17.5 m corresponds to ☐.

Yellow The math sentence, $5 \times 0.6 = 3$, expresses the relationship that if we consider 5 m as ☐, 3 m corresponds to ☐.

Math sentences mean the same whether you multiply by a whole number or decimal number.

Haruto

Misaki

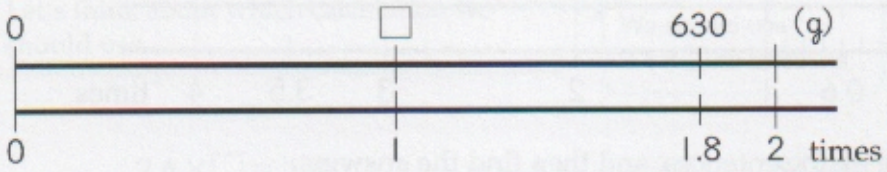
We thought about what the math sentences meant based on the number line diagram.

4

At Rena's house, there is a 10 day old puppy. It now weighs 630 g, which is 1.8 times as heavy as its weight at birth.
How much did the puppy weigh at birth?



Let's think about how you can find the answer.



If we consider the weight at birth as 1, we can consider the weight at 10 days old as 1.8, can't we?

1 If the puppy's weight at birth was □ g, write a multiplication sentence to express the relationship between its weight at birth and its current weight.

□ × □ = □

2 Write a math sentence to find the number in the □ first, then find the number.

□ = □ ÷ □

= □

Answer □ g

Summary

When you are finding the base amount, it may be easier to think about it if, **you write a multiplication sentence using a □.** This is true even when □ is multiplied by a decimal number.

□ × 1.8 = 630, so □ = 630 ÷ 1.8.

2 The area of Town A is 13.8 km², which is 0.6 times as much as the area of Town B. What is the area of Town B?

Additional Problems
→Page 135 U

Riku: Today, we found the amount that corresponded to 1.

5

A store is having a sale on rice balls and hamburgers. Which are discounted more? Think by comparing their discounted prices with their original prices.

Rice ball 

〈Original price〉 〈Discounted price〉

160 yen ➡ 110 yen

Hamburger 

〈Original price〉 〈Discounted price〉

200 yen ➡ 150 yen


Kota

The difference of the original and discounted prices are both ☐ yen, but...

The original prices are different.


Shiho

Let's think about how to compare how the prices are discounted.

1 How many times as much is the discounted price of the rice ball above as its original price? What about the hamburger above?

Rice ball Math Sentence ▸ _____ Answer ☐ times

Hamburger Math Sentence ▸ _____ Answer ☐ times

Summary

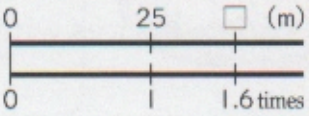
When two different quantities have different base amounts, like the discounted prices above, **you can sometimes compare the size of the discounts by using times as much the quantities are as their base amounts.**

Even when decimal numbers express how many times as much quantities are as their base amounts, you can compare the quantities using times as much just like we did with whole numbers.

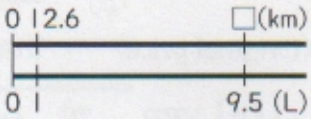
The number to which a discounted price corresponds when the original price is considered as 1 is called a rate, isn't it?

Which Calculation Should We Use?

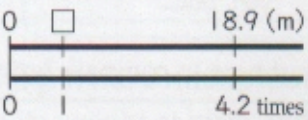
1 A department store building is 25 m tall. The height of a tower is 1.6 times as much as the height of the department store building.
How tall is the tower?



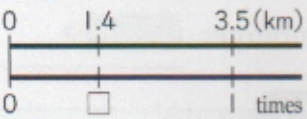
2 A car travels 12.6 km on 1 L of gas.
How many km does this car travel on 9.5 L?

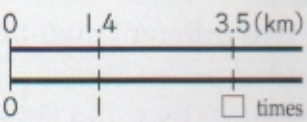


3 There are green houses that have rectangular floors. The length of each of the green houses is 18.9 m and that is 4.2 times as long as their width.
How long is the width of each of the green houses?

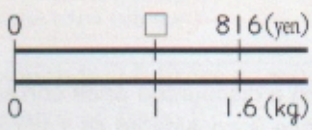


4 The length of South Tunnel is 1.4 km. The length of North Tunnel is 3.5 km.
The length of South Tunnel is how many times as long as the length of North Tunnel?
How many times as long is the length of North Tunnel as the length of South Tunnel?





5 At a market on the beach, you bought 1.6 kg of little clams for 816 yen.
What was the price for 1 kg of clams?

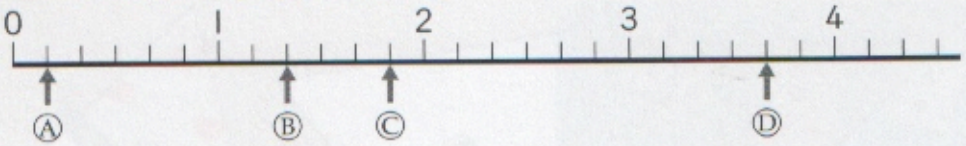


Do You Remember?

Answers→ Page 147

1 What fractions are represented by arrows Ⓐ through Ⓓ? If the fraction is greater than 1, write it as both a mixed number and an improper fraction.

How to Express Fractions and the Structure of Fractions Page 150 ⑥



2 Write the appropriate number in each □.

① $4.385 = 1 \times \square + 0.1 \times \square + 0.01 \times \square + 0.001 \times \square$

② $51.6 \times 10 = \square$　　③ $51.6 \div 10 = \square$

④ $2.4 \times 100 = \square$　　⑤ $2.4 \div 100 = \square$

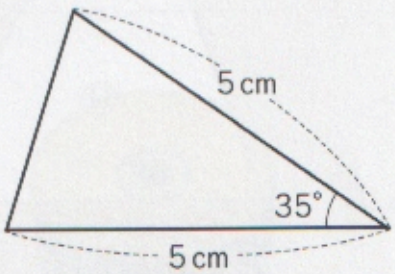
3 ① $6.43 + 1.4$　② $0.48 + 1.52$　③ $37.2 + 1.08$　④ $22 + 3.859$

⑤ $0.8 - 0.29$　⑥ $5.45 - 4.5$　⑦ $6 - 1.74$　⑧ $1 - 0.092$

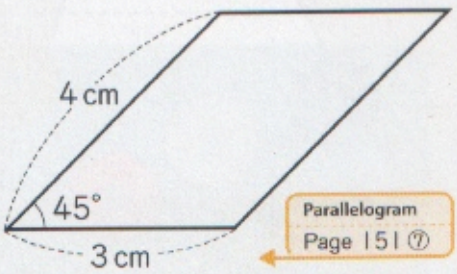
Warm-up

4 Draw an isosceles triangle and a parallelogram as shown below.

①



②



Parallelogram Page 151 ⑦

Playing with Numbers and Calculations

Using 100 or 1,000

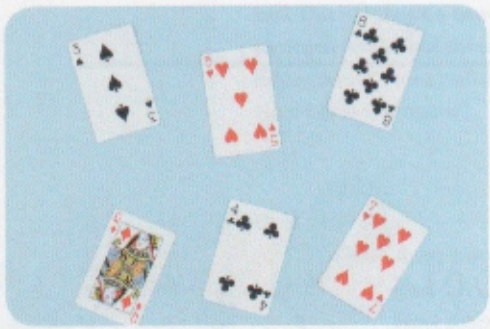
First, find the products of ① through ④. Then, use the answers to find the products of ⑤ through ⑧.

① 25×4　② 50×2　③ 125×8　④ 250×4

⑤ $97 \times 25 \times 4$　⑥ $50 \times 93 \times 2$　⑦ $13 \times 8 \times 125$　⑧ 44×250

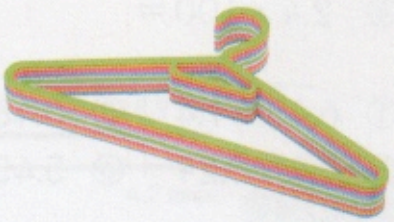
Which of the multiplication sentences ①~④ help to easily calculate ⑤~⑧?

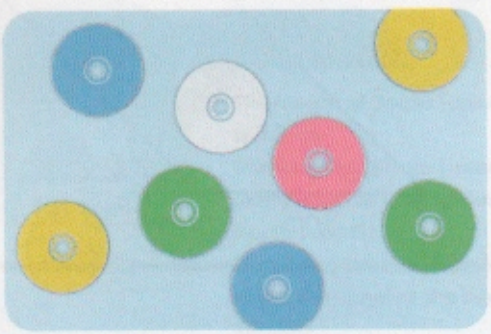
What are the same?













Why is it that each of the above three things can fit on top of each other perfectly? Discuss with your classmates.



Shiho

They have the same shape.

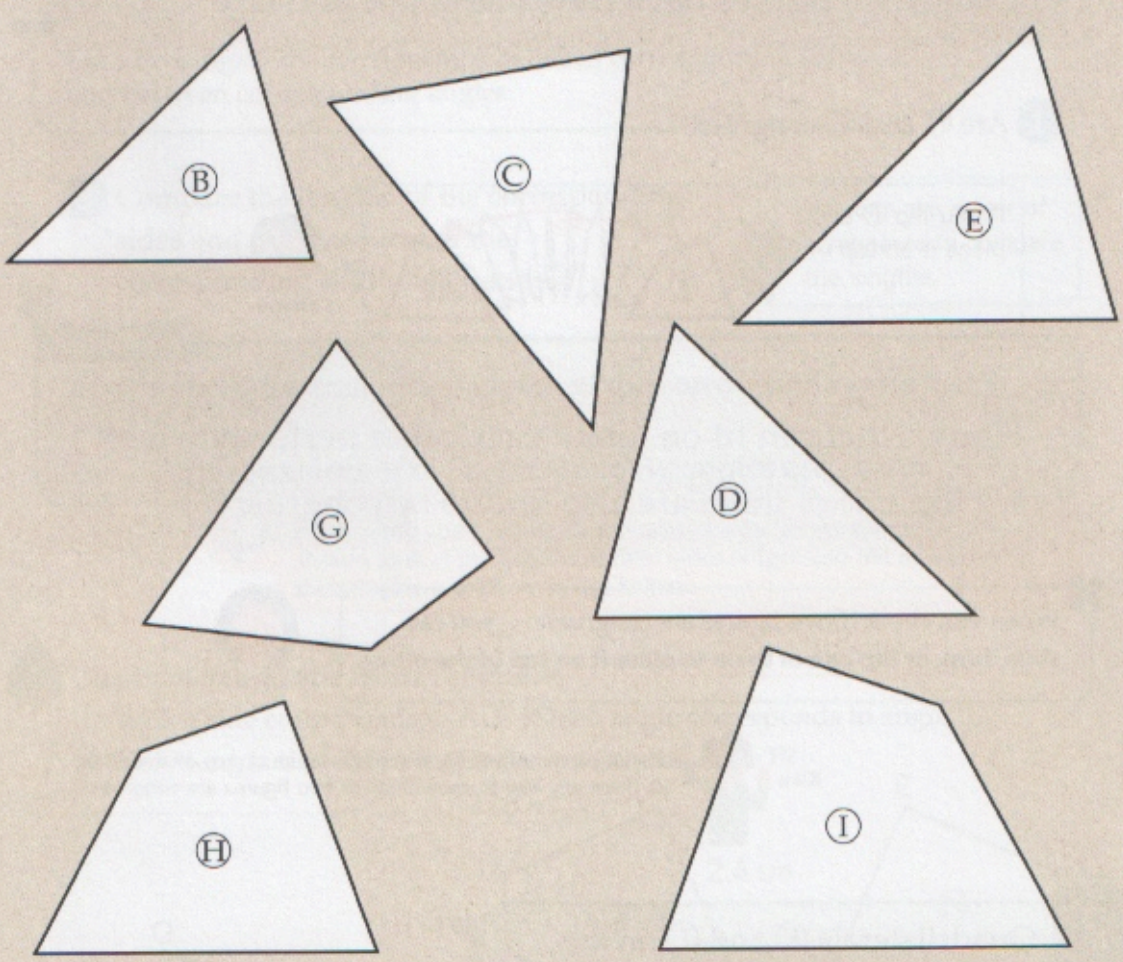


Kota

They need to have not only the same shape but also the same ____ ...

Congruent Shapes

6 Let's Investigate Figures That Have the Same Shape and Size



1 Among the figures above, which ones have the same shape and size as Ⓐ and Ⓕ on page 153?

Let's think about how to find the figures that have the same shape and size.

Cut out Ⓐ and Ⓕ on page 153 to use them.

The figures have the same shape and size. So, if you place one on top of the other...

Misaki

When two figures can fit on top of each other perfectly, we say that **they are congruent.**
Congruent figures have the same shape and size.

1 Are Ⓗ and Ⓕ congruent?

If you flip Ⓕ and place it on top of Ⓗ...




Haruto

Figures that can fit on top of each other perfectly when we flip one of them are also said to be congruent.

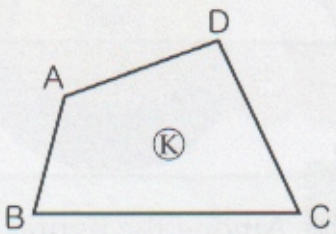
When you check if two figures are congruent, **you can slide, turn, or flip one of them to place it on top of the other.**


Ami


Riku

If you cannot physically place one figure on top of the other, is there any way to show that the two figures are congruent?

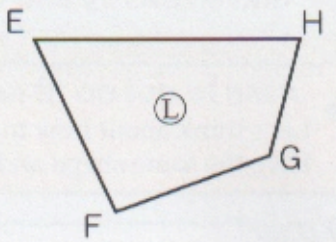
2 Quadrilaterals Ⓚ and Ⓛ on the right are congruent.
Use Ⓚ and Ⓛ to find out the properties of congruent figures in detail.



2 What properties of Ⓚ and Ⓛ should you look at carefully?

When Ⓚ is placed on top of Ⓛ or the other way around, which parts match up...?


Misaki



In congruent figures, the sides, angles and vertices that match up when the figures are placed on top of each other are called **corresponding sides**, **corresponding angles** and **corresponding vertices.**

Let's investigate the relationships between corresponding sides, and between corresponding angles.

3 Compare the lengths of the corresponding sides and the measures of the corresponding angles in Ⓚ and Ⓛ.

You can use a pair of compasses to compare the lengths.

Summary

In congruent figures, **the lengths of the corresponding sides are equal.**

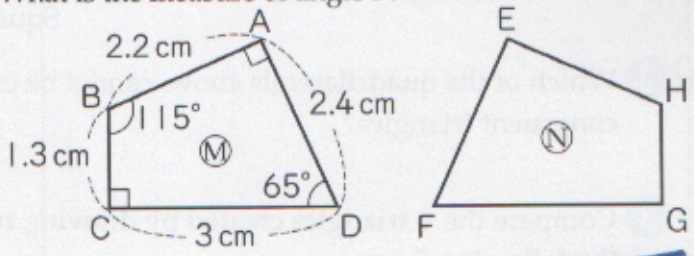
Also, **the measures of the corresponding angles are equal.**

Even when you investigate the relationship between figures, you should look at the lengths of their sides/edges and the measures of their angles, just like you did before.

1 Quadrilaterals Ⓜ and Ⓝ are congruent.

① Which side corresponds to AD? Which angle corresponds to angle B?

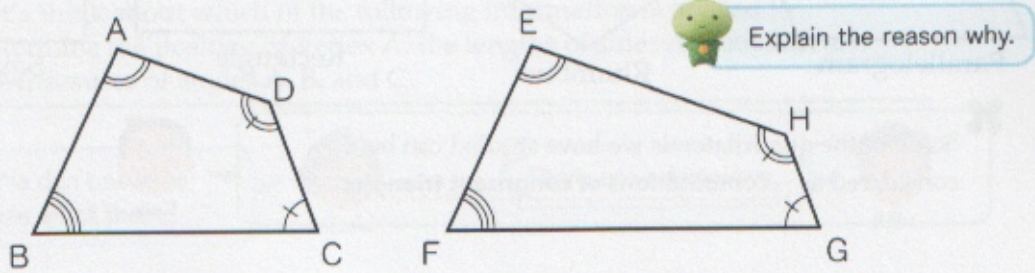
② How long is side EH? What is the measure of angle F?



Additional Problems
→Page 135 V

2 Can we say that the two quadrilaterals below are congruent?

Explain the reason why.





Shiho: Figures that have the same angles are not necessarily congruent.

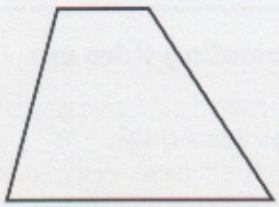
3 Split each of the following quadrilaterals that you have studied into 2 triangles by a diagonal. Investigate if the two triangles in each quadrilateral are congruent.

Let's investigate the properties of the quadrilaterals by focusing on whether the 2 triangles along the diagonal are congruent.

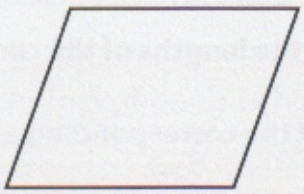
Diagonals
Page 151 ⑨

Use the quadrilaterals given on page 153 to investigate.

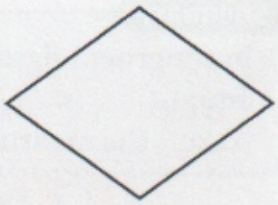
1 Compare the 2 triangles created by drawing a diagonal in each of the following figures.



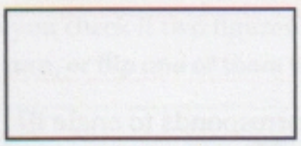
Trapezoid



Parallelogram



Rhombus



Rectangle

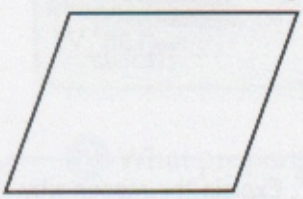


Square

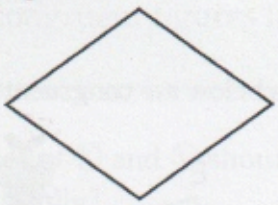
Various Quadrilaterals
Page 151 ⑦

2 Which of the quadrilaterals above cannot be created by combining 2 congruent triangles?

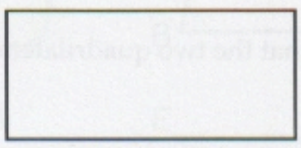
3 Compare the 4 triangles created by drawing two diagonals in each of the following figures.



Parallelogram



Rhombus



Rectangle



Square

Some of the quadrilaterals we have studied can be considered as **combinations of congruent triangles.**



Kota

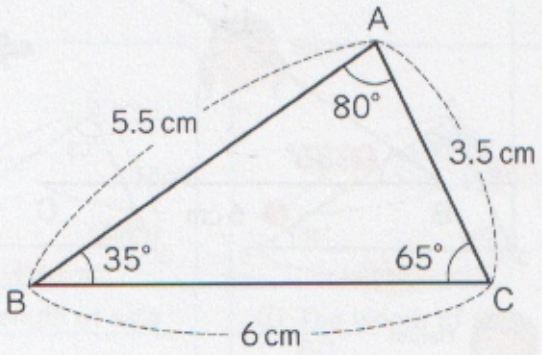


Ami

In 4th grade, we focused on the lengths of diagonals and how they intersect. Today, we focused on the triangles along the diagonal.

4 Think about how to draw a triangle that will be congruent to triangle ABC below.

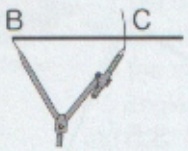
If you know the location of the 3 vertices you can draw a triangle. Which side lengths and which angle measures are needed to locate the 3 vertices?

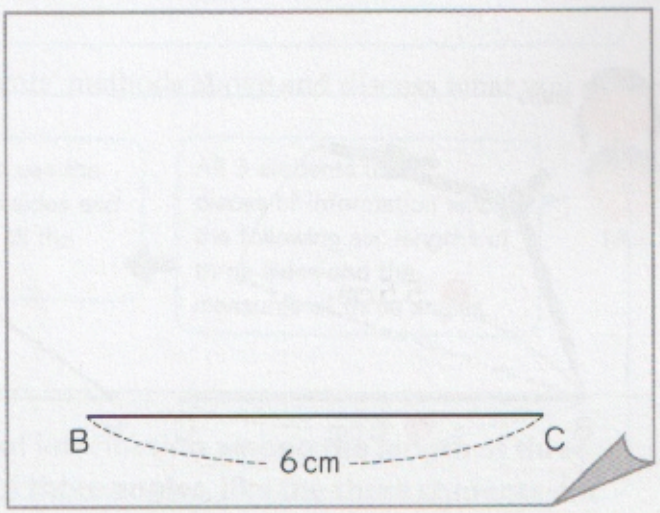


1 We drew side BC first, and now vertices B and C are fixed. The length of which side and the measure of which angle do you need to decide where the last vertex, A, should be?

We used the length of side BC first to decide where vertices B and C were.

You can use a pair of compasses to copy the length of side BC.





Let's think about which of the following information is needed to determine the position of vertex A: the lengths of sides AB and AC, and the measures of angles A, B, and C.

We don't need to use all of them...



Misaki

There are a few ways...



Riku

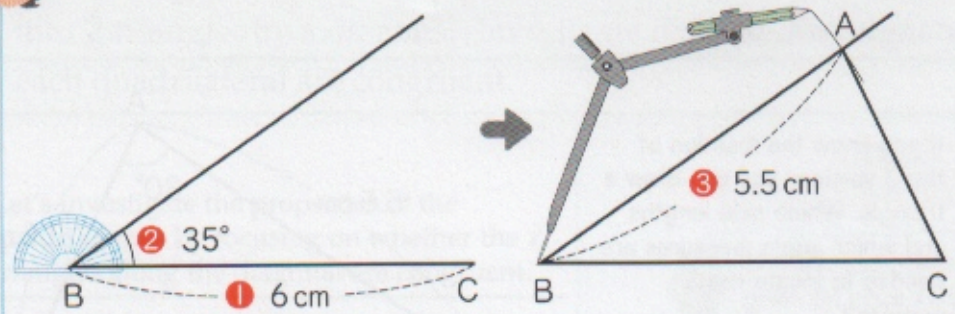
Haruto



Once you can do it in one way, maybe you should try to think of another way.

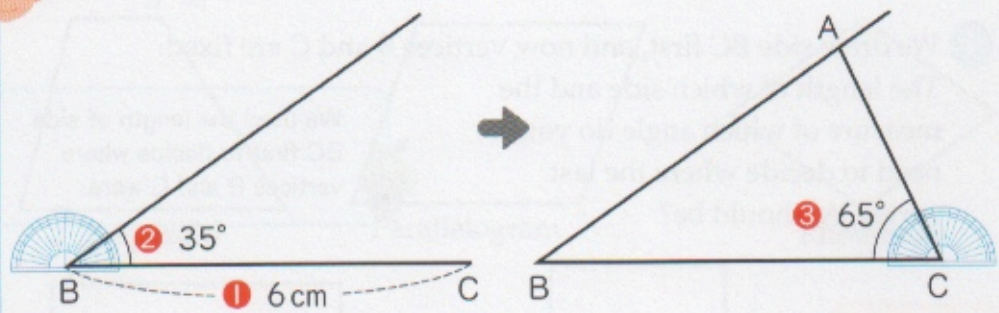
〈Watch〉

Ami

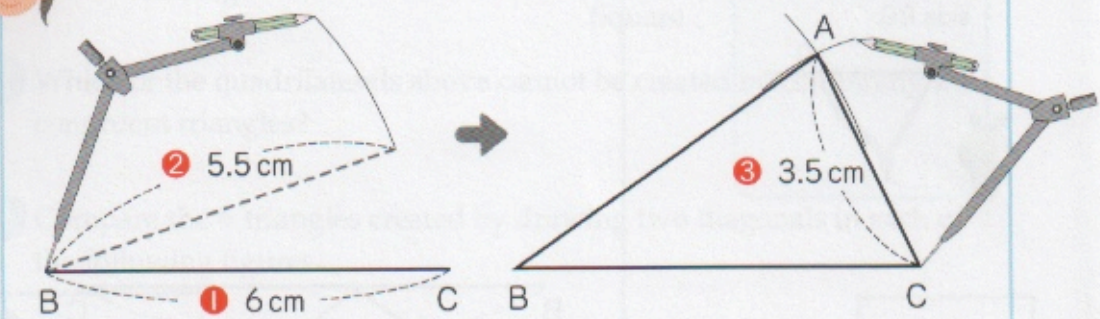


How to Draw an Angle
Page 151 ⑧

Haruto



Shiho



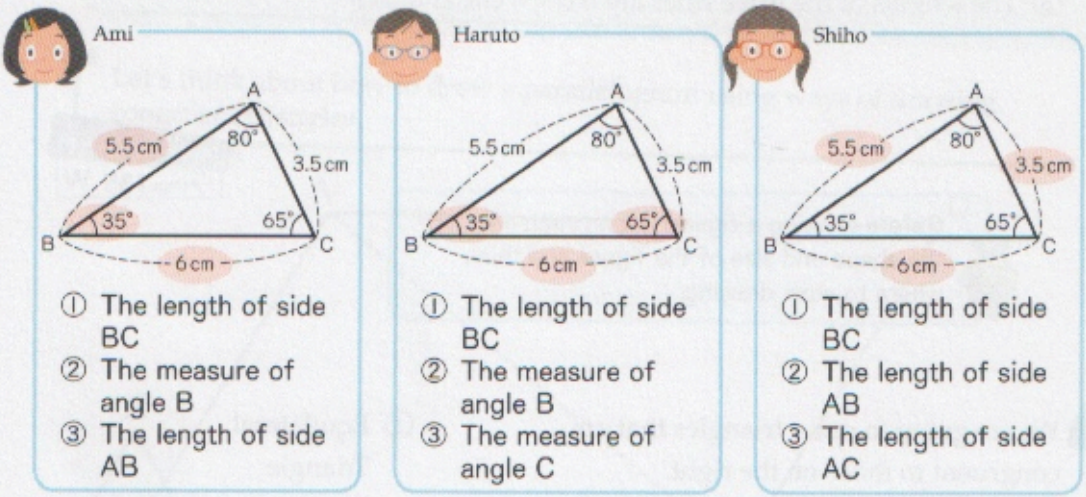
2 Using the three students' methods above, draw a triangle that is congruent to triangle ABC.

Leave all the marks you used in the process of drawing.

3 Check to see if the triangle that you drew is congruent to the original triangle ABC.

Are the lengths of the corresponding sides and the measures of the corresponding angles equal to each other?

4 Which information about the lengths of sides and the measures of angles do the three students' methods use?



The lengths of 2 sides and the measure of the angle between them

The length of one side and the measures of the two angles on the ends of the side

The lengths of the three sides

5 Look at the three students' methods above and discuss what you notice.


Misaki

We don't need to use the lengths of all the sides and the measures of all the angles...

All 3 students used ☐ pieces of information among the following six: lengths of three sides and the measures of three angles.


Riku

Summary

If you use three pieces of information among the length of three sides and the measures of three angles, like the three students did above, you can decide the position of vertex A and draw a triangle congruent to triangle ABC.



As shown in the three students' methods above, the shape and size of a triangle are fixed if you decide three pieces of information among the lengths of three sides and the measures of three angles.


Kota

In 4th grade, we studied the way Haruto drew the triangle. He used one of the conditions that determine the shape and size of a triangle.

3 Draw the following triangles in your notebook.

① Two sides are 4 cm and 7 cm and the angle between them is 60°.

② A side is 4 cm and the two angles on the ends of the side are 45° and 30°.

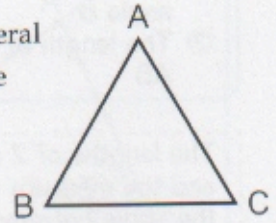
③ The lengths of the three sides are 5 cm, 4 cm, and 3 cm.

Additional Problems
→Page 135 W

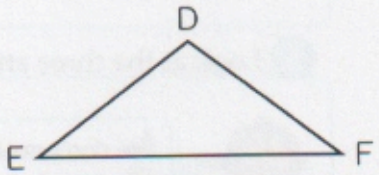
Before drawing a figure, get a rough idea of the shape and size of the figure and think where to start drawing.

4 We are going to draw triangles that are congruent to those on the right.
Other than the types of figures to draw, what information do we need to draw them? Get the minimum information you need by measuring the side(s) and angle(s) of the triangles on the right.

① Equilateral Triangle



② Isosceles Triangle



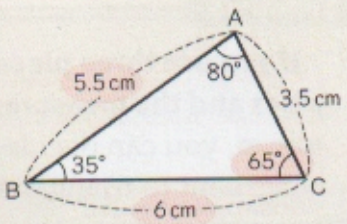


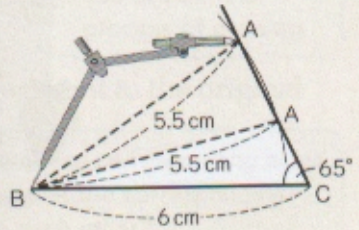
Misaki: I want to try to draw a congruent quadrilateral, too.

Mathlin Newsletter

Two vertex A's?

If you use the lengths of two sides and the measure of an angle that is not between the two sides to draw a triangle, the triangle may not be congruent to the original one.



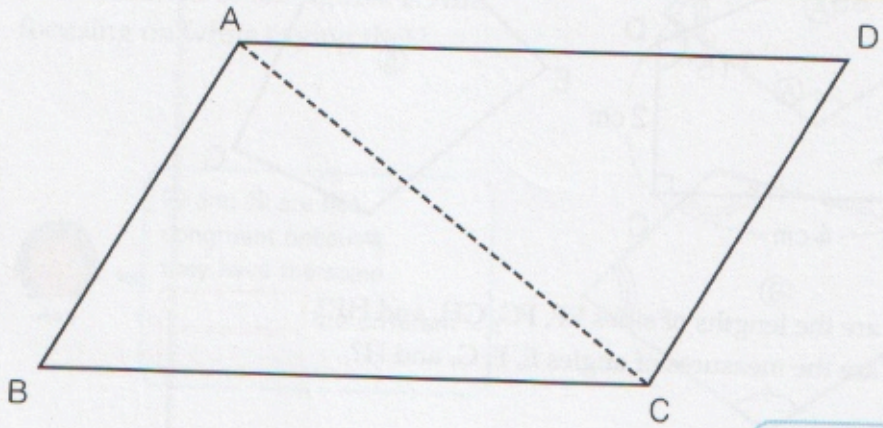


That's because the position of vertex A cannot be fixed. There can be two positions.

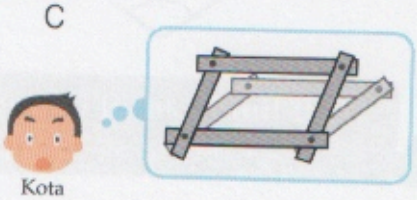
5

Using the known ways for drawing congruent triangles, draw a parallelogram that is congruent to parallelogram ABCD below.

Let's think about how to draw a parallelogram using ways of drawing congruent triangles.



1 Can we draw congruent quadrilaterals by using only the lengths of the four sides?



Kota

2 Measure lengths and angles to get the information necessary. Then, use the information to draw the parallelogram.

To use the ways of drawing congruent triangles, a diagonal should...



Shiho

3 Make it clear which lengths of sides and which measures of angles you used to draw the parallelogram and explain the way you drew it.



Ami

First, I measured the lengths of sides AB and BC and diagonal AC and drew triangle ABC.
Next, I used the lengths of sides AD and CD to...

If you draw a diagonal to split the parallelogram into 2 triangles and examine them, you can draw a parallelogram using the ways of drawing congruent triangles.



Riku

Additional Problems
→Page 136

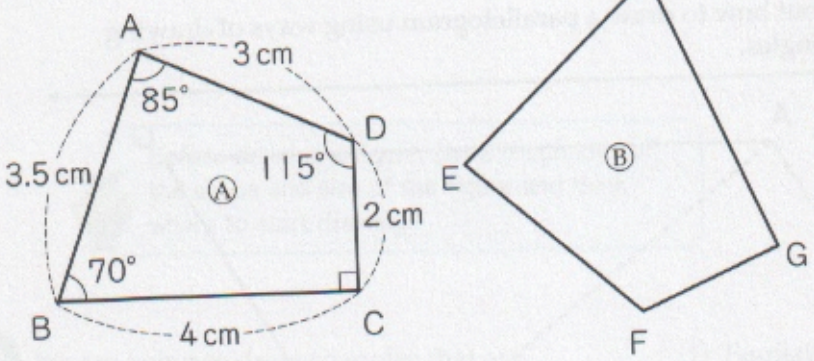
Haruto



I wonder if we can draw squares, rectangles, and rhombuses in the same way.

Check Your Understanding

1 Quadrilaterals Ⓐ and Ⓑ below are congruent.



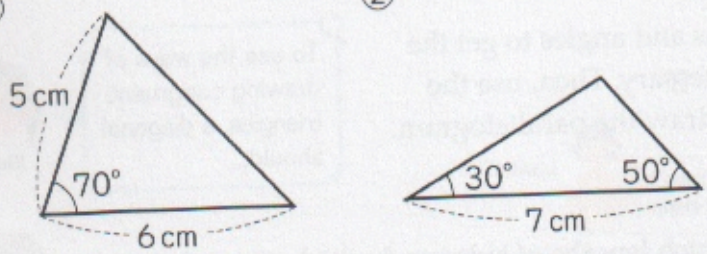
① What are the lengths of sides EF, FG, GH, and HE?
② What are the measures of angles E, F, G, and H?

Do you understand the properties of the corresponding sides and angles of congruent figures?

Page 74 2

2 Draw the triangles below.

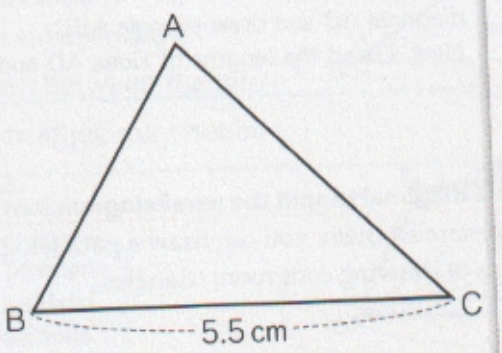
① ②



Can you draw congruent triangles?

Page 77 4

3 Make necessary measures of angles and draw a triangle that is congruent to triangle ABC.



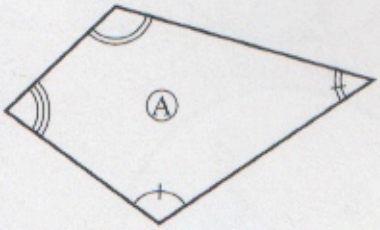
Do you understand the conditions that determine the shape and size of a triangle?

Page 77 4

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

1 Focus on the sides and angles of figures and investigate the relationship between figures

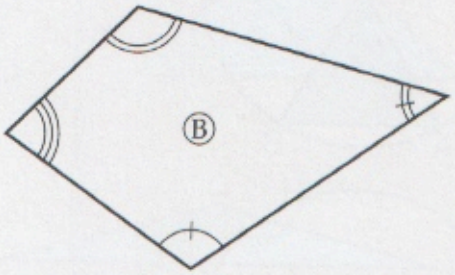
Ami says that quadrilaterals Ⓐ and Ⓑ on the right are not congruent to each other. What element of the figures is Ami focusing on while saying that?





Ami

Ⓐ and Ⓑ are not congruent because they have the same [ ] but different [ ].



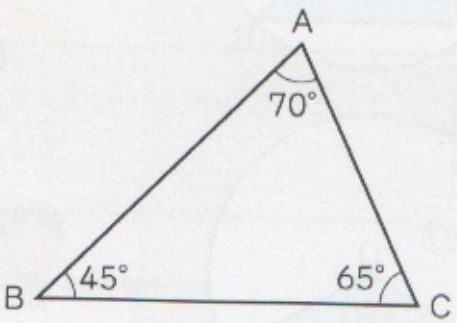
2 Focus on the sides and angles of figures and think about the conditions that determine the shapes and sizes of figures

Is it possible to draw a triangle that is congruent to the one on the right without measuring anything else? Explain the reason why.



Kota

It is [ ] to draw one, because you only know the measures of the angles. To determine the size, you...



Look back on what you have learned in "Let's Investigate Figures That Have the Same Shape and Size" and discuss.



Misaki

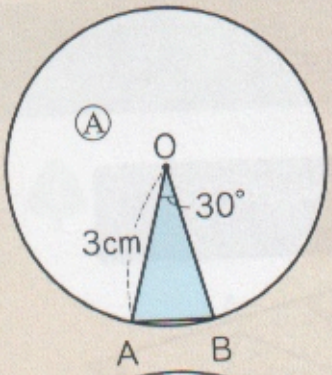
Now I can investigate the relationship between figures based on whether they are congruent or not.
When investigating the relationship between figures, we focused on the lengths of their sides and the measures of their angles.

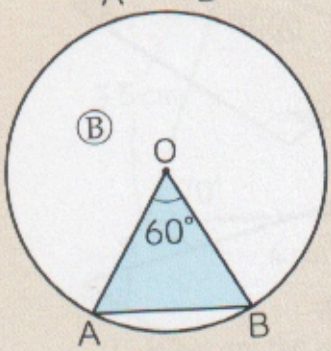


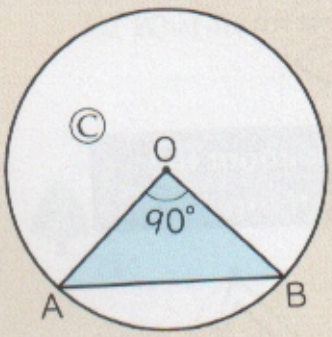
Haruto

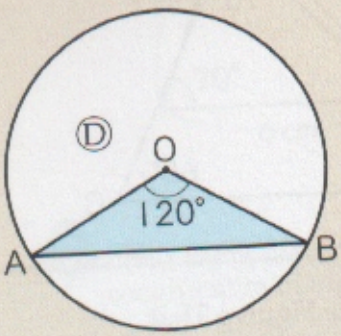
I also want to think about the conditions that determine the shapes and sizes of many different figures.

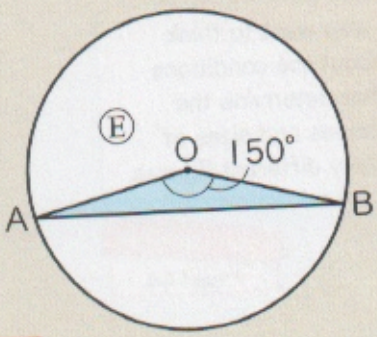
Challenge Yourself
→Page 144











Look for the secrets behind the measures of angles of triangles

Shiho and Kota are drawing different kinds of isosceles triangles using the radius of a circle.

Isosceles triangles have two equal angles.



Shiho

Kota

The shape of Ⓒ is...



ojo

Discuss what you notice about angles of triangles.

Riku: If angle O increases, angles A and B...

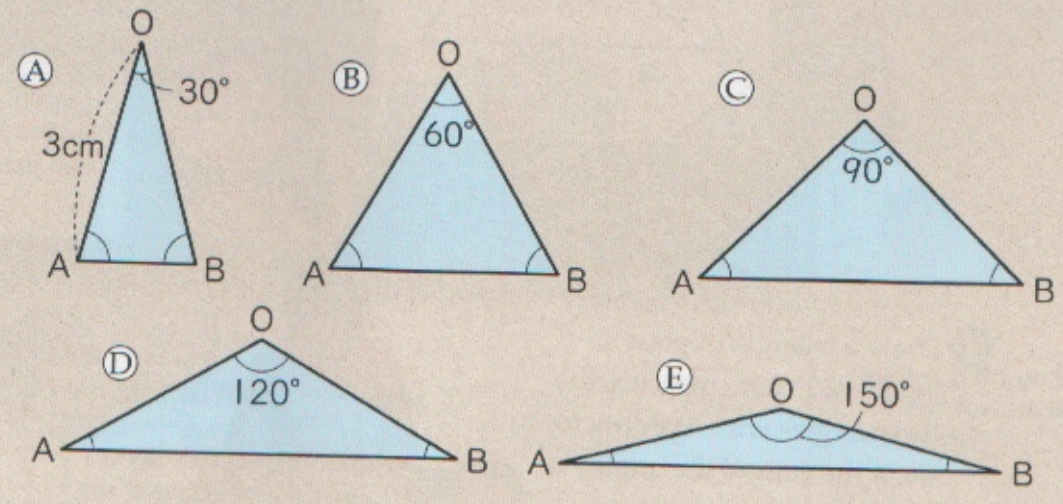
Ami: If angle O decreases, angles A and B...

Misaki: Ⓒ has the shape of a set square. So, angles A and B are both 45°.

Haruto: It looks like there is a pattern in the sizes of angles in triangles.

7 Let's Investigate Angles of Geometric Figures

Use a protractor to measure the angles of the isosceles triangles below.

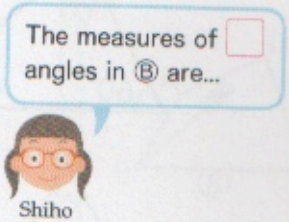


1 Angles of Triangles and Quadrilaterals

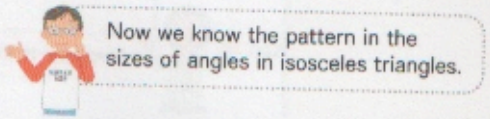
1 Investigate if there is any pattern in the sizes of angles in triangles.

1 List the measures of the angles in the table below.

	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
Angle O	30°	60°	90°	120°	150°
Angle A			45°		
Angle B			45°		



Let's think about the sum of the 3 angles of a triangle.




Shiho

In each of isosceles triangles Ⓐ through Ⓔ on the previous page, the sum of the 3 angles is ☐°.

I wonder if this pattern applies to other triangles, too.


Kota

2 Draw many different triangles, and find the sum of the 3 angles in each as shown in the diagram below.

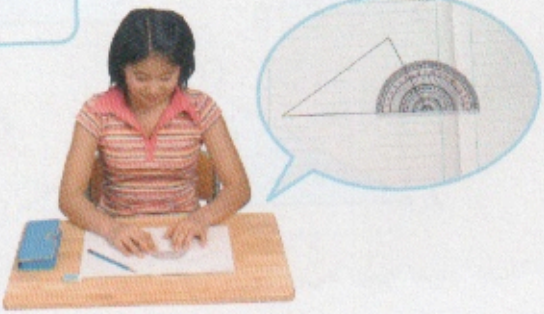



If you put the three angles around a point, what happens?


Riku

We have examined many different triangles. Triangles in all shapes and sizes...

3 Draw a triangle in your notebook. Then, measure the three angles with a protractor to see if the sum of the angles is 180°.



Summary

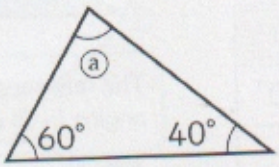
The sum of the 3 angles of a triangle is 180°.


If you put the three angles of a triangle around a point, they form a line.

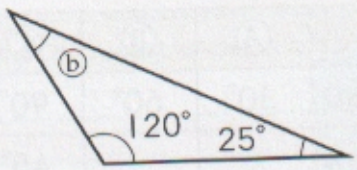
This pattern applies to triangles in all shapes and sizes.

1 What are the measures of angles ⓐ, ⓑ, ⓒ, and ⓓ below? Calculate to find their measures.

①



②

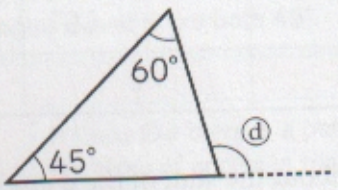


③



Equilateral Triangle

④

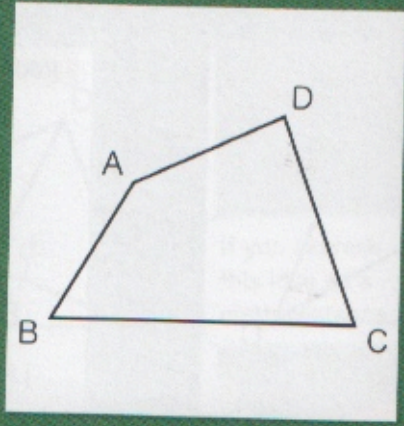


Additional Problems
→Page 136 Y


Ami

I wonder if there is a pattern in the sizes of angles of quadrilaterals.

2 What is the sum of the 4 angles of a quadrilateral?





Grasp the problem.

- What problem are we going to work on today?

1 Plan how to find the answer.

I know what the sum of the 3 angles of a triangle is, but...



Misaki

- What idea may be useful to solve the problem?
- Is there anything you have learned before that you can use to solve this problem?

Let's think about how to determine the sum of the 4 angles of a quadrilateral.

2 Write down your ideas using diagrams and math sentences.

Write down your ideas.

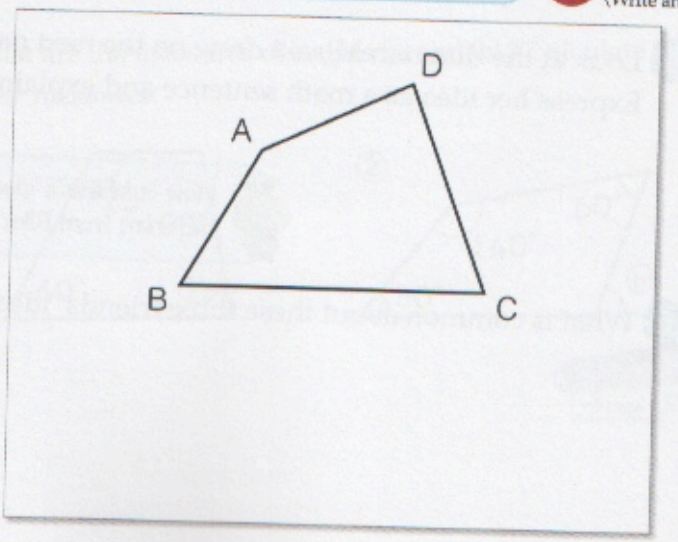


There are diagrams you can use on page 155.



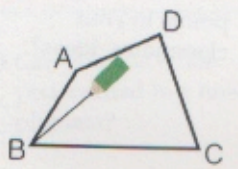
〈Write and Move〉

- Is your idea clear to others?





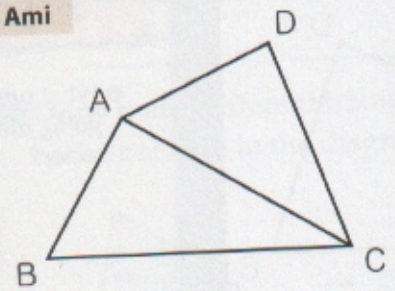
Kota



- If you figured it out one way, try to think of another way.

Haruto and some other students are explaining their classmates' ideas.

Ami



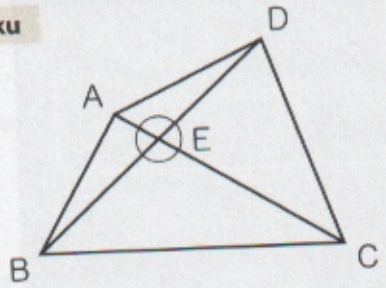
$180 \times 2 = 360$

Answer $360°$



Haruto

Riku



$180 \times 4 - 360 = 720 - 360$

$= 360$

Answer $360°$

Learn with your classmates.

- Can you understand your classmates' ideas based on their diagrams and math sentences?
- What is common and what is different about your own idea and your classmates' ideas?
- What are the good points in your classmates' ideas?

3 Look at the diagram and math sentence Ami wrote. Explain her thinking.



Haruto

First, split quadrilateral ABCD into 2 triangles with diagonal AC. Since the sum of the 3 angles of a triangle is 180°, ...

4 Look at the diagram and math sentence Riku wrote. Explain his thinking.



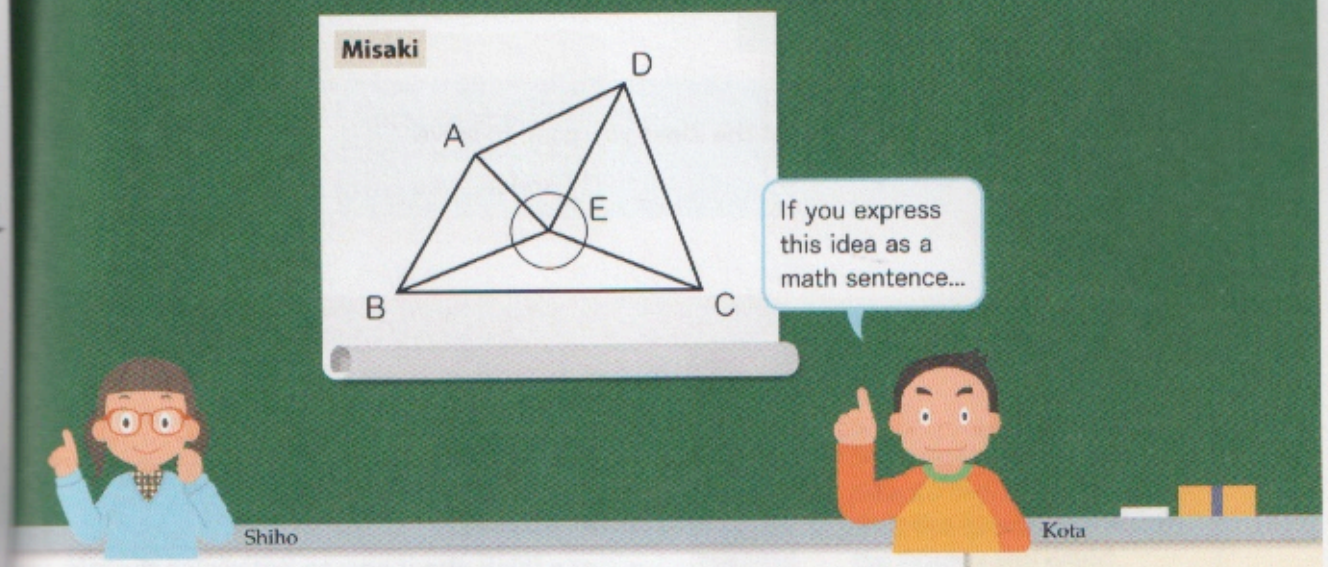
Why did he subtract 360°?

5 Look at the diagram Misaki drew on the next page. Express her idea as a math sentence and explain it.



How is Misaki's idea different from Riku's?

6 What is common about these three friends' ideas?



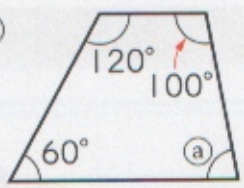
7 Look back and summarize today's lesson.

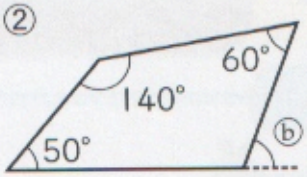
Summary

The sum of the angles of a quadrilateral can be found by **splitting the quadrilateral into triangles.**
The sum of the 4 angles of a quadrilateral is 360°.

We can explain this by using the fact that the sum of the 3 angles of a triangle is 180°, can't we?

2 What are the measures of angles ⓐ and ⓑ ? Calculate to find their measures.

① 

② 

Additional Problems
→Page 137 Z

Look back and summarize today's lesson.

- What did you learn from today's investigation?
- Which way of thinking was useful?
- What do you want to investigate next?

Put it into use.

- Can you use what you learned in a new ploblem?



Look back at the ideas you used to solve problems.



Ami

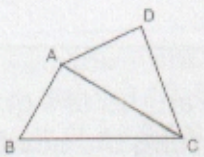
September 10

<Problem>

What is the sum of the 4 angles of a quadrilateral?

- Let's think about how to determine the sum of the 4 angles of a quadrilateral.

<My idea>



① 180 × 2 = 360

I know what the sum of the 3 angles of a triangle is 180°.
I learned it on September 9.

Answer 360°

Because I knew that the sum of the three angles of a triangle is 180°, I drew a diagonal to divide the quadrilateral into two triangles. Since there are two triangles, it becomes 180 × 2.

She thought based on triangles because she knew what the sum of the angles of a triangle amounted to.

When you work on problems, not only write math sentences and answers but also consider using:

- **Diagrams**
- **Tables**
- **Graphs**

Classmates' reflections.



Haruto

There are many ways to divide the quadrilaterals, but all involved dividing the quadrilateral into triangles.



He noticed what some students' ideas had in common and wrote about it.

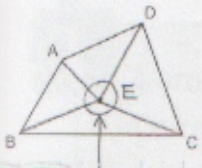
Note Taking Tip ①

Today's lesson was based on what she had learned before. So, she added the date of the lesson so that she can go back to the page of her notebook.

Note Taking Tip ②

She used an arrow to connect her diagram with what she wrote in words.

<Misaki's idea>



$180 \times 4 - 360 = 720 - 360$
$= 360$

Answer 360°

② I subtracted 360° of this part.

She added a point and lines to the diagram to split the quadrilateral into the figures she could handle easily.

<Summary>

The sum of the angles of a quadrilateral can be found by splitting the quadrilateral into triangles.
The sum of the 4 angles of a quadrilateral is 360°.

She explains logically based on the fact that she already knows (the sum of the 3 angles of a triangle).

<My Reflection>

It turns out that the sum of the four angles of a quadrilateral can be obtained by calculation without having to measure each angle.



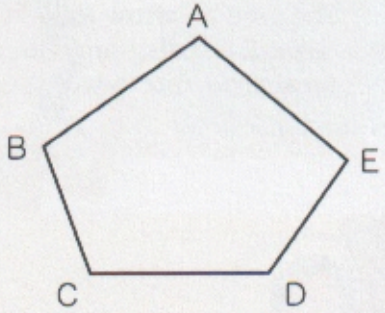
Shiho

Misaki's idea suggests that point E can be anywhere inside the quadrilateral. I want to check if that's the case.



She wrote about further ideas she would like to investigate.

3 Find the sum of the angles in the figures below.



Pentagon



Hexagon

- Pentagon···A geometric figure surrounded by 5 lines.
- Hexagon···A geometric figure surrounded by 6 lines.

Geometric figures that are surrounded by lines, like triangles, quadrilaterals, pentagons, and hexagons, are called **polygons.**

Let's think about how to find the sum of the angles of a polygon.

1 Find the sum of the angles of a pentagon and a hexagon.





Ami

2 Summarize what you found in a table.

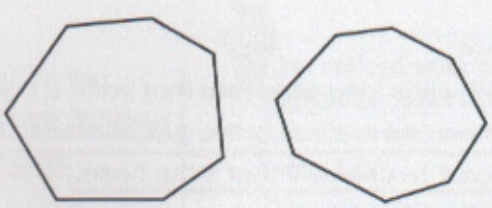
	Triangle	Quadrilateral	Pentagon	Hexagon			
Number of triangles	1						
Sum of angles	180°						

We can think based on figures **whose sum of angles we already know.**



Haruto

3 Find the sum of the angles of a heptagon and an octagon.



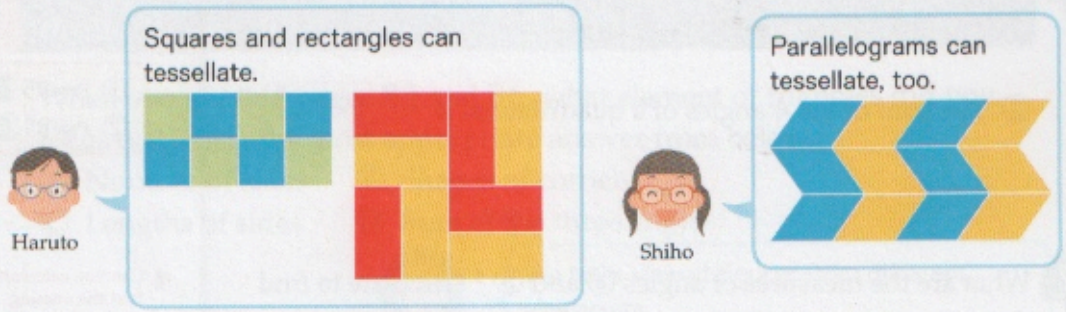


Shiho

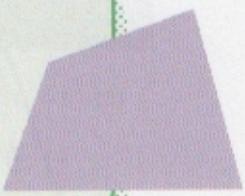
It looks like we can even find the sum of the angles of an icosagon.

2 Tessellations

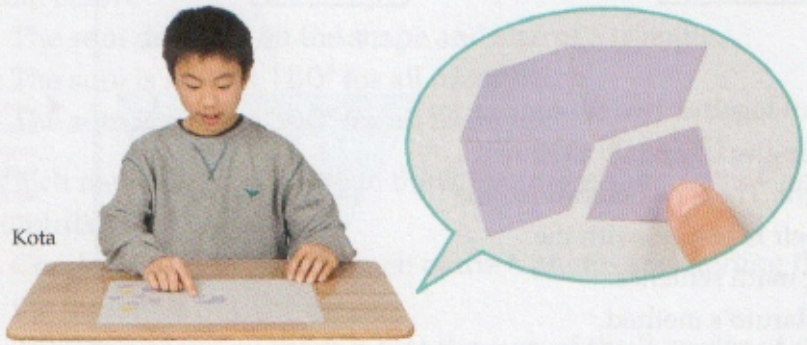
Students are talking about what kinds of quadrilaterals can tessellate.



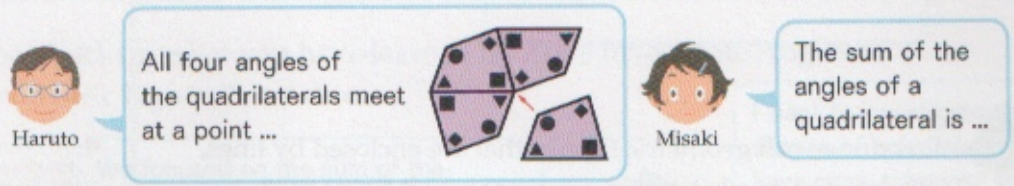
1 Can the quadrilateral on the right tessellate? Cut out the quadrilaterals on page 155 and investigate.



Let's find out if all quadrilaterals can tessellate.

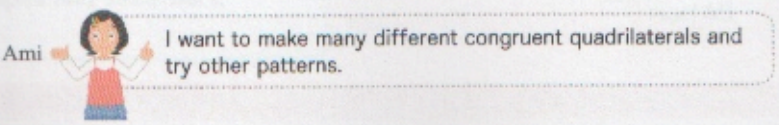


1 What do you notice about the way Kota tessellated with the quadrilateral?



The sum of the 4 angles of a quadrilateral is 360°. So, if you put the 4 angles around a point, any quadrilateral can tessellate.





Check Your Understanding

1 Write the appropriate number in each □.

① The sum of the 3 angles of a triangle is ⋯ □°.

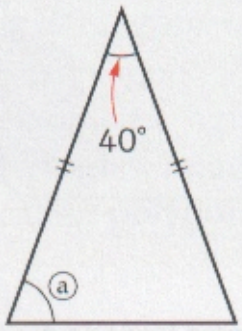
② The sum of the 4 angles of a quadrilateral is ⋯ □°.

Do you understand the sum of the angles of a triangle and a quadrilateral?

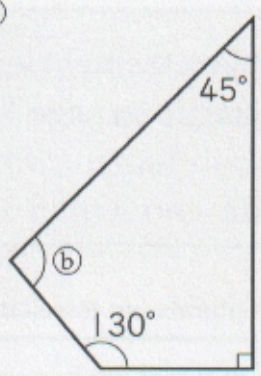
① Page 85 1
② Page 87 2

2 What are the measures of angles ⓐ and ⓑ? Calculate to find them.

① Isosceles Triangle



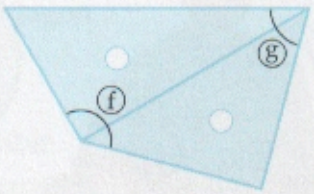
②



Can you calculate to find the missing angle measures?

① Page 85 1
② Page 87 2

3 Haruto put together two set squares to create angles ⓕ and ⓖ as shown on the right. Then, he determined the sum of their measures with the following math sentence. Explain Haruto's method.





Haruto

$360-(60+90)=210$

Can you explain how the measure of an angle was found by looking at a math sentence?

Page 87 2

4 What do we call geometric figures that are enclosed by lines, like triangles, quadrilaterals, pentagons, and hexagons?

Do you understand the name of the shapes that are enclosed by lines?

Page 92 3

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

1 Focus on the measures of angles of figures and think about properties of the figures

When you studied pages 85 and 86, what element of triangles did you focus on? Select the most appropriate answer from below.

Ⓐ Number of sides Ⓑ Shapes of corners
Ⓒ Lengths of sides Ⓓ Sum of the three angles



Ami: We have viewed figures from different viewpoints.

2 Explain the properties of figures logically

① When you learned about the sum of the 3 angles in a triangle, which method did you use to check the sum?
Ⓐ Checking the sum with only one triangle
Ⓑ Checking the sum with more than one triangle

② Select the correct statement about the sum of the 3 angles of a triangle from below.
Ⓐ The sum depends on the shape and size of a triangle.
Ⓑ The sum is always 180° for all triangles.
Ⓒ The sum is always 360° for all triangles.

③ Which method did you use to think about the sum of the 4 angles of a quadrilateral?
Ⓐ Checking the measure of each of the 4 angles and finding the sum of the measures
Ⓑ Thinking based on the fact that the sum of the 3 angles of any triangle is 180°

Look back on what you have learned in "Let's Investigate Angles of Geometric Figures" and discuss.



Misaki: We focused on the sum of the angles of a polygon. I was surprised by the fact that the sum of the angles was the same when the number of sides was the same, even if their shapes and sizes were different.



Riku: I want to investigate the shapes of polygons that have more sides and the sum of the angles of such polygons.

Challenge Yourself
→Page 145

Let's play a number quiz

Look at the multiplication table below. Out of the multiplication facts, pick one number as the answer to a quiz.
Think about three whole numbers (from 1 to 9) that fit in the ☐ below to complete 3 hints for the quiz. Make sure that the number you pick is the only answer to the quiz. Create quiz questions with your friends and quiz each other.

Multiplicand \ Multiplier	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81



3 Hints

① From the multiplication facts ☐6☐.
② If you divide by ☐4☐, the quotient is an integer and you can divide it.
③ It is a number you can get by multiplying ☐9☐ by a whole number.

After playing number quizzes, discuss whatever you thought about whole numbers.


Shiho

Some numbers can be divided by many different numbers without a remainder, and others can be divided by only a few numbers without a remainder.


Misaki

I picked 25, but I couldn't make a quiz with the 3 hints above.


Haruto

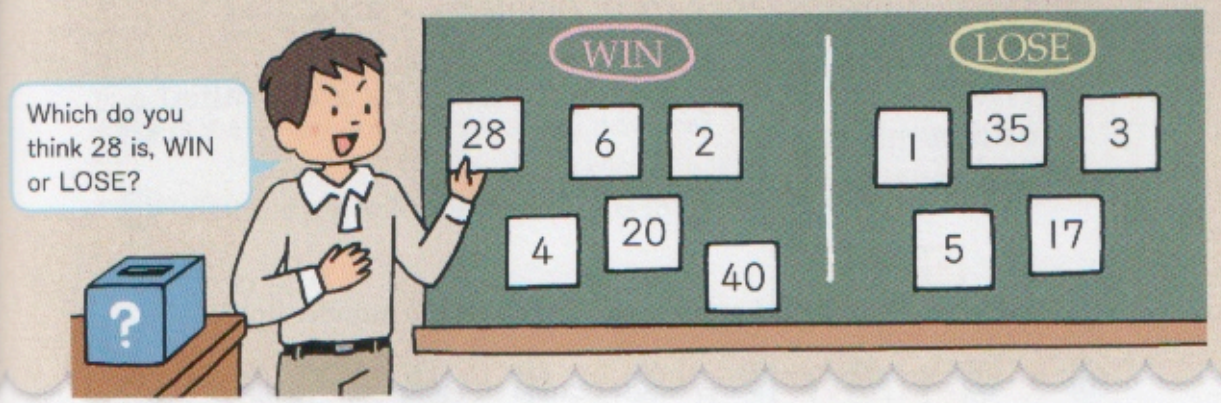
It looks like different numbers have different properties.

ojo

8 Let's Investigate the Properties of Whole Numbers

Even and Odd Numbers, Multi

The teacher is sorting out 40 cards from 1 to 40 into two categories: WIN and LOSE.



1 Even and Odd Numbers

1 Look at the picture above and find out what types of numbers are in WIN and LOSE.

Let's explore the properties of whole numbers and think about how to divide them into two categories.

1 Look at each of the numbers and share what you notice.

Kota: WIN 2, 4, 6, 20, and 40
LOSE 1, 3, 5, 17, and 35
I wonder if he is alternating between the two categories when sorting out 1, 2, 3, 4...

Ami: Numerals in the ones place of the numbers in WIN are 0, 2, 4 and 6. What about those in LOSE? I want to know all the results.

Riku: The numbers in WIN are 2 [...] whole numbers.

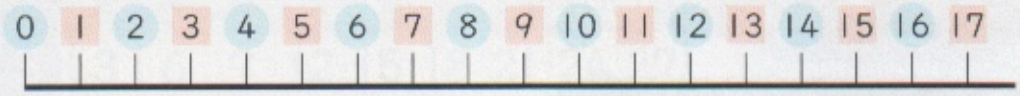
Misaki: I want to find out if we can think the same way with other numbers.

WIN	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ···
LOSE	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, ···

2 Divide the numbers in WIN by 2.
Divide the numbers in LOSE by 2.

- Whole numbers that are divisible by 2 are called **even numbers.**
- Whole numbers that are not divisible by 2 are called **odd numbers.**
- 0 is considered an even number.

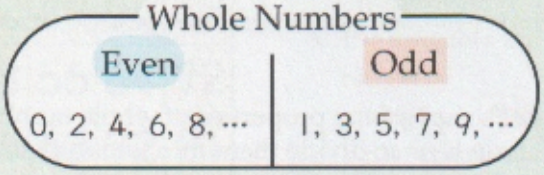
3 How are even and odd numbers arranged on the number line?



4 Is there a whole number that is neither even nor odd?

Summary

Whole numbers can be divided into two categories **if you focus on whether each whole number is even or odd.**



1 Categorize the whole numbers from 0 to 40 as even or odd.

2 What types of numbers are the page numbers on the right and left sides of the math textbook?

3 Select even numbers from the following: 42, 55, 63, and 78.



What digit in which place should we look at to decide whether or not a number is divisible by 2?



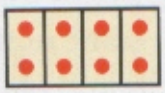
Shiho: I wonder what other properties even and odd numbers have.

2 Investigate even and odd numbers in more detail.

1 Is 8 an even number or an odd number?

2 What is the whole number that fits in □?

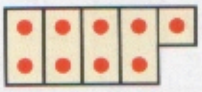
$8 = 2 \times \square$



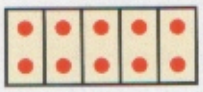
3 Is 9 an even number or an odd number?

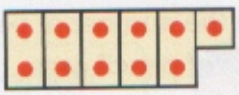
4 What is the whole number that fits in □?

$9 = 2 \times \square + 1$



5 Is 10 an even number or an odd number? What about 11?
Express 10 as a math sentence like the one in 2. Express 11 as a math sentence like the one in 4.





Let's think about what you can tell from the math sentences above that express even and odd numbers.

You get even numbers when you multiply 2 by a whole number.



Kota

You get odd numbers when you [...] to the products of 2 and whole numbers.



Misaki

Even numbers can be expressed as $2 \times \square$ and odd numbers as $2 \times \square + 1$, where numbers in □ are whole numbers.
You can say that even numbers are what you can get by multiplying 2 by whole numbers.



Ami

4 Is 50 an even number or an odd number? What about 51?
Complete the math sentences to answer the questions.

① $50 = 2 \times$ _____

② $51 = 2 \times$ _____



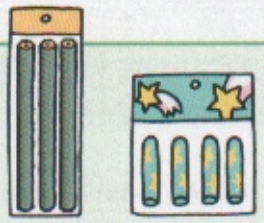
Riku

There are many numbers that can be expressed as $3 \times \square$ or $4 \times \square$, but ...

2 Multiples and Common Multiples

1

A store sells packs of 3 pencils and packs of 4 pencil caps. You are going to buy some packs of pencils and some packs of pencil caps to get the same number of pencils and pencil caps.
To have the same number of pencils and pencil caps, how many pencils and pencil caps do you need to get?



1 Find the number of pencils you will have when you buy 1, 2... packs of pencils.

Number of packs	1	2	3	4	5	6	7	8	9	10	11
Number of pencils											

2 Find the number of pencil caps you will have when you buy 1, 2... packs of pencil caps.

Number of packs	1	2	3	4	5	6	7	8	9	10	11
Number of pencil caps											

3 What type of number is the number of pencils?
What about the number of pencil caps?


Shiho

The number of Pencils is 3 [...] by a whole number.

We call the numbers we get when 3 is multiplied by a whole number **multiples** of 3.
There are infinite multiples of 3, such as 3, 6, 9, and 12 .....


The number 0 is not considered a multiple.

4 Multiples of what number are the number of pencils? What about the number of pencil caps?


Haruto

As you can see from the table, the number of pencils is the same as the number of pencil caps when you...

Let's think about what type the numbers are when you get the same number of pencils and pencil caps.

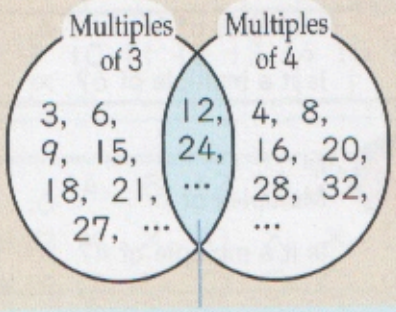
5 To have the same number of pencils and pencil caps for the first time, how many pencils and pencil caps do you get?
To have the same number of pencils and pencil caps for the second time, how many pencils and pencil caps do you get?

6 What type of numbers are the answers for **5**?


Ami

They are multiples of 3 and [...] of 4.

Numbers that are multiples of both 3 and 4 are called **common multiples** of 3 and 4. The smallest common multiple is called the **least common multiple.**



Common Multiples of 3 and 4

When you get the same numbers of pencils and pencil caps, the numbers are **multiples of 3 and multiples of 4 at the same time.**
So, the numbers are common multiples of 3 and 4.


Riku

7 What is the least common multiple of 3 and 4?

8 What is the remainder when you divide multiples of 7 by 7?


Multiples of 7 can be divided by 7 without a remainder.

1 On the number lines below, put a ○ around the multiples of 2, multiples of 3, and multiples of 4.
Also, find common multiples of 2 and 3 among the numbers from 1 to 20.




Misaki

I wonder if there is any way of finding common multiples without using a number line.

2 Find the 5 smallest common multiples of 4 and 6.

Let's think about how to find common multiples without using number lines.



Misaki

Multiples of 4: 4, 8, 12, 16, 20, 24, …

Multiples of 6: 6, 12, 18, 24, 30, 36, …



Haruto

Multiples of 4	4,	8,	12,	16,	20,	24,	…
Is it a multiple of 6?	×	×	○	×	×	○	…



Ami

Multiples of 6	6,	12,	18,	24,	30,	36,	…
Is it a multiple of 4?	×	○	×	○	×	○	…

Haruto lists multiples of 4 first. What about Ami?



1 What is the least common multiple of 4 and 6?

2 What do you notice if you compare the least common multiple of 4 and 6 to other common multiples of 4 and 6?

The common multiples are [...] of 12, the least common multiple.



Shiho

Summary

To find common multiples of 4 and 6, find the multiples of 12, which is the least common multiple of 4 and 6.

You can use what you noticed in 2.

2 Find the 5 smallest common multiples of the numbers in each ( ).

① (6, 9) ② (5, 10) ③ (3, 7) ④ (8, 12)

Additional Problems → Page 137 AA

3 We are going to stack boxes that are 5 cm in height and boxes that are 7 cm in height separately.
At how many cm do these two stacks become level for the first time?



Additional Problems → Page 138 AB



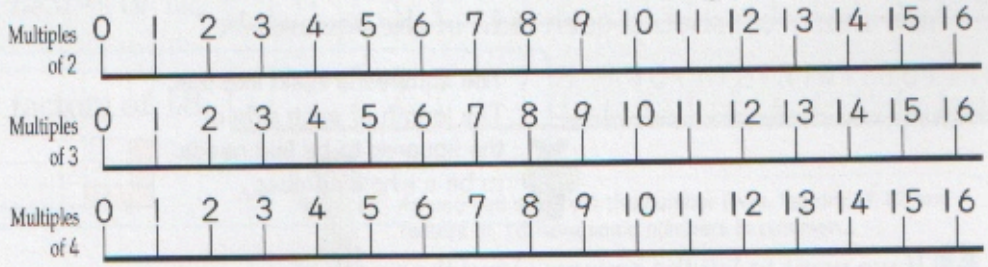
Kota

I wonder if we can find common multiples of three numbers.

3 Find the 3 smallest common multiples of 2, 3, and 4.

Let's think about how to find common multiples of three numbers.

1 Circle the multiples of 2, 3, and 4.



2 Explain how common multiples of 2, 3, and 4 are found on the right.

Multiples of 4	4,	8,	12,	16,	20,	24, …
Is it a multiple of 3?	×	×	○	×	×	○
Is it a multiple of 2?	○	○	○	○	○	○

3 What is the least common multiple of 2, 3, and 4?

4 Find the 5 smallest common multiples of 2, 3, and 4.

You can find common multiples of 3 numbers **in a similar way to the way you find common multiples of 2 numbers.**


Shiho

4 Find the 3 smallest common multiples of the numbers in each ( ).

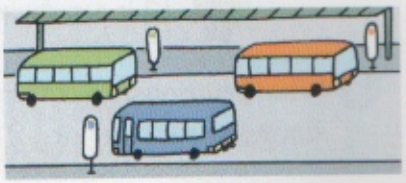
① (2, 3, 5) ② (2, 7, 8) ③ (3, 10, 15)

Additional Problems → Page 138 AC

5 Buses Ⓐ, Ⓑ, and Ⓒ leave from the station following the schedules as shown on the right.

Ⓐ Bound for the hospital leaving every 5 min.
Ⓑ Bound for the city hall leaving every 12 min.
Ⓒ Bound for the zoo leaving every 18 min.

Buses Ⓐ, Ⓑ, and Ⓒ left the station at the same time at 9:10 a.m.
When will buses Ⓐ, Ⓑ, and Ⓒ leave the station at the same time next?



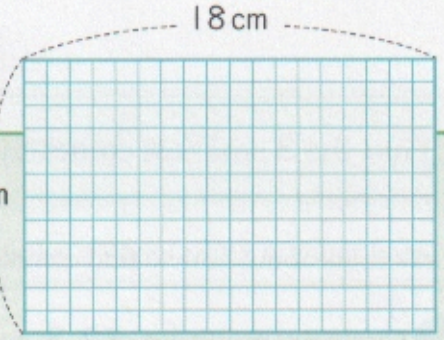
You will learn about programming on page 126.


Riku

It looks like we can find common multiples of 4 numbers in a similar way.

3 Factors and Common Factors

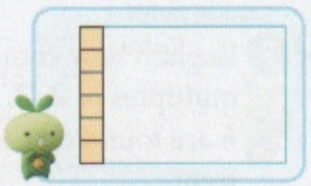
1 We are going to lay congruent paper squares to fill a rectangle that is 12 cm wide and 18 cm long.
If we want to cover the rectangles without any gaps or overlap, how many cm should each side of the squares be?



The squares are laid like this.
The length of each side of the squares to be laid needs to be a whole number.



1 If we want to lay the squares along the width of the rectangle without any gaps or overlap, how many cm should each side of the squares be?
In that case, how many paper squares are used?



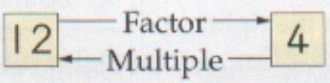
Each side of square (cm)	1	2	3	4	5	6	7	8	9	10	11	12
No gaps or overlap···○ With gaps or overlap···×	○				×							
Number of squares (pieces)	12				—							

—: Not applicable because there are gaps or overlap.

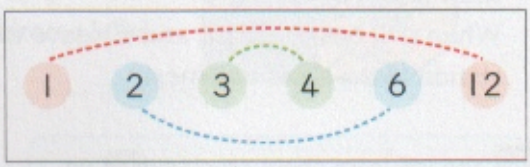
2 When squares can be laid along the width of the rectangle without any gaps or overlap, what type of number is the number that expresses the length of each side of the squares?

12 can be divided by 1, 2, 3, 4, 6 and 12. 1, 2, 3, 4, 6 and 12 are called **factors** of 12.

4 is a factor of 12.
12 is a multiple of 4.



3 What relationship is there among the factors of 12?

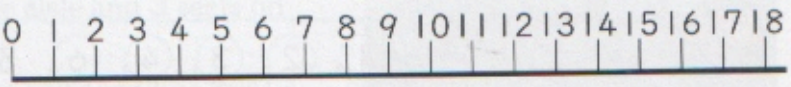


4 When squares can be laid along the length of the rectangle without any gaps or overlap, what type of number is the number that expresses the length of each side of the squares?

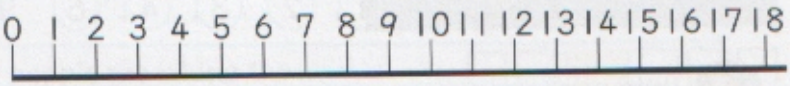


5 Circle the factors of 12 and the factors of 18.

factors of 12



factors of 18





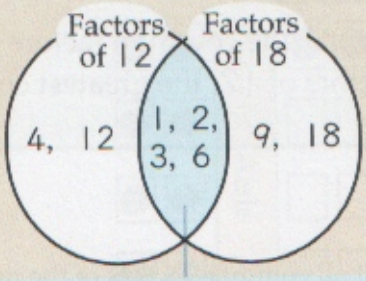
Ami: As you can see from the number lines, factors of 12 and factors of 18 have some numbers in common...

Let's think about what type of numbers are the numbers that express the lengths of each side of the squares that can cover the rectangle.



Kota: The numbers are factors of 12 and [...] of 18.

Factors such as 1, 2, 3, and 6 that are factors of both 12 and 18 are called **common factors** of 12 and 18.
The largest common factor is called the **greatest common factor.**



The numbers that express the lengths of each side of the squares that can cover the rectangle are **factors of 12 and factors of 18 at the same time**, so they are common factors of 12 and 18.



Misaki

6 What is the greatest common factor of 12 and 18?

1 List all common factors of 6 and 9.

2 If the calculation on the right gives a quotient that is a whole number and leaves no remainder, what type of whole number is the one that is in □?

$40 \div \square$



Riku: I wonder if there is any way of finding common factors without using number lines.

2 Find all common factors of 24 and 36.

Let's think about how to find common factors without using number lines.



Shiho

Factors of 24: (1), (2), (3), (4), (6), 8, (12), 24

Factors of 36: (1), (2), (3), (4), (6), 9, (12), 18, 36



Haruto

Factors of 24	1,	2,	3,	4,	6,	8,	12,	24
Is it a factor of 36?	○	○	○	○	○	×	○	×

1 What is the greatest common factor of 24 and 36?

2 What do you notice when you compare the greatest common factor and other common factors of 24 and 36?

The common factors are [...] of 12, the greatest common factor.



Ami

Summary

To find the common factors of 24 and 36, you need to find the factors of 12, the greatest common factor of 24 and 36.

You can use what you noticed in 2.

3 List all common factors of the numbers in each ( ).
Also, find the greatest common factor.

① (12, 20) ② (28, 42) ③ (18, 36)

Additional Problems → Page 138 AD

4 What is the greatest common factor of 6, 9, and 12?

First, find the factors of 6, the smallest number.



Riku

5 Find the greatest common factor of the numbers in each ( ).

① (8, 16, 20) ② (15, 18, 30) ③ (12, 36, 60)

Additional Problems → Page 138 AE



Misaki

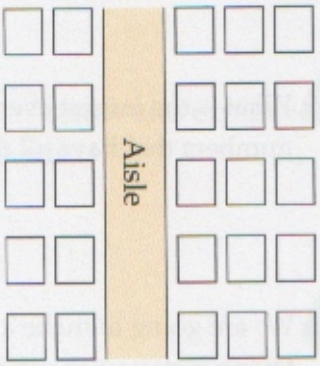
We can find common factors of 3 numbers in the same way, can't we?

Use What You Have Learned

In the Tokaido Shinkansen bullet train, seats are arranged as shown in the picture on the right. There are 2 seats on one side of the aisle and 3 seats on the other side.
Why are the seats arranged like this?



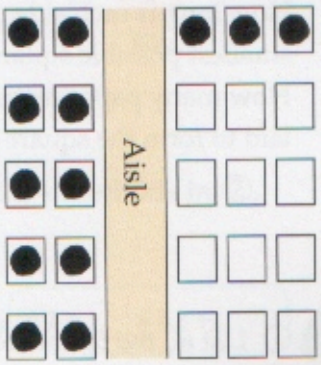
① A group of 13 people are taking the bullet train.
How can they be seated without any empty seat next to any of the 13 people?
Show a seat allocation by writing ● in the diagram on the right.





If people are seated like ●●□, there will be an empty seat next to someone.

② Akira allocated seats to the 13 people as shown on the right.
Express his idea as a math sentence.



$13 = 2 \times \square + 3 \times \square$

③ Express your seat allocation idea in ① as a math sentence as you did Akira's in ②.

$13 = \underbrace{2 \times \square}_{\text{Multiples of 2}} + \underbrace{3 \times \square}_{\text{Multiples of 3}}$



Kota

13 is expressed as the sum of a multiple of 2 and a multiple of 3.

④ For a group size between 2 and 25, is there a certain sized group that must have an empty seat?



Is there any number that is not a multiple of 2, a multiple of 3, or the sum of a multiple of 2 and a multiple of 3?

Check Your Understanding

1 Answer the following questions.

① Is 14 an even number or an odd number?

② List the 3 smallest multiples of 4 and the 3 smallest multiples of 10 in order.

③ List the 3 smallest common multiples of 4 and 10.

Do you understand even and odd numbers? Do you understand multiples?

① Page 97 1
② Page 100 1
③ Page 102 2

2 What is the largest even number among the 3-digit whole numbers that have all the following numerals: 1, 2, and 3?

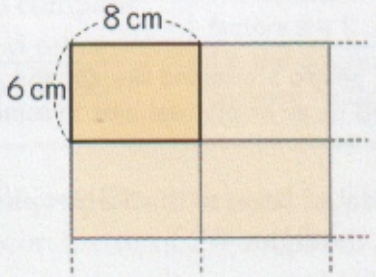
Can you tell the properties of even numbers?

Page 99 2

3 We are going to make a square by laying rectangular papers that are 6 cm long and 8 cm wide without any gaps or overlap.

How long is each side of the smallest possible square?

How many paper rectangles are laid to form the square?



Can you solve problems using common multiples?

Page 102 2

4 ① List all the factors of 32 and all the factors of 48.

② List all the common factors of 32 and 48.

Do you understand factors?

① Page 104 1
② Page 106 2

5 There are 12 pieces of square paper with 1 cm sides. If we want to make a rectangle using all of the squares with no holes or overlap, what should the length and the width of the rectangle be?

Is there only one answer?

Can you solve problems using factors?

Page 104 1

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Think about properties of whole numbers based on multiplication and division

① If the calculation on the right gives a quotient that is a whole number and leaves no remainder, what type of whole number is the one that is in □?

$36 \div \square$

② 60 is an even number.
Think about the appropriate number that fits in □ below and explain the reason why based on Shiho's and Riku's methods.



Shiho

60 can be divided by □ without a remainder.
That's why 60 is an even number.

$60 = \square \times 30$
60 can be expressed as a math sentence "2 × a whole number," so 60 is an even number.



Riku

③ Is 87 an even number or an odd number?
Explain the reason why based on Shiho's and Riku's methods in ②.



Haruto

If you divide 87 by □...



Misaki

$87 = \square \times 43 + \ldots$

Look back on what you have learned in "Let's Investigate the Properties of Whole Numbers" and discuss.



Ami

Now I know that whole numbers can be divided into two categories: even and odd numbers, because every whole number is either even or odd.



Kota

We have found many properties of whole numbers. I want to find other properties.

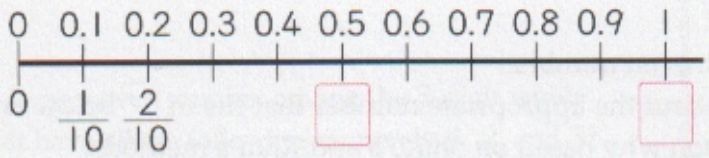
Challenge Yourself
→Page 146

What are the relationships between fractions and decimal / whole numbers?

Shiho and Kota are reviewing what they learned about the relationships between fractions and decimal / whole numbers.



Shiho



When we review the relationship between decimal numbers and fractions, 0.1 is $\frac{1}{10}$, 0.2 is $\frac{2}{10}$, and...



Kota

If we share 1 L of juice among 2, 4, or 5 people equally, each person will get...

How to Express Fractions
Page 150 ⑥

If you think using fractions	$\frac{1}{2}$ L	$\frac{1}{4}$ L	$\frac{1}{5}$ L
If you think using division	Since $1 \div 2 = 0.5$, each person will get 0.5 L.	Since $1 \div 4 = 0.25$, each person will get 0.25 L.	Since $1 \div 5 = 0.2$, each person will get 0.2 L.

You can express the answers both in fractions and in decimal numbers.

Discuss what you notice about the review by Shiho and Kota.



Ami

Sometimes the same size can be expressed both as a decimal number or a fraction.



Haruto

$\frac{1}{2}$ is equal to 0.5, so I wonder if you can also write $1 \div 2 = \frac{1}{2}$.



Misaki

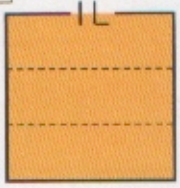
It looks like you can express fractions in decimal numbers and the other way around.

9 Let's Investigate Relationships between Fractions and Decimal/Whole Numbers.

If we share ☐ L of juice among 3 people equally, how much will each person get?

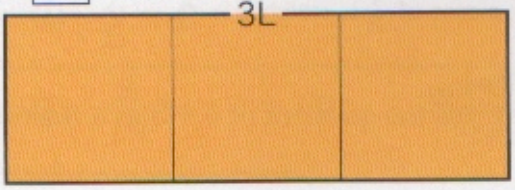


If 1 L is shared,



1 L is equally divided into 3, and each person will get ☐ L.

If 3 L is shared,



3 ÷ 3 = 1, so each person will get ☐ L.

If 2 L is shared...

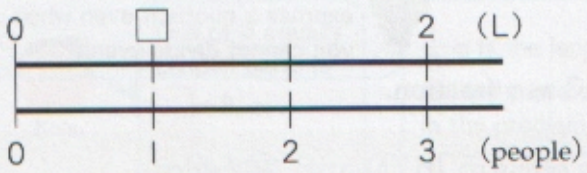


Misaki

1 Division and Fractions

1 If we share 2 L of juice among 3 people equally, how much will each person get?





Math Sentence



Riku

Each person's share will be more than when 1 L is shared but... than when 3 L is shared...

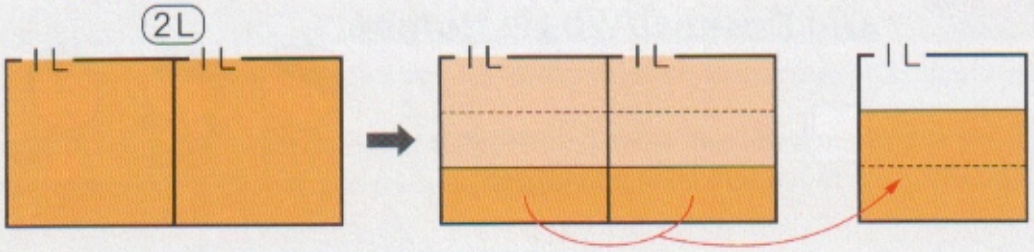


Kota

2 ÷ 3 = 0.666...
You cannot divide evenly.

Let's think about how to express quotients.

1 Look at the diagram and think about how to express the quotient of $2 \div 3$ as a fraction.



Consider 2 L as 2 pieces of ☐ L.
One of 3 equally divided parts of 2 L is ☐ pieces of $\frac{1}{3}$ L, which is ☐ L.

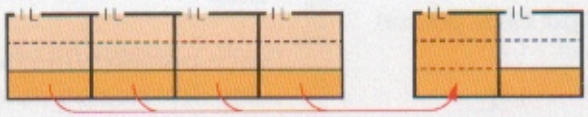
Misaki

$2 \div 3 = \frac{\square}{\square}$ (L)

2 Express the quotient of $4 \div 3$ as a fraction.

$4 \div 3 = \frac{\square}{\square}$

Since one of 3 equally divided parts of 4 L is ☐ pieces of $\frac{1}{3}$ L, ...



Haruto

3 Look at the math sentences and answers in **1** and **2** and discuss what you notice.

In both cases, the divisor and the dividend become ...

Riku

Summary

A quotient can be expressed as a fraction.
The dividend becomes the numerator, and the divisor becomes the denominator.

■ ÷ ● = $\frac{■}{●}$

If you use a fraction, you can express a quotient even when you cannot divide evenly.

4 Express the quotient of $1 \div 3$ as a fraction.

$\frac{2}{3}$ can be considered as shown in Ⓐ and Ⓑ below.

Ⓐ $\frac{2}{3}$ consists of 2 pieces of $\frac{1}{3}$.

Ⓑ $\frac{2}{3}$ is the quotient of $2 \div 3$.

Kota

You can express quotients using fractions.

Ami

After examining a few examples, now we now know that quotients can be expressed as fractions.

1 Express the quotients of $5 \div 4$ and $4 \div 5$ as fractions.

2 Express the following quotients as fractions.

① $6 \div 7$ ② $5 \div 12$ ③ $11 \div 17$ ④ $9 \div 2$

3 Write the appropriate number in each □.

① $\frac{5}{9} = 5 \div \square$ ② $\frac{1}{4} = \square \div 4$

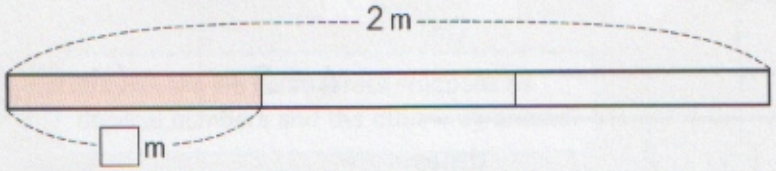
③ $\frac{7}{2} = \square \div 2$ ④ $\frac{2}{5} = 2 \div \square$

⑤ $\frac{13}{6} = \square \div 6$ ⑥ $\frac{8}{19} = \square \div 19$

Additional Problems
→Page 139 AF

4 Shiho looked at the problem below and answered as follows.

We are equally dividing a 2 m long piece of tape into 3 parts. How long is each of them?




Shiho

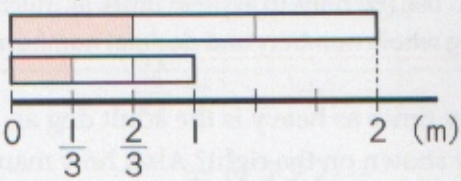
Since it's one of 3 equally divided parts, it's $\frac{1}{3}$ m.

Is Shiho's reasoning right or wrong?
Explain your answer using a diagram and a math sentence.


Kota

One of 3 equally divided parts is □ ÷ 3, or □ m, so...

$\frac{1}{3}$ m is the length of one of 3 equally divided parts of 1 m.
In the problem above, 2 m is...




Ami


Riku

Now we know how to use a fraction to express the quotient of any division of a whole number by a whole number.

Fractions and times as much

2 There are pieces of ribbon whose lengths are shown in the table on the right.
How many times as long is the white ribbon as the red ribbon?

Lengths of ribbon

	Length(m)
Red	5
White	4
Blue	3

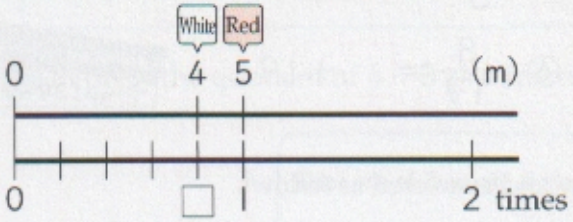
We can use decimal numbers to express times as much. I wonder if we can do that using fractions, too.



Shiho

Times as Much
Page 151 ⑩

Let's think about how to express times as much using fractions.



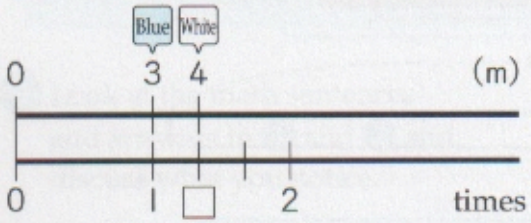
$4 \div 5 =$ ☐ (times)



Haruto

If you express it using a decimal number, it's 0.8 times.

1 How many times as long is the white ribbon as the blue ribbon?



$4 \div 3 =$ ☐ (times)

We can use fractions to express times as much, such as $\frac{4}{5}$ times as much or $\frac{4}{3}$ times as much.
$\frac{4}{5}$ times as much means if we consider 5 m as 1, 4 m will correspond to $\frac{4}{5}$.

We can also use fractions to express times as much, **just like we do using whole numbers and decimal numbers.**



Misaki

5 How many times as heavy is the adult dog as the puppy shown on the right? Also, how many times as heavy is the puppy as the adult dog?



Ami



We can use fractions to express times as much, too.

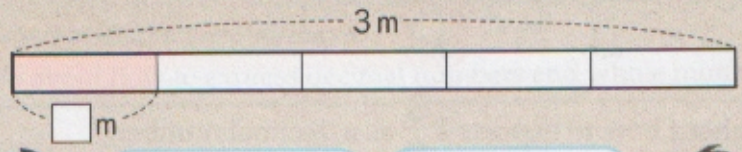
Kota



We are expanding the world of times as much from whole numbers and decimal numbers to fractions.

2 Let's Investigate Relationships between Fractions, Decimals, and Whole Numbers

We are thinking about how long each person's share will be if a 3 m long piece of tape is equally divided into 5 parts.




Riku

It's 3 ÷ 5, so I'll use a fraction...

It's 3 ÷ 5, so I can use a decimal number...


Misaki

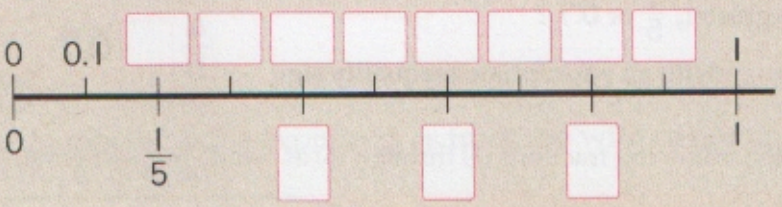
$3 \div 5 = \frac{\square}{\square}$ (m)　　$3 \div 5 = \square$ (m)


Haruto

The size must be the same regardless of whether it is expressed using a fraction or a decimal number.

Let's see if it is with number lines.






Ami

It looks like we can express fractions as decimal numbers and the other way around.

1 Express $\frac{3}{4}$ and $\frac{2}{9}$ as decimal numbers.

Let's think about how to express fractions as decimal numbers.

Just like $\frac{3}{5} = 3 \div 5$ and $3 \div 5 = 0.6$...


Shiho

$\frac{3}{4} = \square \div \square$

$= \square$

$\frac{2}{9} = \square \div \square$

$= 0.222\cdots$

Summary

To express a fraction as a decimal number, you divide the numerator by the denominator.

You can consider fractions as quotients.

1 Think about how to express $2\frac{3}{4}$ as a decimal number.



Ami

$2\frac{3}{4} = 2+\frac{3}{4}$

Since $\frac{3}{4} = 0.75$

$2\frac{3}{4} = 2.75$



Kota

$2\frac{3}{4} = \frac{11}{4}$

$= 11 \div 4$

$= 2.75$

1 Which is greater, $\frac{4}{5}$ or 0.7 ?

Fill in the ☐ with an appropriate inequality sign. $\frac{4}{5}$ ☐ 0.7

Inequality Signs
Page 150 ⑤

2 Express the following fractions ① through ⑥ as whole numbers or decimal numbers.

① $\frac{1}{4}$ ② $\frac{12}{5}$ ③ $\frac{18}{6}$ ④ $\frac{56}{8}$ ⑤ $3\frac{2}{5}$ ⑥ $1\frac{1}{8}$

Additional Problems
→Page 139 AG



Haruto

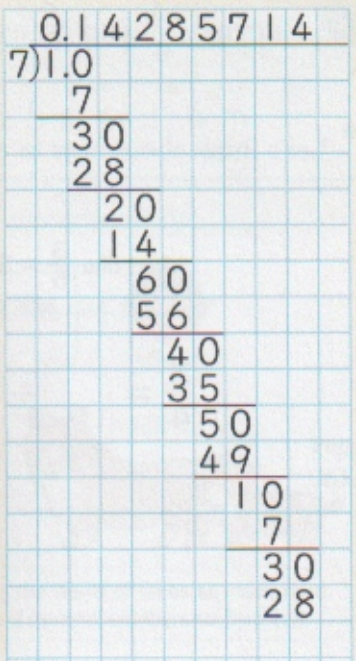
I wonder if we can go the opposite way and express decimal numbers as fractions.



If you try to express $\frac{1}{7}$ as a decimal number...

In order to express $\frac{1}{7}$ as a decimal number, we are calculating $1 \div 7$.

If we keep calculating it, what numerals will appear repeatedly in the quotient?



2 Express 0.3, 0.29, 1.57, 4, and 12 as fractions.

Let's think about how to express decimal numbers and whole numbers as fractions.

1 Express 0.3, 0.29, and 1.57 as fractions.



Misaki: 0.3 means that there are 3 pieces of 0.1.



Riku: 0.29 means that there are [...] pieces of 0.01.

$0.1 = \frac{1}{\square}$ So, $0.3 = \frac{3}{\square}$

$0.01 = \frac{1}{\square}$ So, $0.29 = \frac{29}{\square}$

$0.01 = \frac{1}{\square}$ So, $1.57 = \frac{157}{\square}$

Summary

Decimal numbers can be expressed as fractions with denominators of 10, 100, and so on.

You can find how many pieces of $\frac{1}{10}$ or $\frac{1}{100}$ there are.

2 Express 4 and 12 as fractions.

$4 = 4 \div 1$

$= \frac{\square}{\square}$

$12 = 12 \div 1$

$= \frac{\square}{\square}$

Summary

Whole numbers can be expressed as fractions with denominators of 1 and other ways.

3 Express the following decimal numbers and whole numbers ① through ⑥ as fractions.

① 0.2 ② 0.49 ③ 3 ④ 3.14 ⑤ 5 ⑥ 7.06

Additional Problems → Page 139 AH



Shiho: We were able to express different decimal numbers and whole numbers as fractions.

Check Your Understanding

1 Write the appropriate number in each □.

① $\frac{5}{6} = \square \div 6$ ② $\frac{9}{4} = 9 \div \square$

③ $7 \div 5 = \frac{\square}{\square}$ ④ $11 \div 14 = \frac{\square}{\square}$

Do you understand the relationship between division and fractions?

Page 111 1

2 Answer the following with fractions.

① How many times as long is 20 m as 15 m?
② How many times as heavy is 9 kg as 20 kg?
③ If we consider 3 cm as 1, what does 2 cm correspond to?
④ If we consider 2 cm as 1, what does 3 cm correspond to?

Do you understand times as much with fractions?

Page 114 2

3 Express the following fractions ① through ⑥ as whole numbers or decimal numbers.

① $\frac{3}{8}$ ② $\frac{16}{5}$ ③ $\frac{7}{4}$

④ $\frac{5}{2}$ ⑤ $\frac{8}{2}$ ⑥ $\frac{21}{7}$

Do you understand how to express fractions as whole numbers or decimal numbers?

Page 115 1

4 Express the following decimal numbers and whole numbers ① through ⑥ as fractions.

① 0.5 ② 0.03 ③ 1.6

④ 0.78 ⑤ 7 ⑥ 4.08

Do you understand how to express decimal numbers and whole numbers as fractions?

Page 117 2

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

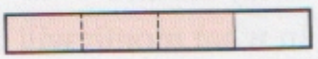
Focus on how to express fractions and summarize what fractions show

Take $\frac{3}{4}$ as an example, and review what fractions show.

Misaki

Fractions show some equally divided parts of a certain amount.
The colored part consists of 3 pieces of

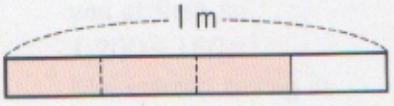
$\frac{1}{4}$ of Ⓐ, so it is $\frac{\square}{\square}$ of Ⓐ in length.

Ⓐ 

Haruto

Fractions show length and other types of quantity.

$\frac{3}{4}$ of 1 m is $\frac{\square}{\square}$ m.



It consists of $\square$ pieces of $\frac{1}{\square}$ m.

Ami

Fractions show times as much.

The blue ribbon is $\frac{\square}{\square}$ times as long as the white ribbon.

If you consider the length of the white ribbon as 1, the length of the blue ribbon corresponds to $\frac{\square}{\square}$.

Lengths of Ribbons

	Length (m)
White	4
Blue	3

Riku

Fractions show quotients. $3 \div 4 = \frac{\square}{\square}$

Review what you have learned in "Let's Investigate Relationships between Fractions and Decimal / Whole Numbers" and discuss.

Shiho

Now we know how to express the same number both as a decimal number and as a fraction by considering fractions as a quotient.

Kota

Now we know that fractions can be used like whole numbers and decimal numbers to express times as much.

Challenge Yourself
→Page 146

Focusing on differences and sums

ojo

1

Last year, Tsuyoshi saved 1,200 yen. Since January of this year, he has been saving 200 yen every month.
Mai had no savings last year. Since January of this year, she has been saving 350 yen every month.
In what month will their savings become equal?




Ami

First, let's find out how their savings will increase within the next several months.

1 Find out how their savings will increase from January to April and discuss what you notice.

	Last year	January	February	March	April
Tsuyoshi (yen)	1,200	1,400			
Mai (yen)	0	350			

I can tell when their savings will become equal if we extend this table...


Kota


Shiho

The difference was as big as 1,200 yen last year, but in April, it reduces to ☐ yen.

2 Think of a simple way to find out in what month their savings will become equal.


Riku

If we focus on the difference like Shiho did...

To make the difference clear, what data should you write down in the extra row of the table?



	Last year	January	February	March	April
Tsuyoshi (yen)	1,200	1,400			
Mai (yen)	0	350			

Kota

	Last year	January	February	March	April				
Tsuyoshi(yen)	1,200	1,400	1,600	1,800	2,000				
Mai (yen)	0	350	700	1,050	1,400				
Difference(yen)	1,200	1,050	900	750	600	450	300	150	0

Decreasing by 150 yen

I'll find out in what month the difference, which starts from 1,200 yen and decreases by 150 yen every month, becomes 0 yen.

Answer

Shiho

	Last year	January	February	March
Tsuyoshi(yen)	1,200	1,400	1,600	1,800
Mai (yen)	0	350	700	1,050
Difference(yen)	1,200	1,050	900	750

Reducing by 150 yen

The difference was 1,200 yen at first, so
1,200 ÷ 150=
Mai started saving in January, so...

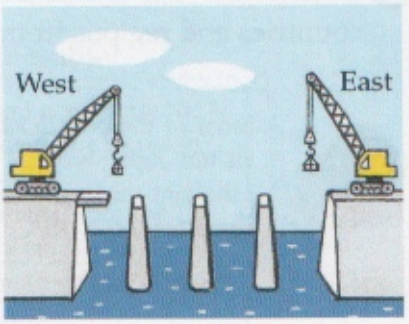
Answer

3 Explain Kota's and Shiho's methods.

2

A contractor is building a 285 m long bridge from both east and west ends. The contractor built 30 m of the west part of the bridge through yesterday, and will continue to build 6 m more each day. The contractor starts building the east part today, and will build 9 m every day.
How many days does it take for the east and west parts to meet?

	Through yesterday	Day 1 (today)	Day 2	Day 3	Day 4
West part(m)					
East part(m)					




Misaki

If you focus on the sum of the lengths reached at the end of each day...

International Visitors to Japan

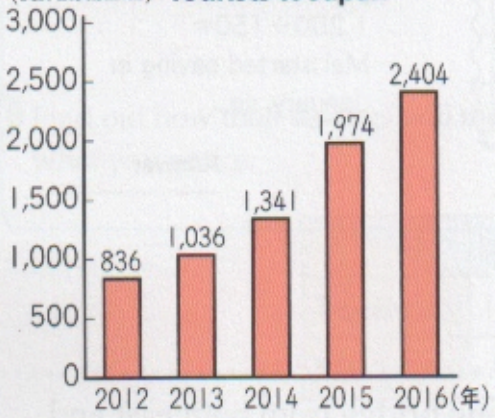


Aiming for an increase in tourism, Japan has set a goal of having 60 million international tourists in 2030.

1

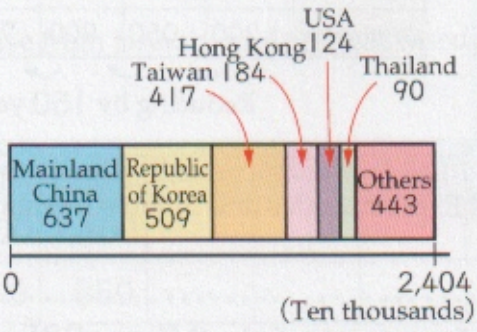
Searching for how many people visited Japan and from which countries and regions they are from, Misaki and her classmates found data 1 and 2 below.

Data 1 **Number of International Tourists to Japan**



(Developed by Tokyo Shoseki based on the data released by the Japan National Tourism Organization (JNTO))

Data 2 **Breakdown of Countries and Regions from which Tourists Came** (in 2016)



(Developed by Tokyo Shoseki based on the data released by the JNTO)

1. Data 2 shows that the largest group of tourists came from mainland China in 2016. About what fraction of the total number of tourists is the number of tourists from mainland China?

2. Look at data 2 and discuss what you notice about the breakdown of the countries and regions from which tourists visited Japan in 2016.



Misaki

Most of the people who visited Japan were... I thought that more tourists came from the USA...



Riku

To increase the number of international tourists to Japan, we need to [...] from more parts of...

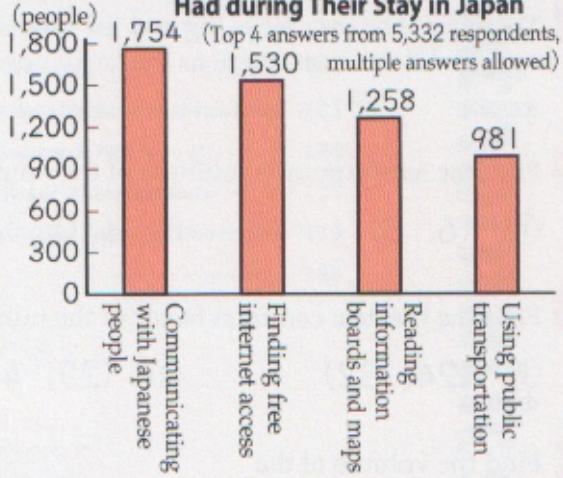
2

Kota found the results of a survey of difficulties tourists had during their stay in Japan.

1 Based on data 3, what do you think Japan needs to do for international tourists to stay comfortably in the country?
Share your suggestions with your classmates.

Data 3 **Difficulties International Tourists Had during Their Stay in Japan**

(Top 4 answers from 5,332 respondents, multiple answers allowed)



(Developed by Tokyo Shoseki based on the results of the "Questionnaire on Creating a More Welcoming Environment for International Visitors to Japan" (conducted by the Japan Tourism Agency in 2017))

Advanced 8th Grade

3

Shiho and Haruto looked at data 1 on the previous page and thought about whether Japan would be able to achieve the target of 60 million international tourists to Japan in 2030.

1 According to Shiho's idea, is the target achievable?

In 2016, the number of tourists increased from 2015 by about 4.3 million.
If you assume that the number of tourists will increase by 4.3 million every year from 2016 on, in 2030...



Shiho

2 According to Haruto's idea, is the target achievable?

After 2012, the number of tourists increased constantly for 4 years. Even if it keeps rising for years to come, the increase will not necessarily remain this big.
Between 2012 and 2016, the smallest increase from the previous year was about 2 million in 2013 from 2012.
So, if you assume an annual increase of 2 million from 2016 on, in 2030...



Haruto

Do You Remember?

Answers → Page 147

1 Calculate. For division problems, divide completely.

① 27×1.9 ② 0.8×1.6 ③ 0.7×0.9 ④ 2.4×0.5

⑤ $78.4 \div 3.5$ ⑥ $4.32 \div 7.2$ ⑦ $0.4 \div 0.5$ ⑧ $5.39 \div 2.2$

2 There are 2 boxes: one is big and the other is small. The big box weighs 40 kg, and it weighs 1.6 times as much as the small one. How much does the small box weigh?

3 Find the least common multiple of the numbers in each ( ).

① (6, 8) ② (10, 15) ③ (3, 4, 9)

4 Find the greatest common factor of the numbers in each ( ).

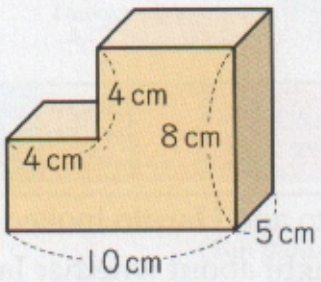
① (24, 32) ② (27, 45) ③ (18, 42, 54)

5 Find the volume of the following figures.

① A cube with 9 cm edges

② A cuboid with a width of 3.5 m, a length of 2.8 m, and a height of 4 m

③



Warm-up

6 ① $\frac{5}{9}+\frac{7}{9}$ ② $2\frac{3}{6}+\frac{2}{6}$ ③ $3\frac{2}{4}+\frac{3}{4}$ ④ $1\frac{3}{5}+2\frac{1}{5}$

⑤ $\frac{9}{7}-\frac{6}{7}$ ⑥ $5\frac{3}{4}-2\frac{1}{4}$ ⑦ $2\frac{4}{5}-\frac{3}{5}$ ⑧ $3\frac{3}{8}-\frac{5}{8}$

Playing with Numbers and Calculations

Express 24 as many different math sentences

To express 24 as many different math sentences, fill each of the □, ○, △, ♡, and ◇ with a whole number that is greater than or equal to 2. The same shape holds the same whole number.

① $24 = \square + \square$ ② $24 = \bigcirc \times \triangle$

③ $24 = \heartsuit \times \heartsuit \times \heartsuit \times \diamondsuit$ ④ $24 = \heartsuit \times \diamondsuit \times \triangle$

Express other numbers as many different math sentences.



New Mathematics 5A Plus

Additional Problems

Need More Practice?
Try These!

1 Let's Summarize the Structures of Whole Numbers and Decimal Numbers ··· 128
2 Let's Think about How to Express the Amount of Space Occupied by Cubes and Cuboids ··· 128
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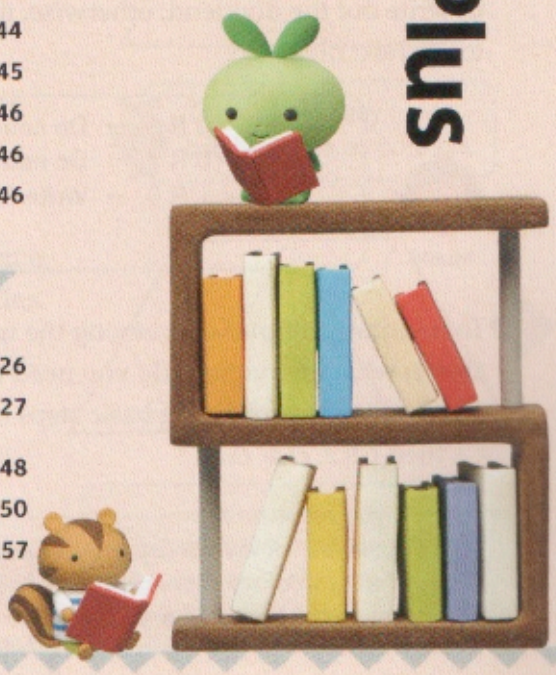
Let's Try WONDERful Problems!

These problems are for you to explore wider and dig deeper. Try them, think hard, and enjoy!

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Notice to Teachers and Parents

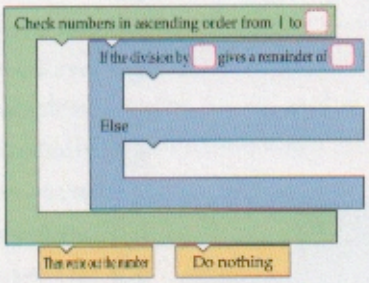
"New Mathematics 5A Plus" is an optional learning material for students who want or need to work on it. Not all students need to use "New Mathematics 5A Plus."

Let's Try Programming!

Think about the process of finding multiples

To find the multiples of 3 using a computer that is capable of doing the following, what instructions do you need to give to the computer?

(A) Checking whole numbers in ascending order from 1
(B) Dividing a given whole number by another given whole number to find a quotient and a remainder in whole numbers
(C) Writing out the whole numbers found based on the results




Riku

We cannot simply give instructions to "find the multiples of 3."



Instructions to computers are sometimes called programs. A process of making programs are sometimes called programming.

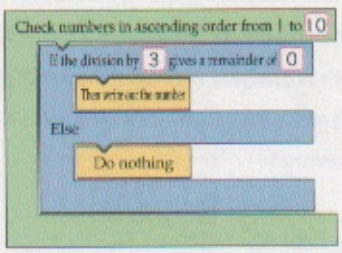
1 If you focus on the remainder of division by 3, what type of numbers can you say multiples of 3 are?

$1 \div 3 = 0$ R □
$2 \div 3 = 0$ R □
$3 \div 3 = 1$ R □
$4 \div 3 = 1$ R □
$5 \div 3 = 1$ R □
$6 \div 3 = 2$ R □
$7 \div 3 = 2$ R □
⋮

2 Do the following as if you were the computer above: divide each of the whole numbers from 1 to 10 by 3; if a dividend gives a remainder of 0, write out the dividend; otherwise, move on to the next dividend.


Misaki

$1 \div 3 = 0$ R 1 → Do nothing
$2 \div 3 = 0$ R 2 → Do nothing
$3 \div 3 = 1$ R 0 → Write out 3
⋮



3 To find the multiples of 4 among the whole numbers up to 20 using the computer above, what instructions do you need to give?
Think using the following basic steps computers take: "check in order" and "if ○ ○, then □□, else △△."



If you focus on the remainder of division by 4, what type of numbers can you say multiples of 4 are?


Kota

- Check numbers in ascending order from 1 to □.
  ↓
- If the division by □ gives a remainder of..., else...

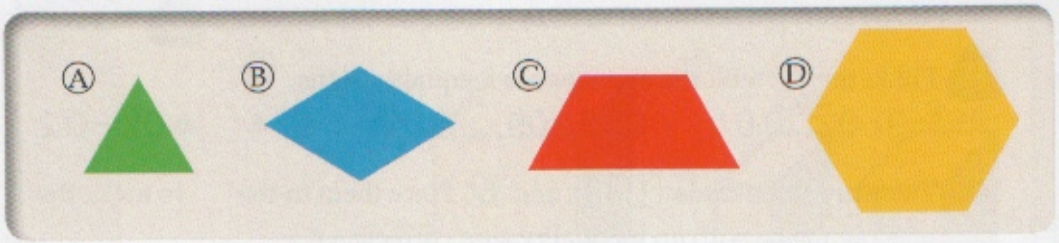
Let's Play with Shapes

Playing with blocks

〈Move〉

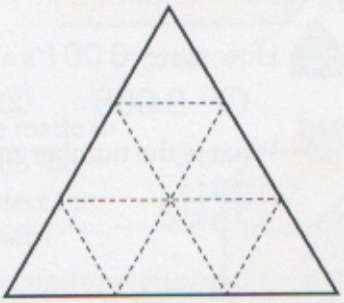
There are 6 shapes of blocks as shown on the right.
Now you are going to use the 4 of them below.





1 How many times as large is Ⓓ as each of the blocks Ⓐ, Ⓑ, and Ⓒ?

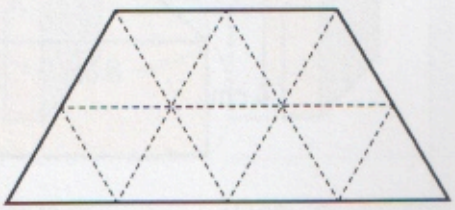
2 Can the shape on the right be formed only with block Ⓑ? Can it be formed only with block Ⓒ?



3 Combine blocks Ⓐ, Ⓑ, and Ⓒ in many ways to form the shape in 2.

Think of as many combinations as possible.

4 Use only the pieces of block Ⓒ to form a shape, as shown on the right, with sides that are 2 times as long as the sides of block Ⓒ.



How many pieces of block Ⓒ will you need?

You may try using all 6 different shapes of blocks to make a variety of shapes.

Additional Problems

Similar Problems Slightly Difficult Problems

1 Let's Summarize the Structures of Whole Numbers and Decimal Numbers

Answers → Page 140

Page 10 1 →

A Write the appropriate number in each □.

$5.069 = 1 \times \square + 0.1 \times \square + 0.01 \times \square + 0.001 \times \square$

A Write the appropriate number in each □.

$7.513 = \square \times 7 + \square \times 5 + \square \times 1 + \square \times 3$

Page 10 2 →

B Fill in the □ with the appropriate inequality signs.

① 0 □ 0.01 ② 3.205 □ 3.09 ③ 4 □ 4.02 − 0.2

B There are three cards: 1, 3, and 6. Place them in the □ to make the math sentence with an inequality sign correct.

□.5□ < 1.□

Page 11 3 →

C How many 0.001's are in the numbers ① through ④ below?

① 0.005 ② 0.037 ③ 0.899 ④ 5.2

C What is the number made of 100 pieces of 0.01 and 10 pieces of 0.001?

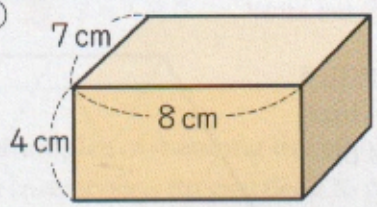
2 Let's Think about How to Express the Amount of Space Occupied by Cubes and Cuboids

Answers → Page 140

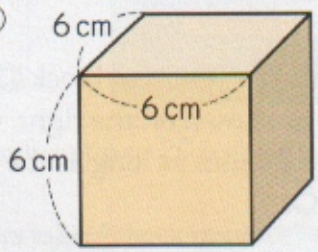
Page 20 3 →

D What are the volumes of the cube and cuboids below in cm^3?

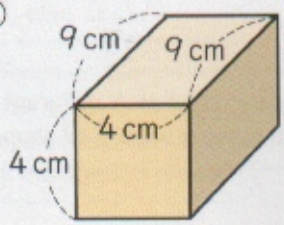
①



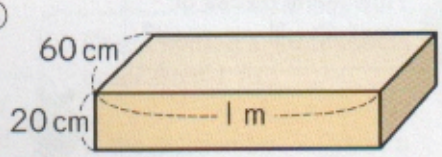
②



③



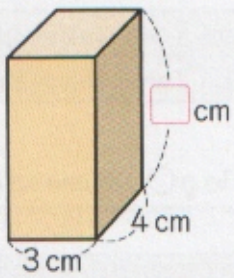
④



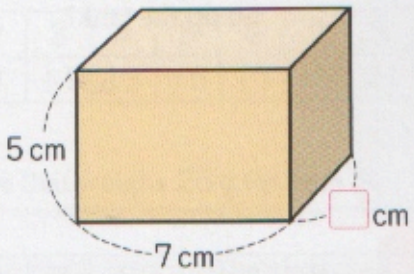
Page20
3 →

D Fill in the □ with appropriate numbers for the following cuboids with the given volumes.

① Volume: 72 cm^3



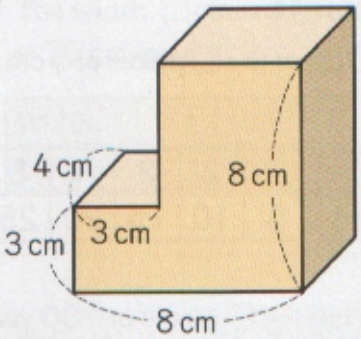
② Volume: 140 cm^3



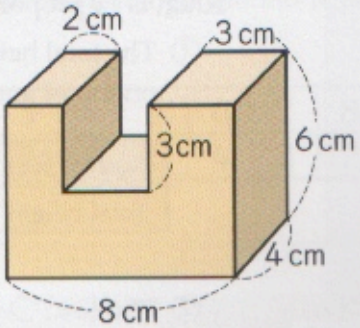
Page23
5 →

E Calculate the volume of each of the following solid figures.

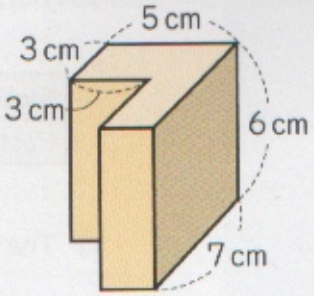
①



②



E Math sentences ① through ③ were made to find the volume of the solid figure on the right. From Ⓐ, Ⓑ, and Ⓒ below, select the diagram that matches each of the math sentences ① through ③.



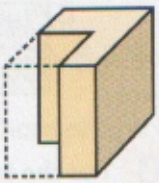
① $3\times5\times6+4\times2\times6$

② $7\times5\times6-4\times3\times6$

③ $3\times3\times6+7\times2\times6$

Ⓐ 

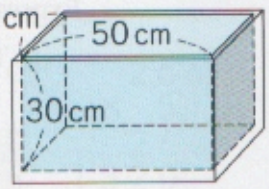
Ⓑ 

Ⓒ 

Page29
3 →

F What is the capacity of the water tank on the right in cm^3?

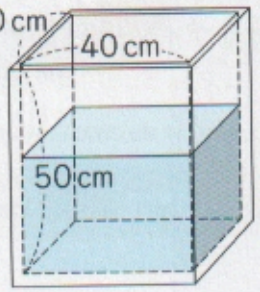
What is it in L?



Page29
3 →

F The water tank on the right contains 36 L of water.

How many more L of water do you need to fill up the tank?



3 Let's Investigate How Quantities Change (1)

Answers → Page 140

Page35
1 →

G For the following two quantities that change in relationship to each other, is ○ proportional to □?

① The total height, ○ cm of the □ pieces of 5 cm high boxes stacked on top of another

Number of boxes □	1	2	3	4	5	6	7	8
Total height ○ (cm)	5	10	15	20	25	30	35	40

② The cost, ○ yen of buying □ pieces of 100 yen cookies in a 80 yen gift box

Number of cookies □	1	2	3	4	5	6	7	8
Cost ○ (yen)	180	280	380	480	580	680	780	880

③ The width, □ cm and area, ○ cm^2 of a rectangle that is 3 cm long

Width □ (cm)	1	2	3	4	5	6	7	8
Area ○ (cm^2)	3	6	9	12	15	18	21	24

G For the following two quantities that change in relationship to each other, is ○ proportional to □?

Express the relationship between □ and ○ as a math sentence.

① The length around a square, ○ cm, with □ cm sides

Each side of square □ (cm)	1	2	3	4	5	6	7	8
Length around square ○ (cm)	4	8	12	16	20	24	28	32

② Total amount of water, ○L in the water tank where there is 5L of water already and 2L of water flows in every minute for □ minutes

Duration of water flow □ (min.)	1	2	3	4	5	6	7	8
Total amount of water ○ (L)	7	9	11	13	15	17	19	21

③ The weight, ○g of □m of wire that weighs 25g for each m

Length of wire □ (m)	1	2	3	4	5	6	7	8
Weight of wire ○ (g)	25	50	75	100	125	150	175	200

④ The width, □cm and length, ○cm of a rectangle around which is 20cm long

Width □ (cm)	1	2	3	4	5	6	7	8
Length ○ (cm)	9	8	7	6	5	4	3	2

4 Let's Expand the World of Multiplication

Answers→ Page 140

Page44 1→

H There is a ribbon that costs 130 yen for 1 m.
How much will this ribbon cost for 2.4 m?

H There is a 2 m wire that weighs 40g.
How many g will 3.9 m of this wire weigh?

Page46 2→

I Find the correct products using $543 \times 26 = 14,118$

① 54.3×2.6 ② 5.43×26 ③ 5.43×2.6

I From Ⓐ to Ⓒ below, select the math sentence whose product is the same as the answer for 27.8×4.1.

Ⓐ 278×41 Ⓑ 2.78×4.1 Ⓒ 2.78×41

Page46 3→

J Place the decimal points in the products.

① $4.3 \times 2.7 = 1161$ (301 + 86)

② $2.64 \times 5.1 = 13464$ (264 + 1320)

Page46 3 →

J Riku calculated 2.1×3.4 using the algorithm based on the following idea, but there is something wrong in his idea.

$$\begin{array}{r} 2.1 \\ \times 3.4 \\ \hline 84 \\ 63 \\ \hline 71.4 \end{array}$$



Riku

Calculation of 2.1×3.4
① Calculate 21×34 as if there were no decimal points.
② The product is 714.
③ Slide the decimal point down to make the product 71.4.

Which part is wrong in Riku's explanation, ①, ②, or ③? Also, find the correct answer.

Page46 4 →

K Estimate first, and then calculate.

① 5.85×6.3 ② 2.46×6.8 ③ 3.1×4.7
④ 60.2×9.54 ⑤ 84×5.1 ⑥ 764×1.4

K Place the four numerals 1, 2, 6, and 7 in the □ on the right to make math sentences ① and ②.

□.□ × □.□

① Math sentence that results in the smallest product
② Math sentence that results in the product that is closest to 10

Page46 5 →

L ① 6.16×7.5 ② 4.2×2.5 ③ 335×5.8
④ 0.59×1.3 ⑤ 0.32×2.9 ⑥ 0.4×1.5

L Fill in the □ with appropriate numerals and place decimal points of the products in the appropriate places to make the calculations with the algorithm correct. If a box is to have a 0 that needs to be crossed out, write a 0 and use \ to cross it out.

① $\begin{array}{r} \square.4 \\ \times \square.5 \\ \hline 1\square\square \\ \square 8 \\ \hline \square 5\square \end{array}$

② $\begin{array}{r} \square.19 \\ \times \quad \square.\square \\ \hline 1\square\square \\ \square 6 \\ \hline \square 9\square 2 \end{array}$

Page47 7 →

M ① 2.7×0.6 ② 18.5×0.3 ③ 0.5×0.7
④ 0.8×0.04 ⑤ 0.5×0.6 ⑥ 1.34×0.5

M In the following math sentences, all the △s contain the same number that is not 0.
Fill in the ☐ with appropriate inequality signs.
① $\triangle \times 1.2 \square \triangle \times 2.5$
② $\triangle \times 1.6 \square \triangle \times 0.9$
③ $\triangle \times 0.04 \square \triangle \times 0.4$

Page49 9 →

N Think of ways to make the calculation simpler, and then calculate.
① $7.2 \times 2.5 \times 4$ ② $3.9 \times 2.6 + 6.1 \times 2.6$
③ 25.7×4 ④ 9.9×6

N Fill in the ☐ with appropriate numbers from
0.25, 2.5, 25, 0.4, 4, 40 to complete math sentences ① and ②.
① ☐ × ☐ = 100
② ☐ × ☐ + ☐ × ☐ + ☐ × ☐ = 1010.1

5 Let's Expand the World of Division

Answers→ Page 140

Page56 1 →

O We bought 2.4 m of ribbon for 360 yen.
How much does 1 m of this ribbon cost?

O Fill in the ☐ with appropriate numbers to complete a math problem that leads to a math sentence of $135 \div 7.5$. Also, find the answers.

On ① ☐ L of gas, a car travels ② ☐ km. How many km does this car travel on 1 L?

Page58 2 →

P Find the following quotients based on the math fact, $171 \div 38 = 4.5$
① $17.1 \div 3.8$ ② $1.71 \div 0.38$ ③ $0.171 \div 0.038$

P Select all the math sentences that result in a quotient of 2.4 among Ⓐ to Ⓓ below, based on the fact that $4.32 \div 1.8 = 2.4$.
Ⓐ $4.32 \div 18$ Ⓑ $43.2 \div 18$
Ⓒ $43.2 \div 1.8$ Ⓓ $432 \div 180$

Page58 3 →

Q Estimate, and then calculate using the division algorithm.

① $5.32 \div 3.8$　② $7.56 \div 2.4$　③ $25.2 \div 5.6$
④ $6.6 \div 1.2$　⑤ $69.7 \div 8.5$　⑥ $95.4 \div 3.18$
⑦ $34.2 \div 5.7$　⑧ $5.88 \div 1.96$　⑨ $45.9 \div 1.7$

Q You are going to divide 28.5 kg of salt into portions of 1.5 kg, and put each portion into a bag which will be sold for 168 yen. How many bags will there be?

Page58 4 →

R ① $5.32 \div 7.6$　② $2.46 \div 4.1$　③ $3.9 \div 5.2$
④ $3.33 \div 7.4$　⑤ $9 \div 7.5$　⑥ $17 \div 6.8$

R Fill in the □ with appropriate numerals and place decimal points of the quotients in the appropriate places to make the calculations with the algorithm correct.

①
$$\begin{array}{r} \square\square \\ 3.2\overline{)2.\square 6} \\ \square\square\square \\ \hline 0 \end{array}$$

②
$$\begin{array}{r} 0\square 5 \\ \square.6\overline{)4.2} \\ \square\square\square \\ \square 8\square \\ \hline 2\square 0 \\ \hline 0 \end{array}$$

Page59 6 →

S ① $37.6 \div 0.4$　② $4.3 \div 0.5$　③ $8.7 \div 0.6$
④ $2.52 \div 0.8$　⑤ $0.78 \div 0.8$　⑥ $3 \div 0.4$

S In the following math sentences, all the △s contain the same number that is not 0.
Fill in the □ with appropriate inequality signs.

① $\triangle \div 1.5 \square \triangle \div 2.5$　② $\triangle \div 0.8 \square \triangle \div 1.2$
③ $\triangle \div 0.12 \square \triangle \div 0.012$

Times As Much with Decimal Numbers

Answers→ Page 141

Page65 1 →

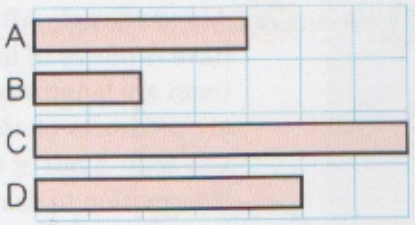
T The lengths of 2 hoses are shown in the table on the right.
If we use the length of hose Ⓐ as the base, how many times as long is hose Ⓑ?
If we use the length of hose Ⓑ as the base, how many times as long is hose Ⓐ?

Hose Lengths

	Length (m)
Ⓐ	5
Ⓑ	2

Page65 1 →

T There are 2 pieces of tape, Ⓐ and Ⓑ. Ⓑ is 0.4 times as long as Ⓐ. In the 4 pieces of tape, A, B, C, and D in the diagram on the right, which ones are Ⓐ and Ⓑ?



Page68 2 →

U Egg Ⓐ weighs 54.9 g and is 0.9 times as heavy as egg Ⓑ. How much does egg Ⓑ weigh?

U The number of students at Elementary School A is 360, and that is 0.8 times as many as the number of students at Elementary School B. The number of students at Elementary School B is 2.5 times as many as the number of students at Elementary School C. How many students are there at Elementary School B and Elementary School C?

6 Let's Investigate Figures That Have the Same Shape and Size

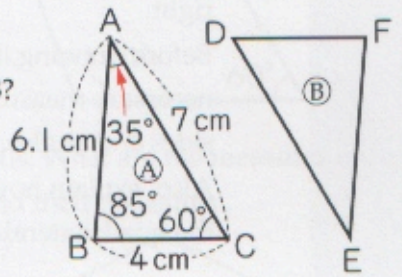
Answers→ Page 141

Page75 1 →

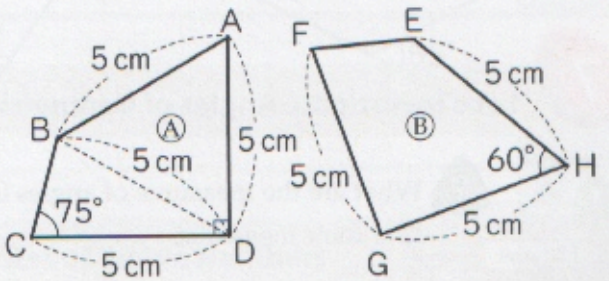
V Triangles Ⓐ and Ⓑ are congruent.

① Which side corresponds to side AB? Which angle corresponds to angle C?

② What is the length of side DF? What is the measure of angle F?



V Quadrilaterals Ⓐ and Ⓑ are congruent. What is the measure of angle E?



Page80 3 →

W Draw the following triangles.

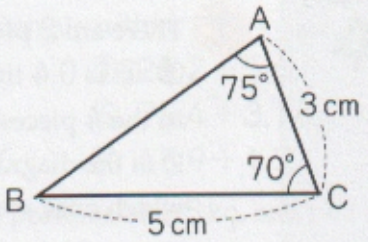
① Two sides are 6 cm and 4 cm, and the angle between them is 55°.

② One side is 5 cm long, and the angles at the ends of the side measure 70° and 40°.

③ Three sides are 7 cm, 6 cm, and 4 cm.

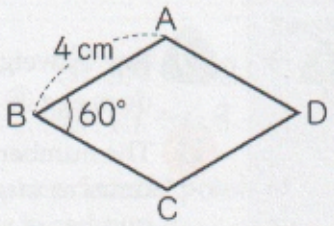
Page 80
3 →

W About triangle ABC on the right, you have 4 pieces of information: some of them are lengths of sides and others are measures of angles.
Use only 3 out of the 4 pieces of information to draw a triangle that is congruent to triangle ABC.

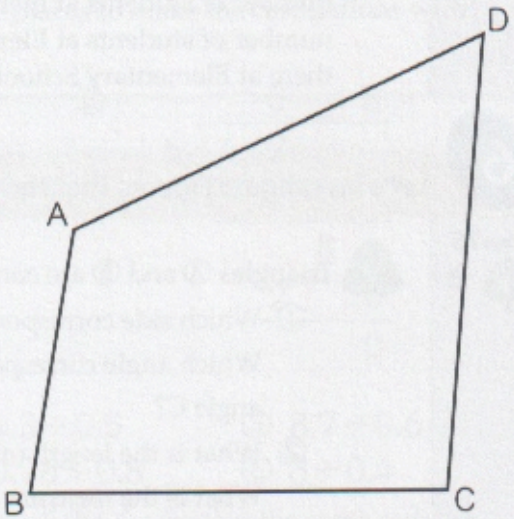


Page 81
5 →

X Using the ways for drawing congruent triangles, draw a rhombus that is congruent to rhombus ABCD on the right.
Also, explain how you drew it.



X Using the ways for drawing congruent triangles, draw a quadrilateral that is congruent to quadrilateral ABCD on the right.
Before drawing it, make necessary measurements of sides and angles.
Also, explain how you drew the quadrilateral.



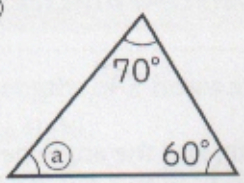
7 Let's Investigate Angles of Geometric Figures

Answers → Page 141

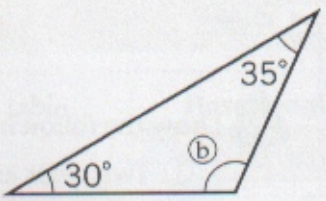
Page 86
1 →

Y What are the measures of angles ⓐ, ⓑ, ⓒ, and ⓓ? Use calculations to find their measures.

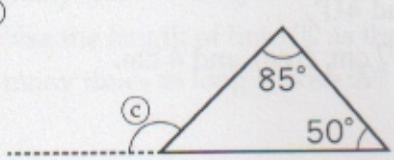
①



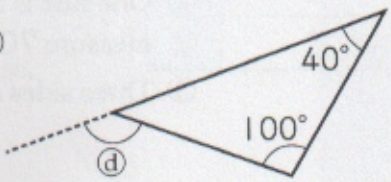
②



③



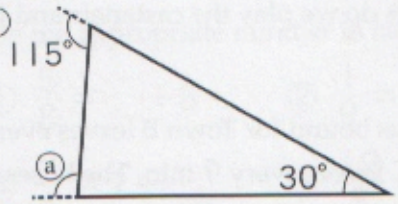
④

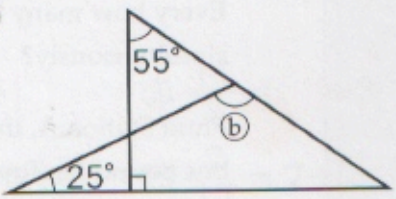


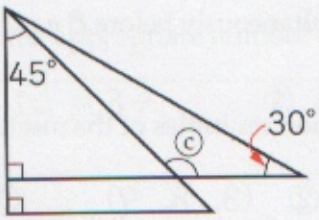
Page 86

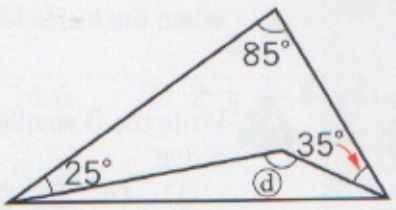
1 →

Y What are the measures of angles ⓐ, ⓑ, ⓒ, and ⓓ? Use calculations to find their measures.

① 

② 

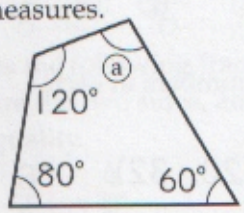
③ 

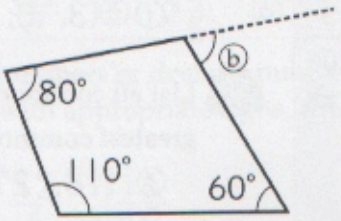
④ 

Page 89

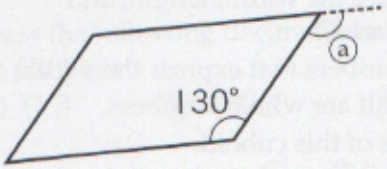
2 →

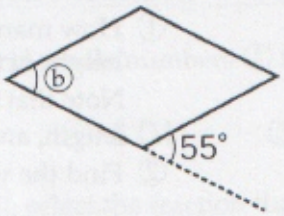
Z What are the measures of angles ⓐ and ⓑ? Use calculations to find their measures.

① 

② 

Z ① is a parallelogram, and ② is a rhombus. What are the measures of angles ⓐ and ⓑ ? Use calculations to find their measures.

① 

② 

8 Let's Investigate the Properties of Whole Numbers

Answers→ Page 141

Page 102

2 →

AA Write the 3 smallest common multiples of the numbers in each ( ) in order.

① (6, 8) ② (7, 14) ③ (5, 7) ④ (9, 12)

AA Find the largest 2-digit number among the common multiples of the numbers in ( ).

① (6, 8) ② (4, 7) ③ (4, 9) ④ (8, 20)

Page 102
3 →

AB Play the castanets and the triangles by following the signals.
Play the castanets every 3 beats and play the triangles every 4 beats.
Every how many beats do we play the castanets and the triangles simultaneously?

AB From Station A, the bus bound for Town B leaves every 6 min. and the bus bound for Town C leaves every 9 min. The buses bound for Town B and Town C left simultaneously at 7 a.m. sharp. Find out all the times when the buses leave simultaneously before 8 a.m.

Page 103
4 →

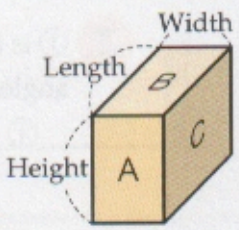
AC Write the 3 smallest common multiples of the numbers in each ( ) in order.
① (4, 5, 8) ② (3, 8, 9) ③ (2, 7, 9)

AC Among the common multiples of the numbers in each ( ), find the one that is closest to 80.
① (3, 5, 6) ② (4, 6, 7)

Page 106
3 →

AD List all common factors of the numbers in each ( ). Also, find the greatest common factor.
① (18, 27) ② (20, 32) ③ (24, 48)

AD Look at the cuboid on the right. Face A has an area of 6 cm^2, face B has an area of 8 cm^2, and face C has an area of 12 cm^2.
① How many cm are the width, length, and height of the cuboid?
Note that the numbers that express the width, length, and height are whole numbers.
② Find the volume of this cuboid.



Page 106
5 →

AE Find the greatest common factor of the numbers in each ( ).
① (9, 18, 21) ② (15, 20, 30)
③ (14, 42, 70)

AE Find the second largest common factor of the numbers in each ( ).
① (12, 18, 42) ② (16, 64, 80)

9 Let's Investigate Relationships between Fractions and Decimal / Whole Numbers.

Answers → Page 141

Page 113 → 3

AF Write the appropriate number in each □.

① $\frac{5}{8} = □ \div 8$ ② $\frac{1}{9} = 1 \div □$ ③ $\frac{11}{3} = □ \div 3$

④ $\frac{7}{4} = 7 \div □$ ⑤ $\frac{19}{8} = 19 \div □$ ⑥ $\frac{7}{17} = 7 \div □$

AF Write the appropriate number in each □.

① $1\frac{1}{2} = 3 \div □$ ② $1\frac{3}{4} = □ \div 4$ ③ $2\frac{□}{3} = 8 \div 3$

Page 116 → 2

AG Express the following fractions ① through ⑥ as whole numbers or decimal numbers.

① $\frac{1}{5}$ ② $\frac{15}{6}$ ③ $\frac{28}{7}$ ④ $\frac{63}{9}$ ⑤ $2\frac{4}{5}$ ⑥ $4\frac{3}{8}$

AG Express the following fractions as whole numbers or decimal numbers, compare the two sides, and fill in the □ with appropriate signs, equal or inequality.

① $\frac{3}{4} □ \frac{4}{5}$ ② $\frac{12}{6} □ \frac{16}{10}$

③ $\frac{12}{8} □ \frac{30}{20}$ ④ $1\frac{2}{5} □ \frac{5}{2}$

Page 117 → 3

AH Express the following decimal numbers and whole numbers ① through ⑤ as fractions.

① 0.8 ② 0.93 ③ 2 ④ 6.01 ⑤ 9

AH Among the choices from Ⓐ to Ⓕ in □, select the fraction that is equal to each of the decimal numbers and whole number in ① to ③.

① 0.4 ② 1.45 ③ 4

Ⓐ $\frac{40}{1}$ Ⓑ $\frac{4}{10}$ Ⓒ $\frac{40}{10}$

Ⓓ $\frac{4}{100}$ Ⓔ $\frac{145}{100}$ Ⓕ $\frac{145}{1,000}$

Additional Problems

Pages 128–139

1 Let's Summarize the Structures of Whole Numbers and Decimal Numbers

A (From left to right) 5, 0, 6, 9

A (From left to right) 1, 0.1, 0.01, 0.001

B ① < ② > ③ >

B 1.53 < 1.6

C ① 5 pieces ② 37 pieces ③ 899 pieces ④ 5,200 pieces

C 1.01

2 Let's Think about How to Express the Amount of Space Occupied by Cubes and Cuboids

D ① 224 cm^3 ② 216 cm^3 ③ 144 cm^3 ④ 120,000 cm^3

D ① 6 ② 4

E ① 196 cm^3 ② 156 cm^3

E ①Ⓐ ②Ⓒ ③Ⓑ

F 45,000 cm^3, 45 L

F 24 L

Ways of Thinking The volume of the water tank is

$30 \times 40 \times 50 = 60{,}000\,(cm^3)$

$60{,}000\,cm^3 = 60\,L$, so

$60 - 36 = 24\,(L)$

3 Let's Investigate How Quantities Change (1)

G ① Yes ② No ③ Yes

G ① Yes (Example) $\square \times 4 = \bigcirc$

② No (Example) $5 + 2 \times \square = \bigcirc$

③ Yes (Example) $25 \times \square = \bigcirc$

④ No (Example) $10 = \square + \bigcirc$

4 Let's Expand the World of Multiplication

H 312 yen

Ways of Thinking $130 \times 2.4 = 312$ (yen)

H 78 g

Ways of Thinking 1 m of this wire weighs $40 \div 2 = 20$ (g)

3.9 m of this wire will weigh $20 \times 3.9 = 78$ (g)

I ① 141.18 ② 141.18 ③ 14.118

I ©

J ① 11.61 ② 13.464

J ③, Correct answer…7.14

K ① 36.855 ② 16.728 ③ 14.57 ④ 574.308 ⑤ 428.4 ⑥ 1069.6

K ① 1.6×2.7 (2.7×1.6)

② 1.7×6.2 (6.2×1.7)

L ① 46.2 ② 10.5 ③ 1943 ④ 0.767 ⑤ 0.928 ⑥ 0.6

L ①

$$\begin{array}{r} 3.4 \\ \times\ 2.5 \\ \hline 170 \\ 68 \\ \hline 8.50 \end{array}$$

②

$$\begin{array}{r} 0.19 \\ \times\ 4.8 \\ \hline 152 \\ 76 \\ \hline 0.912 \end{array}$$

M ① 1.62 ② 5.55 ③ 0.35 ④ 0.032 ⑤ 0.3 ⑥ 0.67

M ① < ② > ③ <

N ① 72 ② 26 ③ 102.8 ④ 59.4

N (From left to right) ①(Example) 25, 4 (2.5, 40)

②(Example) 25, 40, 2.5, 4, 0.25, 0.4

5 Let's Expand the World of Division

O 150 yen

Ways of Thinking $360 \div 2.4 = 150$ (yen)

O ① 7.5 ② 135 Answer…18 km

P ① 4.5 ② 4.5 ③ 4.5

P Ⓑ, Ⓓ

Q ① 1.4 ② 3.15 ③ 4.5 ④ 5.5 ⑤ 8.2 ⑥ 30 ⑦ 6 ⑧ 3 ⑨ 27

Q 19 bags

Ways of Thinking $28.5 \div 1.5 = 19$ (bags)

R ① 0.7 ② 0.6 ③ 0.75 ④ 0.45 ⑤ 1.2 ⑥ 2.5

R ①

$$3.2\overline{)2.56}\quad 0.8,\ 256,\ 0$$

②

$$5.6\overline{)4.2}\quad 0.75,\ 392,\ 280,\ 280,\ 0$$

S ① 94 ② 8.6 ③ 14.5 ④ 3.15 ⑤ 0.975 ⑥ 7.5

S ① > ② > ③ <

Times As Much with Decimal Numbers

T Ⓑ is 0.4 times as long as Ⓐ.
Ⓑ is 2.5 times as long as Ⓑ.

T Ⓐ D Ⓑ B

U 61 g

Ways of Thinking 54.9 ÷ 0.9 = 61 (g)

U Elementary School B···450 students, Elementary School C···180 students

Ways of Thinking

Elementary School B···360 ÷ 0.8 = 450 (students)
Elementary School C···450 ÷ 2.5 = 180 (students)

6 Let's Investigate Figures That Have the Same Shape and Size

V ① Side EF, Angle D
② Side DF···4 cm, Angle F···85°

V 135°

W (Omitted)

W (Figure omitted)
Use 5 cm, 3 cm and 70°.
Or use 3 cm, 75°, and 70°.

X (Figure omitted)
(Example) First, draw side AB (4 cm). Then, draw side BC (4 cm) with angle B of 60°. (You can draw triangle ABC because you know the lengths of 2 sides and the measure of the angle between them.) Next, determine the position of vertex D so that sides AD and CD will be 4 cm long. (You can draw triangle ACD because you know the lengths of 3 sides.)

X (Figure omitted)
(Example) First, draw side BC (5.5 cm). Then, draw side BA (3.5 cm) with angle B of 80°. (You can draw triangle ABC because you know the lengths of 2 sides and the measure of the angle between them.) Next, determine the position of vertex D so that sides AD and CD will be 6 cm long. (You can draw triangle ACD because you know the lengths of 3 sides.)

7 Let's Investigate Angles of Geometric Figures

Y ⓐ 50° ⓑ 115° ⓒ 135° ⓓ 140°

Y ⓐ 95° ⓑ 120° ⓒ 135° ⓓ 145°

Z ⓐ 100° ⓑ 70°

Z ⓐ 130° ⓑ 55°

8 Let's Investigate the Properties of Whole Numbers

AA ① 24, 48, 72 ② 14, 28, 42
③ 35, 70, 105 ④ 36, 72, 108

AA ① 96 ② 84 ③ 72 ④ 80

AB Every 12 beats

Ways of Thinking We play the castanets and the triangles simultaneously when the number of beats are common multiples of 3 and 4, and the least common multiple of 3 and 4 is 12.

AB 7:18, 7:36, and 7:54

Ways of Thinking After 7 o'clock, the buses will leave simultaneously when the numbers that express the minute are common multiples of 6 and 9. Since the least common multiple of 6 and 9 is 18, common multiples of 6 and 9 in whole numbers up to 60 are as follows in ascending order:
18, 18 × 2 = 36, 18 × 3 = 54

AC ① 40, 80, 120 ② 72, 144, 216
③ 126, 252, 378

AC ① 90 ② 84

AD ① Common factors···1, 3, 9 Greatest common factor···9
② Common factors···1, 2, 4 Greatest common factor···4
③ Common factors···1, 2, 3, 4, 6, 8, 12, 24
Greatest common factor···24

AD ① Length···4 cm, Width···2 cm, Height···3 cm
② 24 cm^3

Ways of Thinking Area of face A = Height × Width = 6 (cm^2)
Area of face B = Length × Width = 8 (cm^2)
Area of face C = Height × Length = 12 (cm^2)
Think of the whole numbers for the length, width, and height (cm) that fit in the 3 math sentences above.

AE ① 3 ② 5 ③ 14

AE ① 3 ② 8

9 Let's Investigate Relationships between Fractions and Decimal / Whole Numbers

AF ① 5 ② 9 ③ 11
④ 4 ⑤ 8 ⑥ 17

AF ① 2 ② 7 ③ 2

AG ① 0.2 ② 2.5 ③ 4
④ 7 ⑤ 2.8 ⑥ 4.375

AG ① < ② > ③ = ④ <

AH (Example) ① $\frac{8}{10}$ ② $\frac{93}{100}$ ③ $\frac{2}{1}$
④ $\frac{601}{100}\left(6\frac{1}{100}\right)$ ⑤ $\frac{9}{1}$

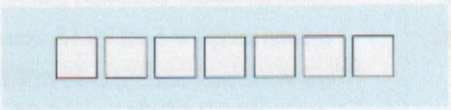
AH ① Ⓑ ② Ⓔ ③ Ⓒ

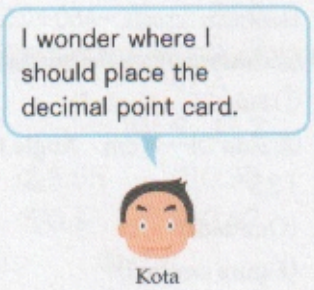
Let's Try WONDERful Problems!

1 Let's Summarize the Structures of Whole Numbers and Decimal Numbers

Answers→ Page 146

1 Using the decimal point card [.] and 6 out of 9 numeral cards [1] to [9], make a decimal number.





① Make the smallest number.

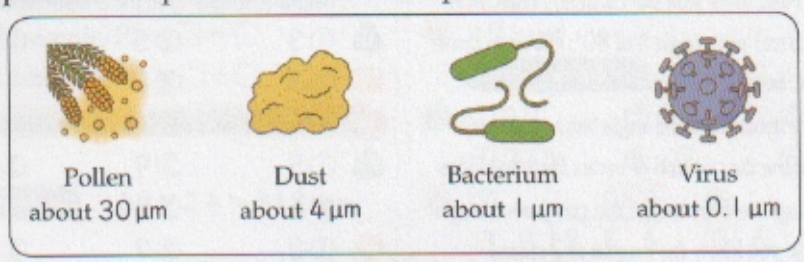
② Make the number that is closest to 50.

2 Small units are useful when you express very short lengths.

$\frac{1}{1,000}$ of 1 mm is equal to 1 μm, micrometer.

① How many mm is 1 μm?

② Medical masks keep you from inhaling the following fine particles and splashes of liquid that contain such particles.



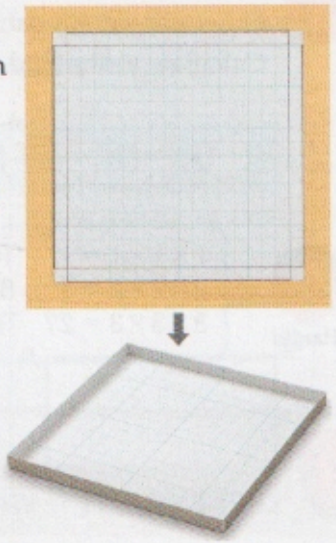
How big is a grain of pollen in mm?
Express the sizes of the other particles in mm.

③ An even smaller unit is nm, nanometer. $\frac{1}{1,000}$ of 1 μm is equal to 1 nm.
How many μm is 1 nm? How many mm is 1 nm?

2 Let's Think about How to Express the Amount of Space Occupied by Cubes and Cuboids

Answers→ Page 146

1 We are going to cut off the 4 corners of a square construction paper with 18 cm sides and make an open box with the side lengths that are whole number cm.
Investigate the relationship between the length of each side of the squares to be cut off from the 4 corners and the capacity of the open box.



① If the length of each side of the squares to be cut off is 1 cm, how much will the volume of the open box be in cm^3?

Length Width Depth Capacity
□ × □ × 1 = □ (cm^3)

② What if we increase the length of each side of the squares to be cut off from 1 cm to 2 cm, 3 cm, ... up to 8 cm?
Find the volume of the open box for each of the lengths and complete the table below.

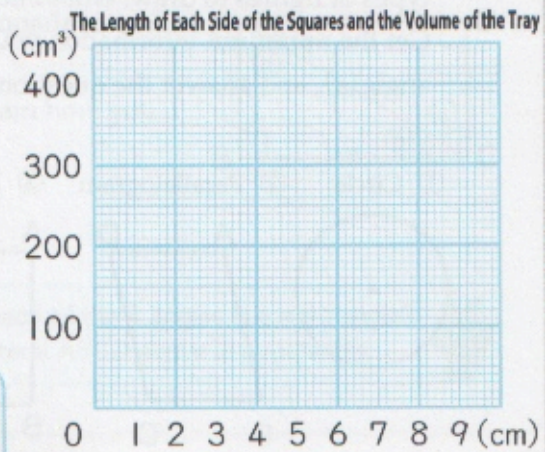
Each side of squares to be cut off (cm)	1	2	3	4	5	6	7	8
Capacity of open box (cm^3)								

③ If the length of each side of the squares to be cut off is 0 cm or 9 cm, we cannot make an open box, but think in the following way.

If the length of each side of the squares to be cut off is 0 cm, the resulting cuboid will be 18 cm in width, 18 cm in length, and 0 cm in depth, and its volume will be $18 \times 18 \times 0 = 0$ (cm^3).
If the length of each side of the squares to be cut off is 9 cm, the resulting cuboid will be 0 cm in width, 0 cm in length, and 9 cm in depth, and its volume will be $0 \times 0 \times 9 = 0$ (cm^3).



Express the relationship between the length of each side of the squares to be cut off and the volume of the open box in a line graph.



④ What is the length of each side of the squares to be cut off in cm that will lead to the largest capacity of the open box?


Ami

If you show the data in a graph, you can recognize the answer at a glance.

4 Let's Expand the World of Multiplication

Answers→ Page 147

Advanced High School

1 How many cm is each side of a cube with a volume of 10 cm^3?
Calculate to the 2nd decimal place.

$\square \times \square \times \square = 10$

You can use a calculator.



Haruto

$1 \times 1 \times 1 = 1$ Too small...
$2 \times 2 \times 2 = 8$ Still too small...
$3 \times 3 \times 3 = 27$ Too big...

$2.5 \times 2.5 \times 2.5 = 15.625$
Still too big.

Misaki

5 Let's Expand the World of Division

Answers→ Page 147

1 Select 4 out of 10 cards from 0 to 9 to make the division math sentence below.

$\square.\square \div \square.\square$

You can place 0 in the 1st decimal place to make 5.0, for example.

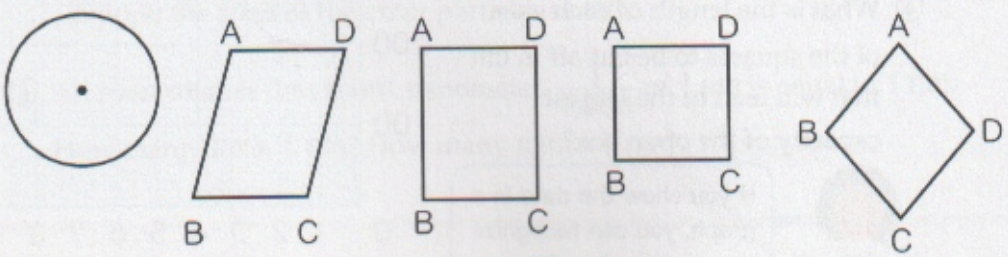
① Make a math sentence that results in the largest quotient. Also, find the quotient.
② Make a math sentence that results in the quotient that is closest to 1. Also, find the quotient.

6 Let's Investigate Figures That Have the Same Shape and Size

Answers→ Page 147

1 Draw a figure that is congruent to each of the ones below. Other than the types of figures to draw, what information do you need to draw them? Get the minimum information you need by measuring the length(s) and angle(s), and answer the question.

① Circle ② Parallelogram ③ Rectangle ④ Square ⑤ Rhombus



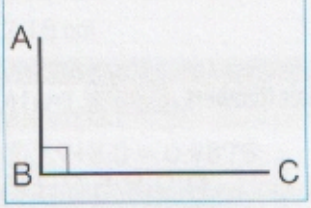
7 Let's Investigate Angles of Geometric Figures

Answers→ Page 147

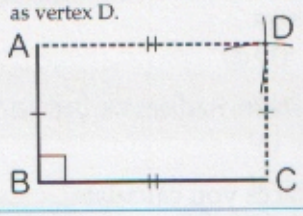
Advanced 8th Grade

1 Takeshi drew quadrilateral ABCD with a right angle at B. He drew it using the way of drawing parallelograms, which he learned in 4th grade.

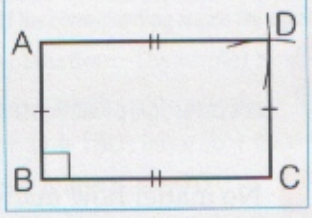
❶ Draw sides AB and BC with a right angle at B.



❷ Set the radius of a pair of compasses to the length of side BC, and then draw an arc from vertex A. Change the radius to the length of side AB, and then draw an arc from vertex C. Name the intersection of the arcs as vertex D.

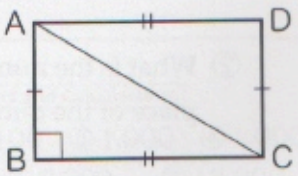


❸ Connect vertices A and D and vertices C and D to form quadrilateral ABCD.



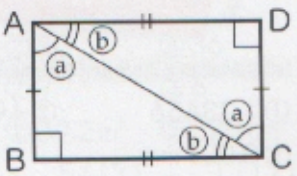
The resulting quadrilateral ABCD is a rectangle.
Explain the reason why by reviewing Takeshi's drawing steps ❶, ❷, and ❸.

① Draw diagonal AC as shown on the right. Explain the reason why triangle ABC is congruent to triangle CDA.



② Since triangle ABC is congruent to triangle CDA, you can tell the measure of angle D. How much is it? Also give the reason why.

③ How much are angles A and C of quadrilateral ABCD?
Using the figure on the right, explain how you found the answers.



Were you able to explain that each of the 4 angles is a right angle? If so, you can say that quadrilateral ABCD above is a rectangle.

8 Let's Investigate the Properties of Whole Numbers

Answers→ Page 147

1 Dividing a certain 3-digit number by 3, 4, 5, 6, 7, or 8 will leave a remainder of 2 each time. What is this 3-digit number?

□□□ ÷ 3 6 8 5 7 4 = ? remainder 2

What if this number is made smaller by 2...?



9 Let's Investigate Relationships between Fractions and Decimal / Whole Numbers

Answers→ Page 147

1 No matter how many places you calculate $2 \div 7$ to, you cannot divide completely.

① If you keep doing the calculation on the right, what numerals will appear repeatedly after the 4 in the 6th decimal place of the quotient? Write down the six numerals that follow the 4.

② What is the numeral in the 50th decimal place of the quotient?

```
  0.285714
7)2.0
  14
   60
   56
    40
    35
     50
     49
      10
       7
       30
       28
```

Answers

Let's Try WONDERful Problems!

Pages 142–146

1 Let's Summarize the Structures of Whole Numbers and Decimal Numbers

1 ① 1.23456 ② 49.8765

2 ① 0.001 mm

② Pollen…0.03 mm

Dust…0.004 mm

Bacterium…0.001 mm

Virus…0.0001 mm

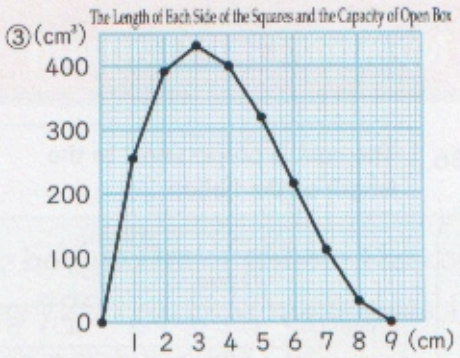
③ 0.001 μm, 0.000001 mm

2 Let's Think about How to Express the Amount of Space Occupied by Cubes and Cuboids

1 ① $16 \times 16 \times 1 = 256\,(\text{cm}^3)$

②

Each side of squares to be cut off (cm)	1	2	3	4	5	6	7	8
Capacity of open box (cm^3)	256	392	432	400	320	216	112	32



④ 3 cm

4 Let's Expand the World of Multiplication

1 2.15 cm

5 Let's Expand the World of Division

1 ① 9.8÷0.1 = 98

② 7.9÷8.0 = 0.9875

6 Let's Investigate Figures That Have the Same Shape and Size

1 (Example) ① Radius (1 cm)

② Side AB (2 cm), Side BC (1.5 cm), Angle B (75°)

③ Side AB (2 cm), Side BC (1.5 cm)

④ Side AB (1.5 cm)

⑤ Side AB (1.5 cm), Angle B (100°)

7 Let's Investigate Angles of Geometric Figures

1 ① (Example) Side BC and side DA have the same length. Side AB and side CD have the same length. The 2 triangles have side AC in common. Since the lengths of all the 3 corresponding sides are equal in length, triangle ABC is congruent to triangle CDA.

② 90° Reason··· (Example) In congruent figures, the measures of the corresponding angles are equal.

③ 90° Explanation··· (Example) Since the sum of the 3 angles of a triangle is 180°, 90° + ⓐ + ⓑ = 180°. Since ⓐ + ⓑ = 90°, both angles A and C are 90°.

8 Let's Investigate the Properties of Whole Numbers

1 842

9 Let's Investigate Relationships between Fractions and Decimal / Whole Numbers

1 ① 285.714 ② 8

Answers Do You Remember?

Page 39

1 Yes

2 ① 2.5 times ② 400 cm

3 (From top to bottom, left to right)

① 3, 3, 84 ② 10, 10, 100, 2,800

③ 280 ④ 2,800

Playing with Numbers and Calculations

Ⓐ 12 Ⓑ 4 Ⓒ 2 Ⓓ 18 Ⓔ 8 Ⓕ 3

Ⓚ 5 Ⓛ 8 Ⓜ 25 Ⓝ 50 Ⓞ 2 Ⓟ 4

Page 71

1 Ⓐ $\frac{1}{6}$ Ⓑ $\frac{8}{6}$, $1\frac{2}{6}$

Ⓒ $\frac{11}{6}$, $1\frac{5}{6}$ Ⓓ $\frac{22}{6}$, $3\frac{4}{6}$

2 ① 4, 3, 8, 5 ② 516 ③ 5.16

④ 240 ⑤ 0.024 (in this order)

3 ① 7.83 ② 2 ③ 38.28 ④ 25.859

⑤ 0.51 ⑥ 0.95 ⑦ 4.26 ⑧ 0.908

4 (Omitted)

Playing with Numbers and Calculations

① 100 ② 100 ③ 1,000 ④ 1,000

⑤ 9,700 ⑥ 9,300 ⑦ 13,000

⑧ 11,000

Page 124

1 ① 51.3 ② 1.28 ③ 0.63 ④ 1.2

⑤ 22.4 ⑥ 0.6 ⑦ 0.8 ⑧ 2.45

2 25 kg

3 ① 24 ② 30 ③ 36

4 ① 8 ② 9 ③ 6

5 ① 729 cm³ ② 39.2 m³ ③ 320 cm³

6 ① $\frac{12}{9}\left(1\frac{3}{9}\right)$ ② $2\frac{5}{6}\left(\frac{17}{6}\right)$ ③ $4\frac{1}{4}\left(\frac{17}{4}\right)$

④ $3\frac{4}{5}\left(\frac{19}{5}\right)$ ⑤ $\frac{3}{7}$ ⑥ $3\frac{2}{4}\left(\frac{14}{4}\right)$

⑦ $2\frac{1}{5}\left(\frac{11}{5}\right)$ ⑧ $2\frac{6}{8}\left(\frac{22}{8}\right)$

Playing with Numbers and Calculations

□ = 12, ○ = 6, △ = 4,

♡ = 2, ◇ = 3

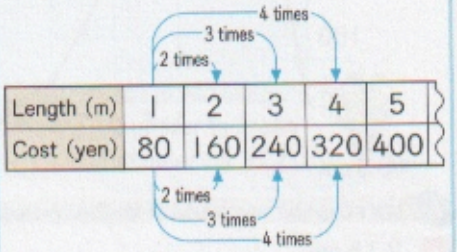
Let's Think Using a Double Number Line Diagram

This is the situation from problem 3 on page 36.

There is a roll of ribbon that costs 80 yen for 1 m. If you buy 9 m of this ribbon, how much will it cost?

- 80 yen for 1 m
- Finding the cost for 9 m

The cost is proportional to the length of the ribbon.



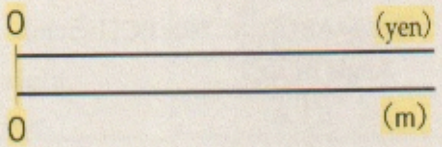


Riku

● Express the multiplication problem above as a number line diagram.

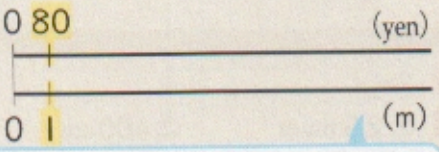
1

At the left end of the diagram, draw a tick mark and write "0."
Draw 2 parallel lines.
At the right end of the bottom line, write "(m)."
At the right end of the top line, write "(yen)."



2

Since it is "80 yen for 1 m," on the bottom line, draw a tick mark that expresses the base amount (1 m) and write "1." On the top line, draw a tick mark and write "80."

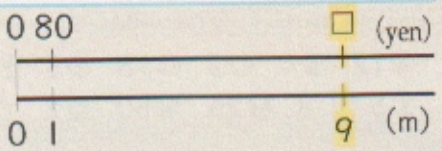


Since you write "1" as the base amount on the bottom number line, you write "(m)" on the bottom number line as well.

3

Express the unknown number as □.

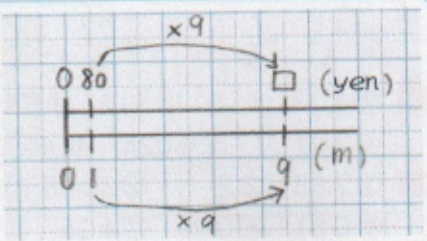
To show "□ yen for 9 m," draw a tick mark that expresses how many times as long 9 m is as the base amount and write "9" on the bottom line. On the top line, draw a tick mark and □.





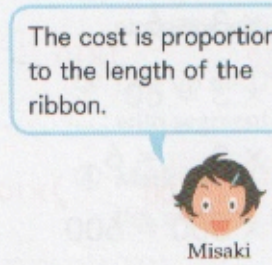
Shiho

The cost is proportional to the length of the ribbon. If you buy the ribbon that is 9 times as long as the base length, the cost becomes 9 times as much as the base cost.
So, the math sentence to find the cost will be:
80×9



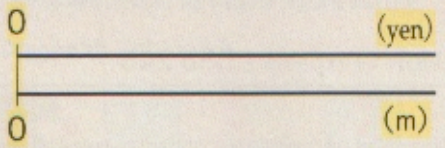
We bought 9 m of ribbon. The cost was 720 yen. How much does 1 m of this ribbon cost?

- 720 yen for 9 m
- Finding the cost for 1 m



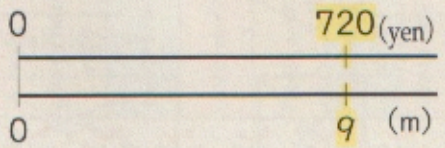
● Express the division problem above as a number line diagram.

1



At the left end of the diagram, draw a tick mark and write "0."
Draw 2 parallel lines.
At the right end of the bottom line, write "(m)."
At the right end of the top line, write "(yen)."

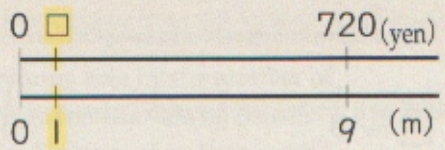
2



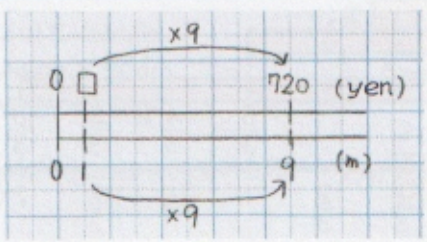
Since it is "720 yen for 9 m," on the bottom line, a tick mark that expresses how many times as long 9 m is as the base amount and write "9." On the top line, draw a tick mark and write "720."

3

Express the unknown number as □.



To show "□ yen for 1 m," draw a tick mark that expresses the base amount (1 m) and write "1" on the bottom line. On the top line, draw a tick mark and □.



The cost is proportional to the length of the ribbon. So, if you buy the ribbon that is 9 times as long as the base length, the cost becomes 9 times as much as the base cost. This can be expressed as the following multiplication sentence:

□ × 9 = 720

So, the math sentence to find □ will be:

□ = 720 ÷ 9


Kota

Review Exercises

1 Properties of Multiplication (4th Grade)

In multiplication, if a multiplicand or a multiplier becomes 10 or 100 times as much, the product also becomes 10 or 100 times as much.

$2 \times 3 = 6$
$\downarrow \times 10 \quad \rangle \times 10$
$20 \times 3 = 60$

$2 \times 3 = 6$
$\downarrow \times 100 \quad \rangle \times 100$
$2 \times 300 = 600$

$2 \times 3 = 6$
$\downarrow \times 10 \ \downarrow \times 100 \quad \rangle \times 1{,}000$
$20 \times 300 = 6{,}000$

2 Properties of Division (4th Grade)

In division, the quotient does not change if we divide both the dividend and the divisor by the same number.

$150 \div 50 = 3$
$\downarrow \div 10 \ \downarrow \div 10$) No change
$15 \div 5 = 3$

In division, the quotient does not change if we multiply both the dividend and the divisor by the same number.

$15 \div 5 = 3$
$\downarrow \times 10 \ \downarrow \times 10$) No change
$150 \div 50 = 3$

3 Approximate Numbers (4th Grade)

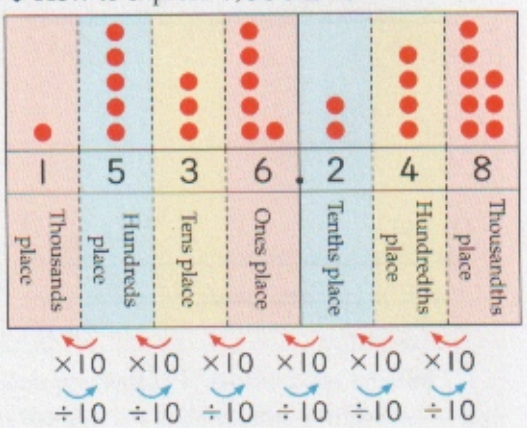
◆ Rounding

If the digit in the place is [0, 1, 2, 3, or 4] → round down.
If the digit in the place [5, 6, 7, 8, or 9] → round up.

To round to the nearest thousand, → round based on the digit in the hundreds place, one place below the thousands.	To round to the highest place in the given number → Round based on the digit in the second highest place.
One thousands — Hundreds: 13,648 ↓ Rounded up 14,000	①② 13,648 ↓ Rounded down 10,000
One thousands — Hundreds: 17,392 ↓ Rounded down 17,000	①② 17,392 ↓ Rounded up 20,000

4 Structure of Whole Numbers and Decimal Numbers (4th Grade)

◆ How to express 1,536.248



5 Equal and Inequality Signs (3rd Grade)

Equal and **inequality signs** show that a number or math sentence is equal to or larger/smaller than the other.

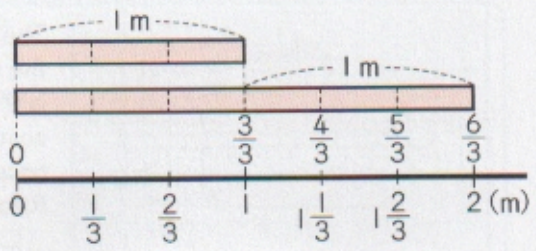
equal sign: Same = Same

inequality signs: Large > Small
Small < Large

6 How to Express Fractions and the Structure of Fractions (4th Grade)

The length that is $\frac{1}{3}$ as long as the base length, 1 m, is written as $\frac{1}{3}$ m.

The length that consists of 5 pieces of $\frac{1}{3}$ m is written as $\frac{5}{3}$ m or $1\frac{2}{3}$ m.

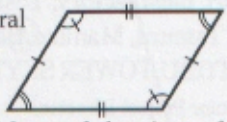


7 Various Quadrilaterals (4th Grade)

Trapezoid: a quadrilateral with one pair of opposite sides that are parallel.



Parallelogram: a quadrilateral with two pairs of opposite sides that are parallel.



The lengths of opposite sides and the sizes of opposite angles of a parallelogram are equal.

Rhombus: a quadrilateral whose sides are equal in length.

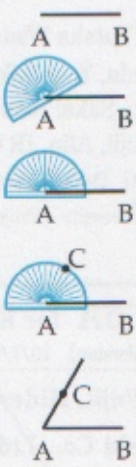


Opposite sides of a rhombus are parallel.

The sizes of opposite angles of a rhombus are equal.

8 How to Draw an Angle (4th Grade)

◆ How to draw a 60° angle



① Draw segment AB.

② Line up the center of the protractor with point A.

③ Line up the 0° line with segment AB.

④ Mark point C at the tick mark for 60°.

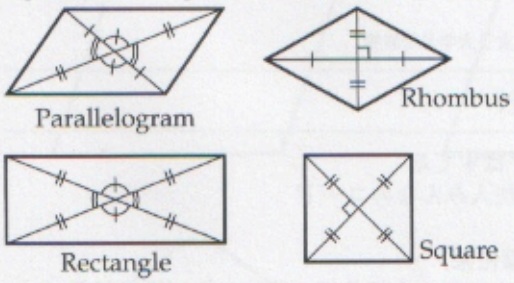
⑤ Draw a line from point A through point C.

9 Diagonals (4th Grade)

Diagonals

Segments that connect the opposite vertices of quadrilaterals.

Diagonals of different quadrilaterals have different properties.



10 Times as Much (4th Grade)

Lengths of the blue ribbon and the white ribbon based on the length of the red ribbon can be expressed as follows.

	Length (m)
Red	2
Blue	6
White	5

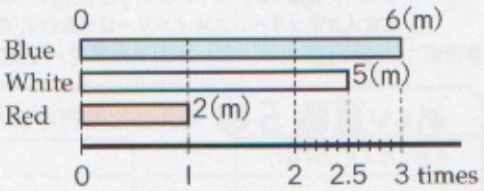
If we consider the length of the red ribbon (2 m) as 1, Blue Red

- The length of the blue ribbon (6 m) corresponds to 3. $(6 \div 2 = 3)$
- The length of the white ribbon (5 m) corresponds to 2.5. $(5 \div 2 = 2.5)$

White Red

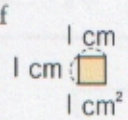
We can use the phrase "times as much" to express these relationships in sentences and a diagram as follows.

- The blue ribbon is 3 times as long as the red ribbon.
- The white ribbon is 2.5 times as long as the red ribbon.

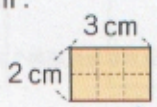


11 How to Express Area and Formulas for Area (4th Grade)

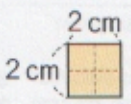
We express area by the number of 1 cm squares that can be placed inside. The area of a square with 1 cm sides is expressed as 1 cm².



Area of Rectangle
= Length × Width
= Width × Length



Area of Squares = Side × Side



12 Units of Quantity (2nd and 3rd Grades)

【Length】
1 cm = 10 mm
1 m = 100 cm
1 km = 1,000 m

【Capacity】
1 L = 1,000 mL

【Weight】
1 kg = 1,000 g
1 t = 1,000 kg

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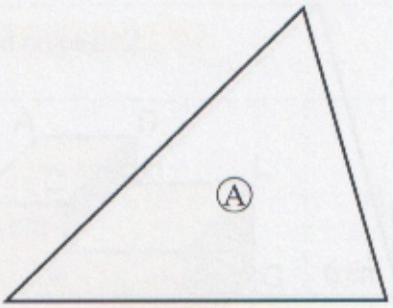
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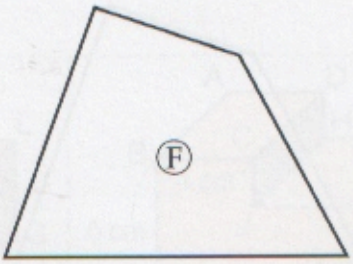
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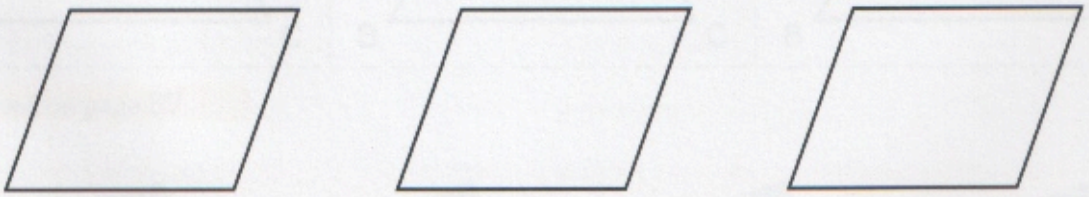


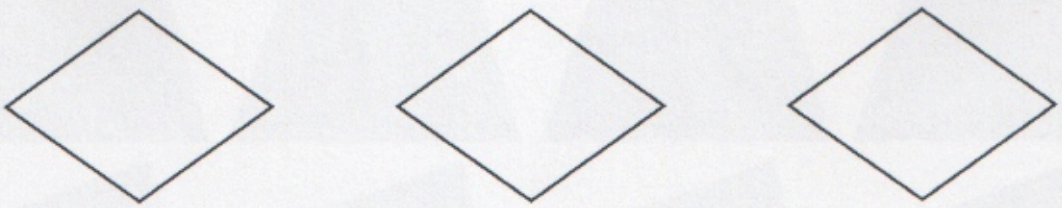


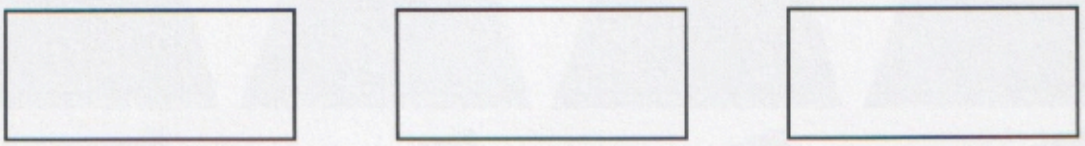
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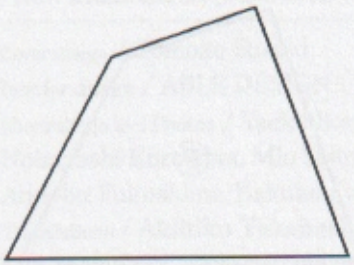


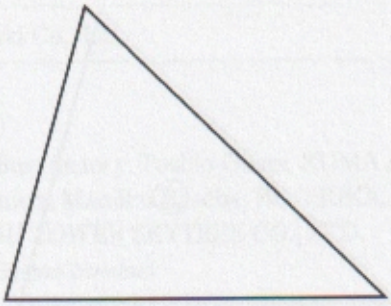








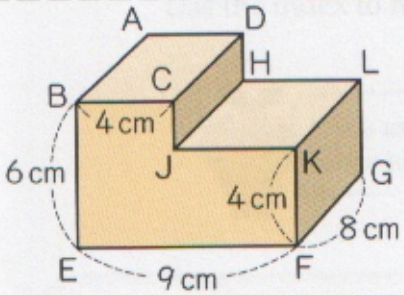


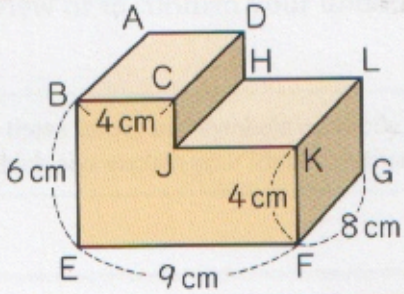


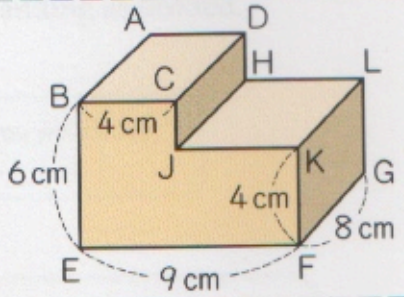




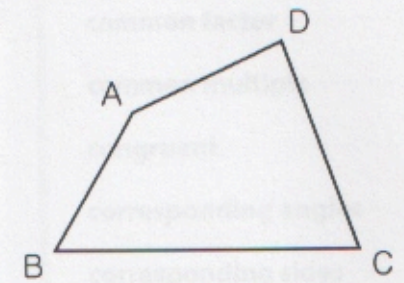
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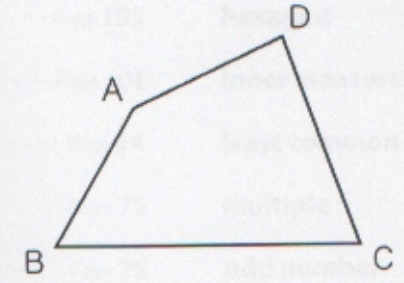


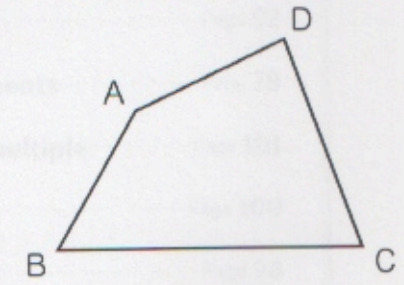




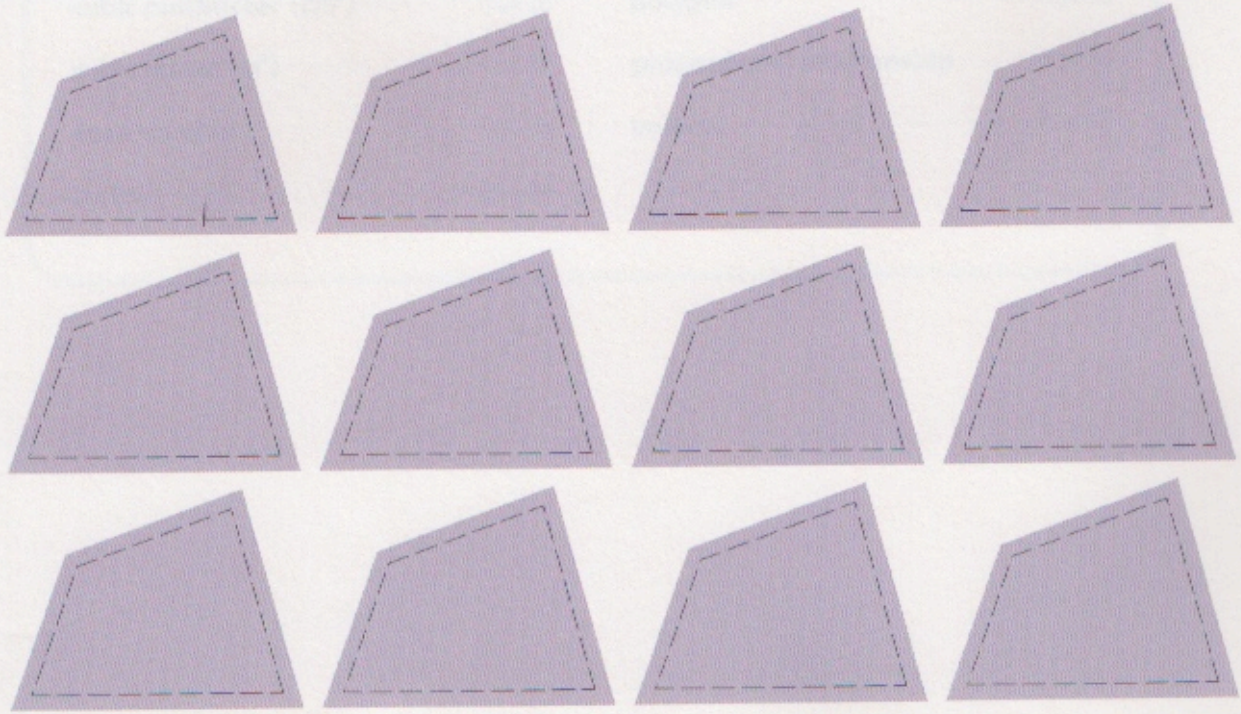
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Index

This index lists the terms and symbols you study in this textbook.
Use the index to review or to confirm your understanding as needed.



If you use these terms and symbols correctly, you will find it easier to think and explain your ideas to others.

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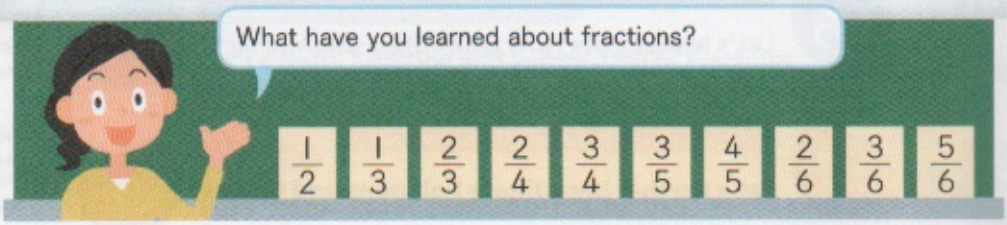
New Mathematics
for Elementary School

5B



TOKYO SHOSEKI

Let's review what you have learned about fractions




Riku

We learned how to write fractions as decimal numbers and the other way around.

$\frac{3}{4} = \square \div \square$

$= \square$

$0.3 = \frac{\square}{\square}$

Relationships between Fractions and Decimal / Whole Numbers
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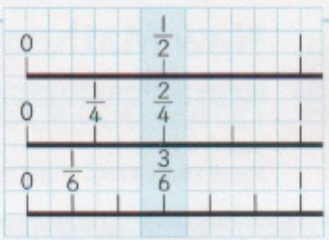
We learned that fractions can be considered as quotients. $\frac{\bullet}{\blacksquare} = \bullet \div \blacksquare$


Shiho


Ami

We found many fractions of the same size. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \cdots$

We used number lines to examine fractions of the same size.




Kota


Haruto

We learned that we can add and subtract fractions that have the same denominators. $\frac{3}{5} + \frac{4}{5} = \frac{\square}{5}$ $\frac{5}{6} - \frac{2}{6} = \frac{\square}{6}$

We found that $\frac{3}{5} + \frac{4}{5}$ can be considered as $3 + 4$ if we use $\frac{1}{5}$ as the base amount.


Misaki

I wonder if there is any type of addition or subtraction of fractions that we haven't studied.


Kota

I wonder if we can add or subtract fractions with different denominators.

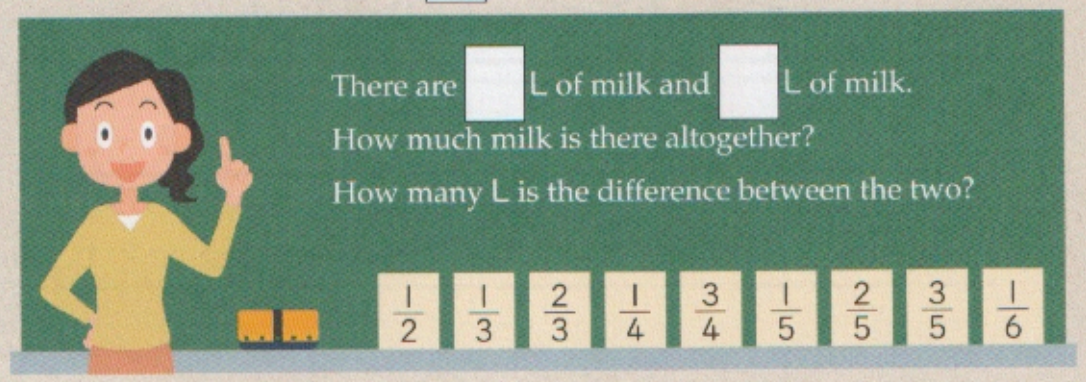

Shiho

Since decimal numbers can be written as fractions, if both decimal numbers and fractions are in an addition or subtraction math sentence...

Addition and Subtraction of Fractions

10 Let's Extend Addition and Subtraction of Fractions

Place various fraction cards in ☐ and think about the problem.



1 Finding a Common Denominator and Adding and Subtracting Fractions

1 There are $\frac{1}{2}$ L of milk and $\frac{1}{3}$ L of milk.
How much milk is there altogether?

Math Sentence ▸ _____


Haruto: Since the denominators are different, we can't calculate as they are...

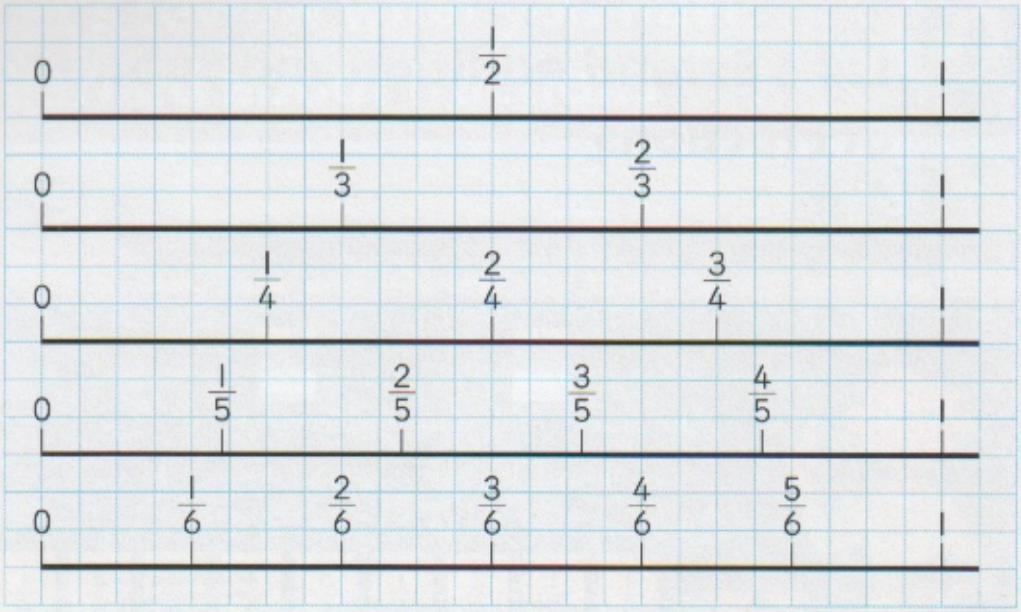
Let's think about how to add fractions with different denominators.


Riku: $\frac{1}{2}$ can be written as a decimal number, but...

Some fractions are the same size as $\frac{1}{2}$ and other fractions are the same size as $\frac{1}{3}$, so...


Ami

1 Use number lines to find some fractions that are the same size as $\frac{1}{2}$ and other fractions that are the same size as $\frac{1}{3}$. Then, find the fractions that have the same denominators.



$\frac{1}{2} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{1}{3} = \frac{\square}{\square}$

2 Explain how to calculate.



If you consider $\frac{1}{6}$ as the base amount...

$\frac{1}{2} + \frac{1}{3} = \frac{\square}{6} + \frac{\square}{6}$

$= \frac{\square}{6}$

Answer $\square$ L

3 Use the number lines above to explain how to calculate $\frac{2}{3} - \frac{1}{2}$.

Summary

You can do addition and subtraction of fractions with different denominators if you **change the fractions into equal size fractions with the same denominator.**

If you change fractions with different denominators into those with the same denominator, you can calculate using a base amount of $\frac{1}{\bullet}$.



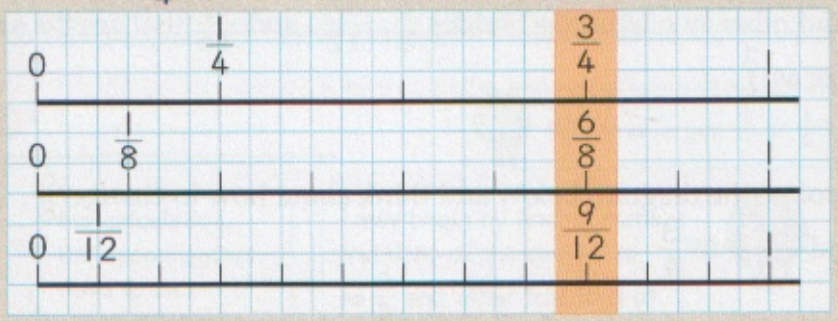
Riku: I wonder if we can find equal size fractions more easily.



Haruto

I wonder if there is any pattern that fractions of the same size have in common.

To find out about fractions of the same size, Haruto is listing fractions that are the same size as $\frac{3}{4}$.



$\frac{6}{8}$ and $\frac{9}{12}$ are the same size as $\frac{3}{4}$.

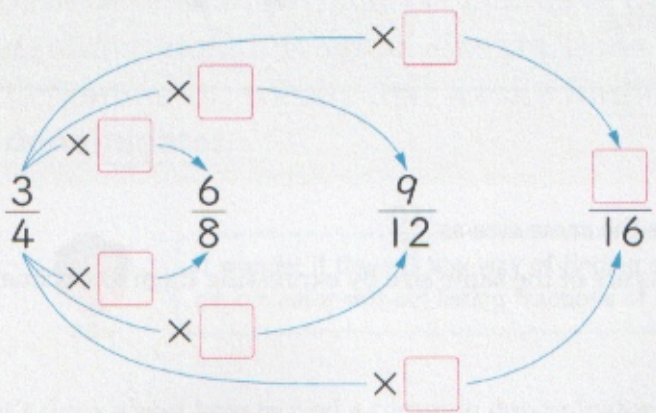
2

Find other fractions besides $\frac{6}{8}$ and $\frac{9}{12}$ that are the same size as $\frac{3}{4}$.

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{\square}{\square} = \frac{\square}{\square} \cdots$$

Let's think about how to find fractions that are the same size.

1 Look at $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$ and discuss what you notice.



$\frac{3}{4} = \frac{3\times\square}{4\times\square} = \frac{6}{8}$

$\frac{3}{4} = \frac{3\times\square}{4\times\square} = \frac{9}{12}$

$\frac{3}{4} = \frac{3\times\square}{4\times\square} = \frac{\square}{16}$

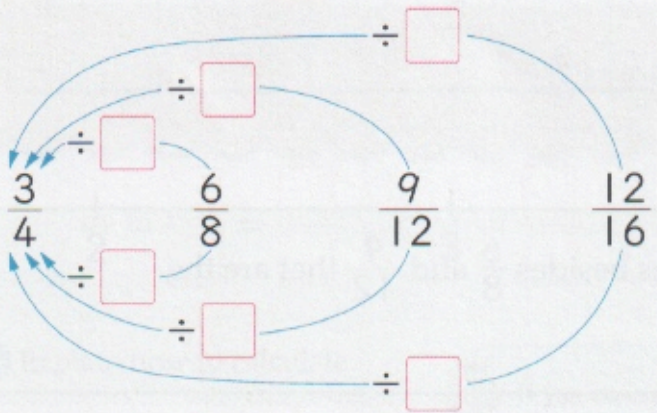


Shiho

2 Check to see if $\frac{12}{16}$ and $\frac{3}{4}$ are the same size by writing them as decimal numbers.

3 Find other two fractions besides $\frac{6}{8}$, $\frac{9}{12}$, and $\frac{12}{16}$ that are the same size as $\frac{3}{4}$.

4 Look at the diagram below and think about how to change $\frac{6}{8}$, $\frac{9}{12}$, and $\frac{12}{16}$ into $\frac{3}{4}$.



$\frac{6}{8} = \frac{6 \div \square}{8 \div \square} = \frac{3}{4}$

$\frac{9}{12} = \frac{9 \div \square}{12 \div \square} = \frac{3}{4}$

...



Misaki

Summary

The size of a fraction does not change if you multiply or divide both the denominator and the numerator by the same number.
If we use this property, we can make fractions of the same size.

$\frac{\bullet}{\blacksquare} = \frac{\bullet \times \blacktriangle}{\blacksquare \times \blacktriangle}$

$\frac{\bullet}{\blacksquare} = \frac{\bullet \div \blacktriangle}{\blacksquare \div \blacktriangle}$

1 Make 2 fractions that are the same size as $\frac{10}{8}$.
Check that the 2 fractions are of the same size by expressing them as decimal numbers.



Shiho

By comparing some fractions of the same size, we were able to find out a property of equal size fractions.

3

There is $\frac{3}{5}$ L of milk and $\frac{1}{4}$ L of milk. How many L is the difference between the two?

Math Sentence



Which is larger, $\frac{3}{5}$ or $\frac{1}{4}$?



Kota

For calculation, the denominators need to be...

I may begin by finding some fractions of the same size as $\frac{3}{5}$ and other fractions of the same size as $\frac{1}{4}$ and...



Ami

1 Make fractions that are the same size as $\frac{3}{5}$. Also, make fractions that are the same size as $\frac{1}{4}$. Then, find fractions from the two groups that have the same denominators.

$\frac{3}{5}$	$\frac{6}{10}$	$\frac{9}{15}$	$\frac{12}{20}$	$\frac{15}{25}$	$\frac{18}{30}$	$\frac{21}{35}$	$\frac{24}{40}$	$\frac{27}{45}$	$\frac{30}{50}$ ...

$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{4}{16}$	$\frac{5}{20}$	$\frac{6}{24}$	$\frac{7}{28}$	$\frac{8}{32}$	$\frac{9}{36}$	$\frac{10}{40}$ ...

$\frac{3}{5} = \frac{\square}{20} = \frac{\square}{40}$ $\quad$ $\frac{1}{4} = \frac{\square}{20} = \frac{\square}{40}$

When we change fractions that have different denominators into equal size fractions with the same denominator, we say that we are **finding a common denominator.**



Riku

If wonder if there is any way of finding a common denominator without listing fractions of the same size.

Let's think about how to find a common denominator.

2 Look at the results of 1 and discuss what you notice.

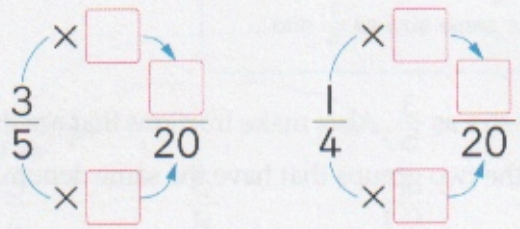
Denominators 20 and 40 are (...) of 5 and 4.



Haruto

Common denominators such as 20 and 40 are common multiples of the original denominators, 5 and 4.

3 Rewrite $\frac{3}{5}$ and $\frac{1}{4}$ to fractions with a common denominator of 20.



To change each of the denominators, 5 and 4, to 20...



Misaki

Summary

To find a common denominator, **find a common multiple of the original denominators.** Then, rewrite the fractions to those with the common denominator.

If you find the least common multiple, the common denominator will be the smallest.



4 Explain how to calculate $\frac{3}{5}-\frac{1}{4}$, and then find the answer.

$$\frac{3}{5}-\frac{1}{4}=\frac{\square}{20}-\frac{\square}{20}$$

$$=\frac{\square}{20}$$

We are going to rewrite the fractions to those with a common denominator of 20.
If we use $\frac{1}{20}$ as the base amount...



Riku

Answer $\square$ L



Ami

We should find a common denominator before calculating.



Kota

By comparing some fractions rewritten to have a common denominator, we were able to figure out how to find a common denominator.

4 Arrange $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{4}$ in ascending order.



Haruto: To compare the 3 fractions in size, their denominators need to be...

Let's think about how to find a common denominator of 3 fractions.



Kota: I wonder what is a common multiple of 2, 3, and 4...

$\frac{1}{2} = \frac{\square}{\square}$ $\frac{2}{3} = \frac{\square}{\square}$ $\frac{1}{4} = \frac{\square}{\square}$

You can find a common denominator of 3 fractions **by finding a common multiple of the denominators just like you did with 2 fractions.**



Shiho

If you find the least common multiple of the 3 denominators, the common denominator will be the smallest, just like the case with 2 denominators.

2 Compare the following fractions by finding a common denominator, and then write the appropriate inequality sign in each $\square$.

① $\frac{7}{9} \square \frac{5}{6}$ ② $\frac{27}{24} \square \frac{9}{8}$ ③ $2\frac{3}{10} \square 2\frac{4}{15}$

3 Rewrite the fractions in the ( ) to fractions with a common denominator.

① $\left(\frac{5}{3}, \frac{7}{4}\right)$ ② $\left(1\frac{1}{3}, 1\frac{2}{5}\right)$ ③ $\left(\frac{2}{3}, \frac{5}{12}\right)$

④ $\left(\frac{3}{2}, \frac{1}{4}\right)$ ⑤ $\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{5}\right)$ ⑥ $\left(\frac{4}{3}, \frac{7}{10}, \frac{8}{15}\right)$

4 ① $\frac{2}{3}+\frac{1}{4}$ ② $\frac{3}{7}+\frac{1}{2}$ ③ $\frac{7}{5}+\frac{2}{3}$ ④ $\frac{4}{3}+\frac{6}{5}$

⑤ $\frac{4}{5}-\frac{2}{3}$ ⑥ $\frac{7}{8}-\frac{3}{5}$ ⑦ $\frac{8}{7}-\frac{1}{2}$ ⑧ $\frac{5}{3}-\frac{9}{7}$



It is easier to tell the size of an answer if we change an improper fraction to a mixed number, isn't it?

Additional Problems → Page 136 A



Misaki: If we find a common denominator, we can add and subtract fractions with different denominators.

2 Simplifying Fractions and Addition/Subtraction of Fractions

1 Think about how to calculate $\frac{1}{3}+\frac{1}{6}$.

1 Explain the following two students' ideas.



Misaki

$$\frac{1}{3}+\frac{1}{6}=\frac{1\times6}{3\times6}+\frac{1\times3}{6\times3}$$
$$=\frac{6}{18}+\frac{3}{18}$$
$$=\frac{9}{18}$$



Haruto

$$\frac{1}{3}+\frac{1}{6}=\frac{1\times2}{3\times2}+\frac{1}{6}$$
$$=\frac{2}{6}+\frac{1}{6}$$
$$=\frac{3}{6}$$



Riku

Misaki's answer was $\frac{9}{18}$, and Haruto's was $\frac{3}{6}$. They expressed their answers differently...

Let's think about how to express answers.

The two answers are the same in size, so I wonder if both of them are okay.



Ami

Compared with $\frac{9}{18}$, $\frac{3}{6}$ has a smaller denominator and it's easier to tell its size.



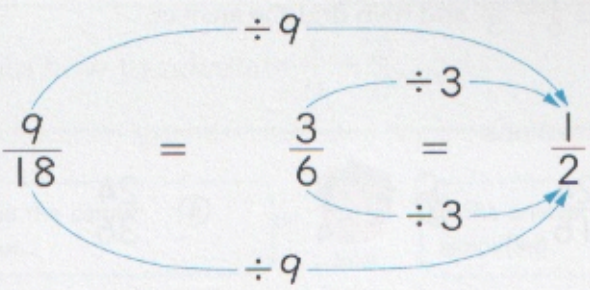
Kota



Shiho

Come to think of it, there is a fraction that has an even smaller denominator than $\frac{3}{6}$.

2 Find the fraction that is of the same size as $\frac{9}{18}$ and $\frac{3}{6}$ and has the smallest denominator.



When we divide both the denominator and the numerator by their common factor, we say we are **simplifying the fraction.**

$\frac{\overset{1}{\cancel{9}}}{\underset{2}{\cancel{18}}} \begin{matrix}\div 9\\ \div 9\end{matrix} = \frac{1}{2}$

$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}}} \begin{matrix}\div 3\\ \div 3\end{matrix} = \frac{1}{2}$

When simplifying a fraction, we usually try to make the denominator as small as possible.

It is easier to tell the size of a fraction when its denominator is smaller. So, we should simplify answers if possible to make their denominators as small as possible.

Ami

3 Kota and Shiho simplified $\frac{18}{24}$ as follows.

Explain the two students' ideas.

Kota

$\frac{\overset{3}{\cancel{18}}}{\underset{4}{\cancel{24}}} = \frac{3}{4}$

Shiho

$\frac{\overset{3}{\cancel{18}}}{\underset{4}{\cancel{24}}} = \frac{3}{4}$

Riku

To make a denominator as small as possible, we should repeat the process of finding a common factor of the denominator and the numerator and dividing the denominator and the numerator by the common factor.

Ami

Shiho (...) by the greatest common factor of the denominator and the numerator.

If you divide the denominator and the numerator by their greatest common factor, you can rewrite the original fraction to a fraction with the smallest denominator without repeating the process of finding a common factor.

1 Explain how to calculate $\frac{7}{6}-\frac{2}{3}$, and then find the answer.

2 Simplify the following fractions.

① $\frac{8}{12}$ ② $\frac{24}{16}$ ③ $2\frac{18}{24}$ ④ $\frac{24}{36}$ ⑤ $\frac{90}{15}$

3 Simplify the following fractions and find the ones that are the same size as $\frac{2}{3}$.

Ⓐ $\frac{4}{6}$ Ⓑ $\frac{6}{8}$ Ⓒ $\frac{9}{12}$

Ⓓ $\frac{10}{15}$ Ⓔ $\frac{12}{18}$ Ⓕ $\frac{12}{20}$

Ⓖ $\frac{15}{21}$ Ⓗ $\frac{16}{24}$ Ⓘ $\frac{20}{30}$

Ⓙ $\frac{30}{45}$ Ⓚ $\frac{32}{48}$ Ⓛ $\frac{50}{75}$



4 ① $\frac{5}{12}+\frac{1}{3}$ ② $\frac{5}{6}-\frac{1}{3}$



If an answer is a fraction, think about whether or not you can simplify it.



Simplifying Fractions and Properties of Division

Let's take a new look at simplifying fractions from the viewpoint of properties of division.

(Example) Simplifying $\frac{3}{12}$ into $\frac{1}{4}$

$\frac{3}{12}$ = 3 ÷ 12

Simplify ↓ ÷3 ÷3 Equal

$\frac{1}{4}$ = 1 ÷ 4

Simplifying a fraction does not change its size. One way you can see why that is the case is because dividing both the dividend and the divisor by the same number does not change the quotient.

Multiplying both the denominator and the numerator of a fraction by the same number does not change the size of the fraction. Take a new look at this fact from the viewpoint of properties of division.

2 Explain how to calculate $\frac{1}{6}+\frac{3}{8}$.



Riku: First, to use the same denominator...



Ami: If the answer can be simplified...

Let's review the process of calculation.

1 Explain the two students' ideas.



Shiho

$$\frac{1}{6}+\frac{3}{8}=\frac{1\times8}{6\times8}+\frac{3\times6}{8\times6}$$
$$=\frac{8}{48}+\frac{18}{48}$$
$$=\frac{26}{48}^{13}_{24}$$



Haruto

$$\frac{1}{6}+\frac{3}{8}=\frac{1\times4}{6\times4}+\frac{3\times3}{8\times3}$$
$$=\frac{4}{24}+\frac{9}{24}$$
$$=\frac{13}{24}$$

To Addition and subtraction of fractions with different denominators, **find a common denominator first and rewrite the fractions to those with the same denominator** before calculating. If the answer can be simplified, make the denominator as small as possible **so that it is easier to tell the size of the fraction**.



Misaki



Kota: Haruto rewrote the fractions using 24, the least common multiple of the denominators, 6 and 8.

5 ① $\frac{2}{5}+\frac{1}{10}$ ② $\frac{1}{4}+\frac{5}{12}$ ③ $\frac{7}{4}+\frac{1}{6}$ ④ $\frac{7}{6}+\frac{5}{8}$

⑤ $\frac{2}{3}-\frac{1}{6}$ ⑥ $\frac{4}{5}-\frac{3}{10}$ ⑦ $\frac{9}{8}-\frac{5}{6}$ ⑧ $\frac{13}{12}-\frac{5}{8}$

B 〈Practice〉

Additional Problems → Page 136 B

6 ① $\frac{2}{3}+\frac{1}{6}+\frac{2}{9}$ ② $\frac{1}{3}-\frac{1}{6}-\frac{1}{9}$



Riku: I wonder if we can add or subtract mixed numbers with different denominators.

3 Addition and Subtraction of Various Fractions

1 Let's think how to calculate $2\frac{3}{4}-1\frac{2}{3}$.

Let's think about how to calculate the mixed numbers.

1 Explain the two students' ideas.



Kota

$$2\frac{3}{4}-1\frac{2}{3}=2\frac{9}{12}-1\frac{8}{12}$$
$$=1\frac{1}{12}$$



Misaki

$$2\frac{3}{4}-1\frac{2}{3}=\frac{11}{4}-\frac{5}{3}$$
$$=\frac{33}{12}-\frac{20}{12}$$
$$=\frac{13}{12}$$

2 Check to see if $1\frac{1}{12}$ and $\frac{13}{12}$ are equal.

3 Calculate $1\frac{3}{5}+2\frac{1}{3}$ using the two students' ideas.



Shiho

First, $\frac{3}{5}$ and $\frac{1}{3}$ should be...



Riku

First, the mixed numbers should be [...] into improper fractions.

To add or subtract mixed numbers with different denominators you can either **find a common denominator with the whole numbers as they are** or **change the mixed numbers into improper fractions and then find a common denominator.**



Haruto

1 ① $3\frac{1}{2}+1\frac{1}{3}$ ② $1\frac{1}{2}+2\frac{1}{6}$ ③ $1\frac{1}{6}+\frac{3}{10}$

Additional Problems → Page 136 C

2 ① $2\frac{2}{5}-1\frac{1}{3}$ ② $3\frac{5}{6}-1\frac{1}{3}$ ③ $1\frac{7}{10}-\frac{1}{6}$

Additional Problems → Page 136 D

Shiho 

I want to think about addition and subtraction with fractions and decimal numbers.

2 Think about how to calculate $\frac{2}{5}+0.3$.

Since these numbers are expressed differently, we can't calculate as they are...



Haruto

Let's think about how to do calculations involving both fractions and decimal numbers.

1 Explain the two students' ideas.



Kota

If we write the decimal number as a fraction ...

$$\frac{2}{5}+0.3=\frac{2}{5}+\frac{3}{10}$$
$$=\frac{4}{10}+\frac{3}{10}$$
$$=\frac{7}{10}$$



Ami

If we write the fraction as a decimal number ...

$$\frac{2}{5}+0.3=0.4+0.3$$
$$=0.7$$

2 Think about how to calculate $\frac{2}{3}+0.5$.



Shiho

$$\frac{2}{3}=2\div 3$$
$$=0.666...$$



Riku

A method we can always use is ...

Summary

When we have both fractions and decimal numbers, we **change them to all fractions or all decimal numbers.** If the fractions cannot be expressed as exact decimal numbers, we change them all to fractions.

If we change numbers to fractions, we can always calculate, can't we?



3 ① $0.6+\frac{4}{5}$ ② $\frac{3}{10}-0.25$ ③ $\frac{1}{3}+0.75$ ④ $\frac{5}{7}-0.5$

Additional Problems
→Page 137 E

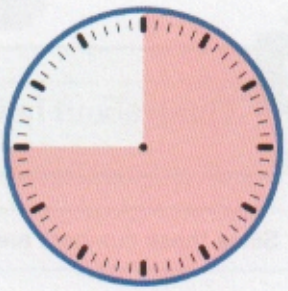


Misaki

If we can change numbers to decimal numbers, we don't need to find a common denominator or simplify a fraction.

4 Time and Fractions

1 How much of an hour is 45 minutes?



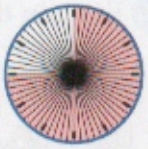
Let's think about how to express time using fractions.

1 Explain the three students' ideas.



Haruto

It is 45 of 60 equally divided parts of 1 hour, so it is $\frac{45}{60}$ of an hour.





Misaki

It is 9 of 12 equally divided parts of 1 hour, so it is ☐ of an hours.





Kota

It is ☐ of 4 equally divided parts of 1 hour, so it is ☐ of an hour.



2 Check to see if the fractions found by the three students are equal.

3 How much of a minute is 40 seconds?



We can express time using fractions by **dividing** 1 hour or 1 minute **into equal parts** and think about **how many parts there are in the time.**



Ami

1 Write the appropriate fractions in each ☐.

① 15 min = ☐ hr ② 40 min = ☐ hr ③ 5 min = ☐ hr

④ 48 sec = ☐ min ⑤ 90 min = ☐ hr ⑥ 100 min = ☐ hr

Check Your Understanding

1 Fill in the □ with the appropriate number.

① $\frac{7}{9}=\frac{\square}{18}=\frac{21}{\square}$ ② $\frac{54}{60}=\frac{27}{\square}=\frac{\square}{20}$

Can you make fractions of the same size?

Page 5 2

2 Rewrite the fractions in the ( ) to fractions with a common denominator.

① $\left(\frac{2}{3}, \frac{3}{5}\right)$ ② $\left(\frac{7}{6}, \frac{9}{10}\right)$

③ $\left(2\frac{1}{3}, 1\frac{1}{8}\right)$ ④ $\left(\frac{1}{3}, \frac{2}{5}, \frac{5}{6}\right)$

Can you find a common denominator?

Page 7 3
Page 9 4

3 There is $\frac{5}{8}$ L of juice and $\frac{5}{6}$ L of juice. How much juice is there altogether?

How many L is the difference between the two?

Based on a situation, can you make a math sentence and find the answer?

Page 7 3

4 Simplify the following fractions.

① $\frac{16}{18}$ ② $\frac{9}{24}$ ③ $\frac{45}{15}$ ④ $\frac{72}{60}$ ⑤ $2\frac{25}{100}$

Can you simplify fractions?

Page 10 1

5 ① $\frac{1}{2}+\frac{2}{5}$ ② $\frac{9}{5}+\frac{8}{15}$ ③ $1\frac{3}{8}+2\frac{1}{2}$

④ $\frac{7}{15}+2\frac{1}{3}$ ⑤ $\frac{3}{4}-\frac{1}{3}$ ⑥ $\frac{13}{12}-\frac{5}{6}$

⑦ $2\frac{5}{6}-\frac{3}{5}$ ⑧ $3\frac{7}{8}-1\frac{7}{10}$ ⑨ $1\frac{13}{14}-\frac{3}{7}$

⑩ $\frac{1}{2}+\frac{1}{3}+\frac{1}{9}$ ⑪ $\frac{3}{4}-\frac{3}{5}+\frac{1}{2}$ ⑫ $0.3+\frac{5}{6}$

Can you add or subtract fractions?

①⑤ Page 7 3
②⑥ Page 10 1
③④⑦⑧⑨ Page 14 1
⑩⑪ Page 13 2
⑫ Page 15 2

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Base Amount and Think about How to Calculate

$\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$

Since $2+3=5$, there are 5 pieces of $\frac{1}{7}$.

$0.2+0.3=0.5$

Since $2+3=5$, there are 5 pieces of 0.1.

Fractions with a numerator of 1, such as $\frac{1}{7}$, are called "unit fractions."



Shiho: We were able to calculate both of the two calculations above as $2+3$, because we thought about how many pieces of base amount there were.

Kota is explaining the reason why he needs to find a common denominator to calculate $\frac{2}{5}+\frac{3}{7}$.

Fill in the $\square$ with the appropriate number.



Kota: We can't simply calculate $\frac{2}{5}$ and $\frac{3}{7}$ because they are based on different unit fractions: $\frac{1}{5}$ for $\frac{2}{5}$ and $\frac{1}{7}$ for $\frac{3}{7}$. So...

Find a common denominator of $\frac{2}{5}$ and $\frac{3}{7}$ and make them $\frac{14}{35}$ and $\frac{15}{35}$.



Kota: Now they are based on the same unit fraction: $\frac{1}{\square}$.

$$\frac{2}{5}+\frac{3}{7}=\frac{14}{35}+\frac{15}{35}$$
$$=\frac{29}{35}$$



Kota: You just have to find how many pieces of $\frac{1}{\square}$ there are.

Look back on what you have learned in "Let's Extend Addition and Subtraction of Fractions" and discuss.



Misaki: We were able to add or subtract fractions with different denominators by finding a common denominator and rewriting the fractions to those with the same denominator.



Haruto: I also want to think about multiplication and division of fractions.



You'll study that topic in 6th grade.

Challenge Yourself
→ Page 144

Do You Remember?

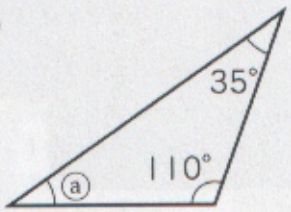
Answers→ Page 149

1 Order the numbers on the right from the smallest to the largest.

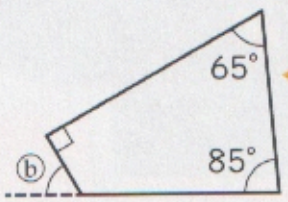
$1\frac{7}{8}$ 1 $\frac{5}{7}$ 0 0.65

2 What are the measures of angles ⓐ and ⓑ below? Calculate to find their measures.

①

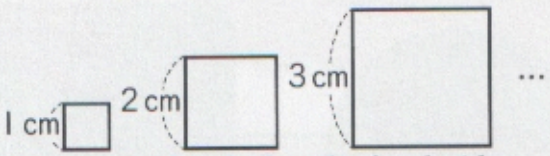


②



Sum of the Angles of a Triangle or a Quadrilateral
Page 151 ⑧

3 We increase the length of each side of a square from 1 cm to 2 cm, 3 cm ... as shown in the diagram on the right.



Is the length around the square, ○cm, proportional to the length of each side, □cm? Find the answer by writing the appropriate numbers in the table below.

Proportional Relationships
Page 151 ⑩

Each side of square □(cm)	1	2	3	4	5	6	7	8
Length around square ○(cm)								

4 There is a 1.2 m wire that weighs 9.6 g.

① How many g will 1 m of this wire weigh?

② How many m will 1 g of this wire be?

Warm-up

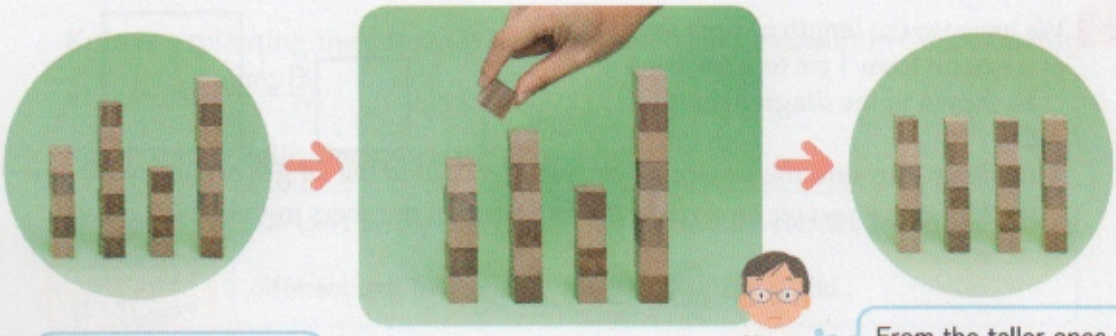
5 Which way of sharing juice will give each person more juice?

Ⓐ Sharing 900 mL of juice among 6 people

Ⓑ Sharing 800 mL of juice among 5 people

What does it mean to "Even something out"?





Haruto

From the taller ones to the shorter ones move...

When we make a group of items that have different sizes into items that are the same size we say we are "evening out" the items.



Can we even out the amounts of juice above? Discuss with your classmates.

Riku

From higher levels to lower levels... We can look at the levels and roughly even them out.

Shiho

To even them out exactly...

Average

11 Let's Think about Evened Out Amounts



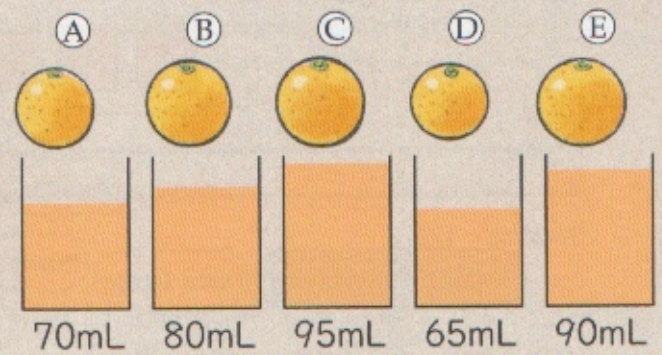
20 pieces



Misaki

I wonder how much juice there will be if we squeeze all of them.

We squeezed 5 oranges.



1 We squeezed 5 oranges, and the results are as shown above. How many mL of juice do you think we can squeeze from one orange?



Kota

The most juice produced was 95 mL, and the least was 65 mL...

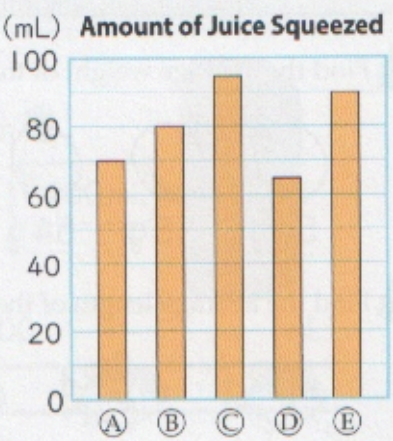
1 Look at the graph on the right. Use it to even out the amount of juice that was squeezed from the oranges.
What is the amount in mL after we even it out?



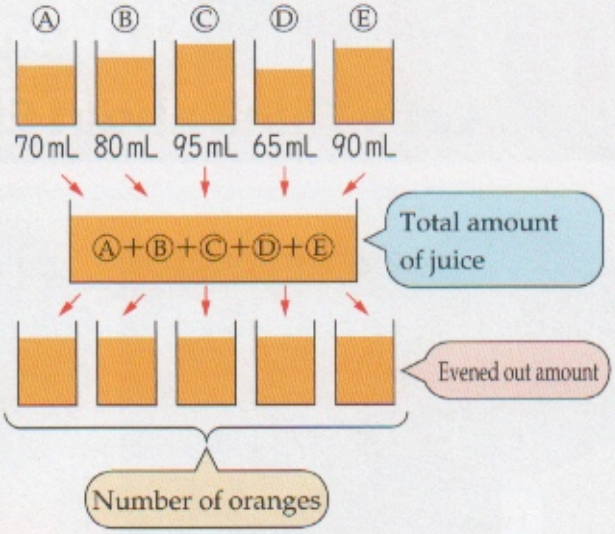
Ami

It would be tedious to make a graph for an answer every time.

Let's think about how to calculate the evened out amount.



2 Look at the diagram on the right and explain how to calculate the evened out amount.



Total amount of juice | Number of oranges | Evened out amount

$(70+80+95+65+90) \div 5 =$ ☐ Answer ☐ mL

When a number of quantities are evened out so that they are all equal, we call the evened out amount their **average**.

Summary

You can calculate the average by **finding the total amount and dividing it by the number of quantities.**

Average = Total ÷ Number of Quantities

1 Find the average weight of the eggs below.



2 Find the average length of the fish below.





Is there any fish that has exactly the same length as the average?

Additional Problems
→Page 137 F



Haruto: The way of calculating for the average is the same as division for finding the amount for each.

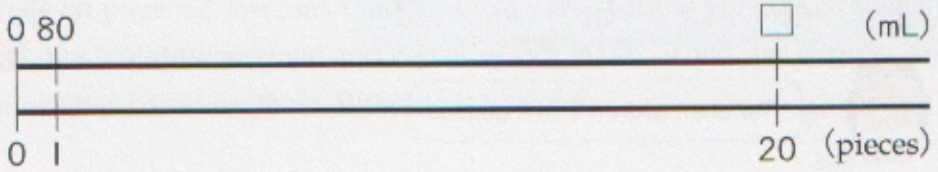
2 How many mL of juice do you think we can get if we squeeze all 20 oranges in problem 1?



Let's think about how to estimate the total amount.

The average amount of juice we can squeeze from one orange is 80 mL...


Misaki



$80 \times \square = \square$

Answer □ mL

Summary

We can estimate the total amount if we use the average.

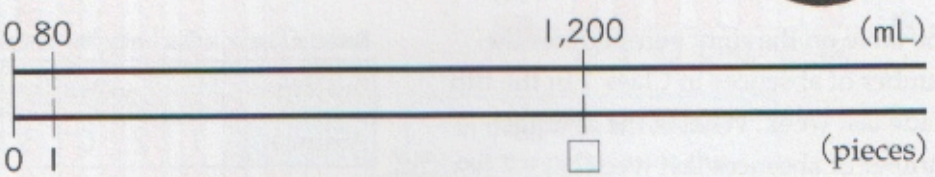
We are assuming we can squeeze 80 mL from each of the oranges, aren't we?

3 For the last month, Hiroko ran 2 km a day on average.
If she keeps running this way for a year, how many km will she run in a year?

Assume the year has 365 days.

4 The water bottle on the right can hold 1,200 mL.
We are going to put the orange juice from problem 1 in this water bottle.
How many oranges do we need to squeeze?






Ami

Using the formula of average = total ÷ number of quantities, the math sentence in 2 can be written as 80 = 1,600 ÷ 20.

3

The numbers below show the number of goals Emi's soccer team scored in the last 6 games.
What is the average number of goals in the last 6 games?



1, 4, 0, 5, 3, 2



Shiho: The team scored 0 in a game...

Let's think about what to do with the game with a score of 0.

1 Discuss the following two student's ideas.



Ami

$(1+4+5+3+2)\div 5=3$

Answer 3 goals



Kota

$(1+4+0+5+3+2)\div 6=2.5$

Answer 2.5 goals

Since it's the average of the last 6 games, 0 should be...



Haruto



Misaki

With Kota's idea, the number of goals becomes a decimal number...

Since **the purpose is to find the average of the last 6 games**, the score of 0 should be included in the number of goals.



Riku

Even though the actual quantities are not decimal numbers, as in soccer scores, we can express the average as a decimal number.

5 The table on the right summarizes the number of absences in Class 1 in the 5th grade last week. What is the average number of absences last week?

Number of Absences in Class 1 in the 5th Grade Last Week

Days of the week	Mon	Tue	Wed	Thurs	Fri
Absences	1	2	0	3	2

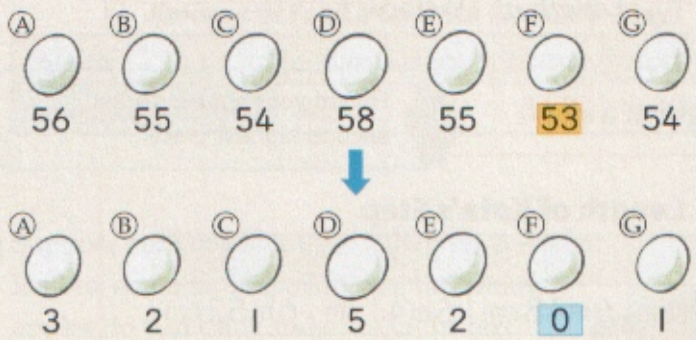


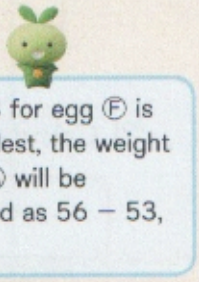
Shiho: It looks like the average can be used in many different situations.

Tentative Average

To calculate an average, we first find the total of the quantities.
However, if the quantities are very large or there is a large amount of data, finding the total can be tedious.
In these situations, the calculation of the average may be simplified by using the following procedure.

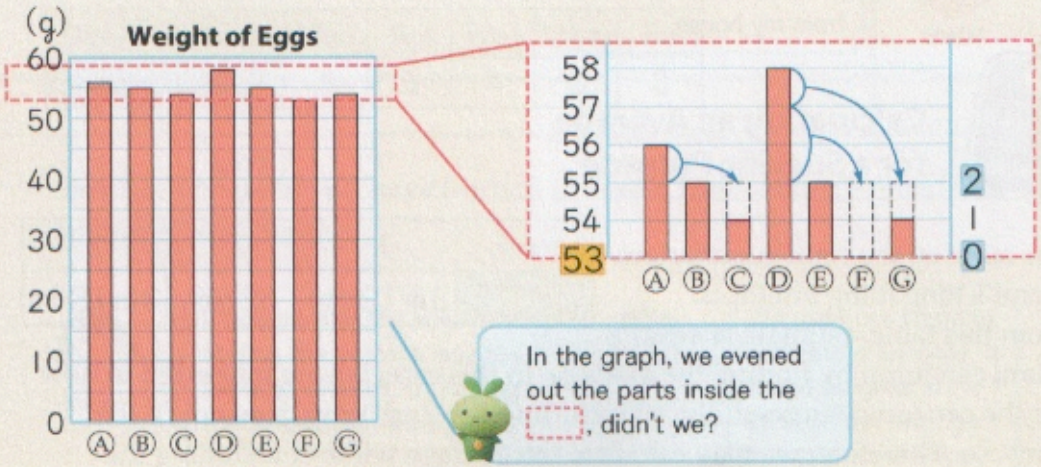
❶ In 4 on page 22, we make the smallest value for the weights of the eggs, 53, the tentative average and consider 53 as 0. Then, we express other quantities based on their differences with 53.





❷ Find the average of the values we determined in step ❶
$(3+2+1+5+2+0+1) \div 7 = 2$

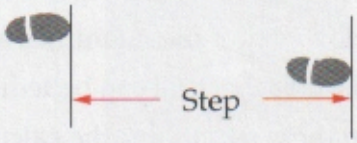
❸ Add the average obtained in step ❷, 2, to the tentative average of 53.
This sum is the correct average.
$53+2=55$



Use What You Have Learned

Let's estimate a distance by using your steps.

① Determine the length of your step.

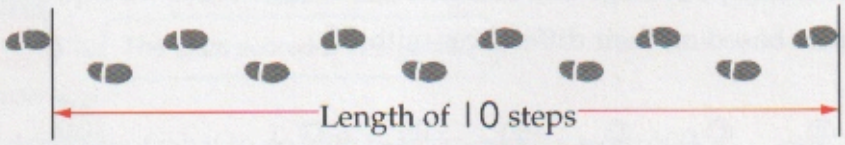



Kota

My steps are not always the same length, so...


Measure it 3 times for this lesson.

(1) Measure the lengths of 10 steps several times.



(2) Calculate the length of a step.


Round your answer to the second highest place.

Length of Kota's Step

Attempt	1	2	3
Length of 10 steps	6 m 48 cm	6 m 41 cm	6 m 52 cm

- Average of 3 attempts···$(6.48+6.41+6.52)\div 3=6.47$(m)
- Step····· $6.47\div 10=0.647$ about 0.65 m

② Walk around and measure various distances.


Misaki

I wonder how far the evacuation center is from my house.


Since we rounded the length of a step to the second highest place, let's round the distances to the second highest place, too.



Calculating an Average for a Specific Purpose

Foul

Attempt	1	2	3
Measured distance	3 m 12 cm	62 cm	3 m 18 cm

The table on the right summarizes Mami's long jump attempts. From this table, estimate how far Mami can jump by finding the average. In this case, we are interested in how far she can jump successfully, so including her 2nd jump does not fit the purpose. Therefore, we may calculate the average without including the second jump.

Check Your Understanding

1 Makoto's family generated 3.5 kg of combustible trash in a week.
How much combustible trash does Makoto's family generate in a day on average?

Can you find the average?

Page 21 1

2 The table below shows the amount of feed a cow ate between January and May.
If the cow eats the same amount of feed for the rest of the year, how much feed will this cow eat in a year?

Can you find the total amount by using the average?

Page 23 2

Amount of Feed a Cow Ate (January–May)

Month	January	February	March	April	May
Amount of feed (kg)	450	480	360	420	390

3 Suppose each apple weighs 300 g on average.
If a box contains 6 kg of apples, how many apples do you think there are in the box?



Can you find the number of pieces by using the average?

Page 23 2

Units of Weight
Page 151 ⑪

4 The table below summarizes the number of people in Class 2 in the 5th grade who borrowed library books last week. What is the average number of people who borrowed books in a day?

Can you find the average of amounts that include 0?

Page 24 3

Number of People Who Borrowed Books Last week, Class 2 in the 5th grade

Days	Mon	Tue	Wed	Thurs	Fri
Number of people	6	5	4	0	3

Look back on what you have learned in "Let's Think about Evened Out Amounts" and discuss.


Misaki

I thought that the way of finding the average was the same as division for finding the amount for each.


Riku

We used our steps to estimate a distance. It would be great if we can use the average in many other ways.

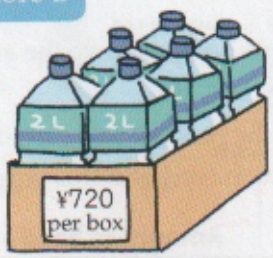
Can you make a comparison?

Two stores sell the same brand of water. Which store offers a better bargain?

Ⓐ Store A

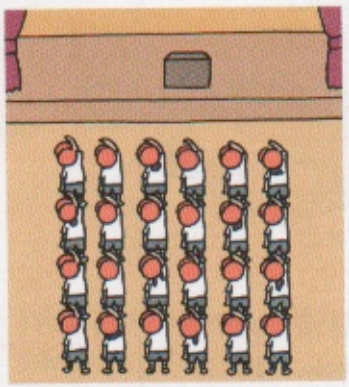


Ⓑ Store B

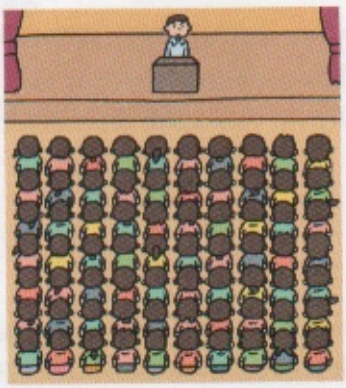


People are gathered in two places. Which place is more crowded?

Ⓒ



Ⓓ



Discuss your reasoning about the two situations above with classmates.



Misaki: Ⓐ and Ⓑ are the same in the number of bottles, but the prices...



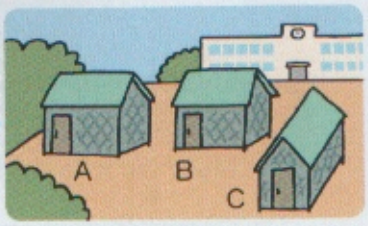
Kota: Ⓒ and Ⓓ are the same in the area, but...

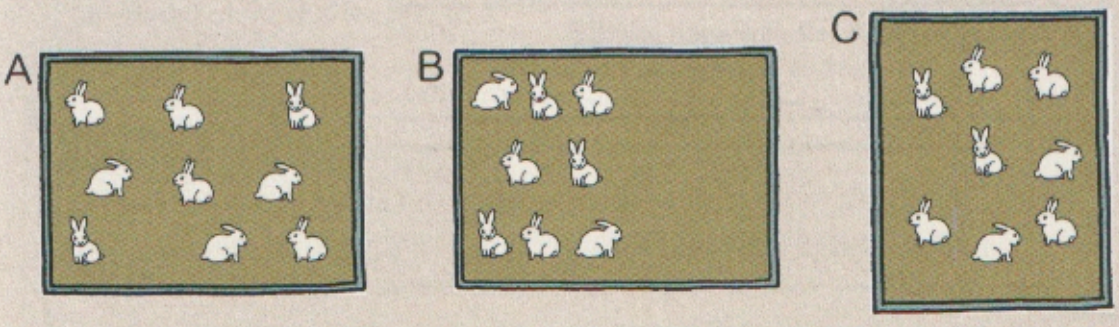


Riku: In each of the two situations above, the number of bottles or the area is the same, so you can simply make a comparison by focusing on the price or the number of people, but...

Per Unit Quantity

12 Let's Think about How to Compare (1)





1 Crowdedness

1 Investigate how to order cages A, B, and C according to how crowded they are.


Ami: I wonder if there is any difference in crowdedness.


Shiho: In B, the rabbits are gathered in one place.


Haruto: I wonder if A and B have the same area.

Let's investigate how to compare crowdedness.

1 To compare crowdedness, what quantities do we need to know?

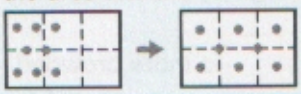
2 Which rabbit cage is more crowded, A or B?


Kota: Since A and B have the same area...

Area of Cage and Number of Rabbits

	Area (m^2)	Number of rabbits
A	6	9
B	6	8
C	5	8

Even if the rabbits were gathered in one place as shown in B, by evening them out, we can still compare using the area and the number of rabbits, can't we?



3 Which rabbit cage is more crowded, B or C?

4 Which rabbit cage is more crowded, A or C?

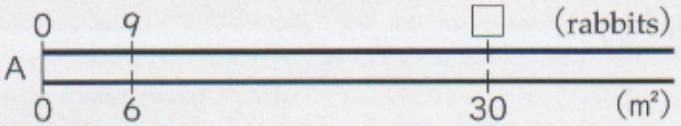

Ami: Both the areas and the numbers of rabbits are different.


Riku: If one of them is the same, ...



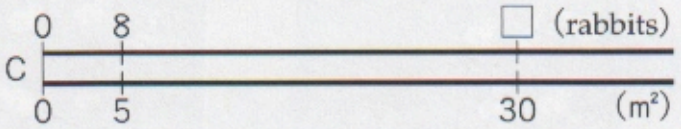
Kota

Make the area the same by using a common multiple of 5 and 6, 30.



$30 \div 6 = 5$

$9 \times 5 =$ □ (rabbits)



$30 \div 5 = 6$

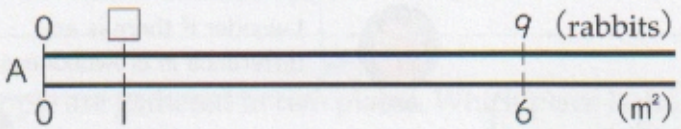
$8 \times 6 =$ □ (rabbits)

□ is more crowded.

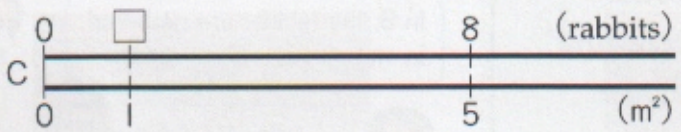


Shiho

Compare based on the number of rabbits in 1 m².



$9 \div 6 =$ □ (rabbits)



$8 \div 5 =$ □ (rabbits)

□ is more crowded.

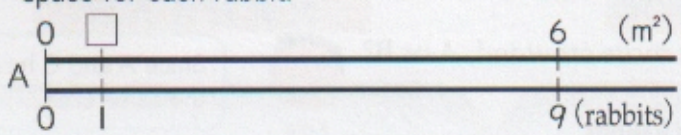


Haruto

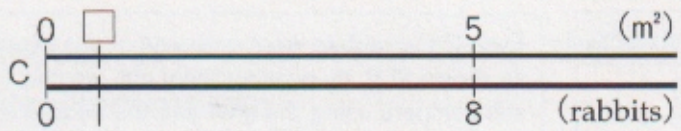
Compare based on the amount of space for each rabbit.



If you can't divide completely, round to the second highest place.



$6 \div 9 =$ □ (m²)



$5 \div 8 =$ □ (m²)

□ is more crowded.

We can compare crowdedness **if you make either the area or the number of rabbits the same.** If you use a double number line diagram, you can easily tell the relationship between the area and the number of rabbits.



Ami



Riku

I wonder which ideas will always work well.

5 The area of cage D and the number of rabbits in there are as shown on the right. Order the cages A, C, and D based on how crowded they are.

Area of Cage and Number of Rabbits

	Area (m^2)	Number of rabbits
D	9	14

If you use a common multiple, you always have to find one.



Riku

Summary

Even if either the area or the number of rabbits is not the same, you can compare crowdedness **by finding the average number of rabbits in 1 m^2 or the average area for 1 rabbit.**



Crowdedness is usually expressed by the quantity in 1 m^2 (the unit for area). That's because the larger number will mean it's more crowded.

A quantity expressed using two quantities are called "per unit quantity." Examples of per unit quantity include the average number of rabbits in 1 m^2 and the average area for 1 rabbit.

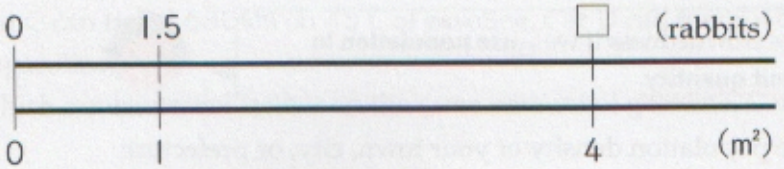
1 Cage A in 1 is as crowded as cage E. How many rabbits are there in cage E?

Area of Cage and Number of Rabbits

	Area (m^2)	Number of rabbits
E	4	□



Cage A has 1.5 rabbits in 1 m^2. The area of cage E is 4 m^2. Cage E is 4 times as large as 1 m^2, so the number of rabbits is...





Misaki

I wonder for what else can we use the per unit quantity.

2 Various Per Unit Quantities

1 Compare the crowdedness of Hokkaido and Okinawa.



Let's think about how to compare.

1 To compare crowdedness, what quantities do we need to know?

The population per unit area is called the "population density."
We use population density to express the crowdedness of cities, prefectures, and countries.

Population density is generally expressed as population in 1 km².

2 Find the population density of Hokkaido and Okinawa.

Round the answers to the second highest place.

Area and Population of Hokkaido and Okinawa (2018)

	Area (km²)	Population (ten thousands)
Hokkaido	83,424	534
Okinawa	2,281	147

(Developed by Tokyo Shoseki based on 2018 Statistical Reports on the Land Area by Prefectures and Municipalities in Japan (the Geospatial Information Authority of Japan) and the Basic Resident Register (as of Jan. 1, 2018, the Ministry of Internal Affairs and Communications))

Hokkaido ····· 5,340,000 ÷ 83,424 = ☐

Okinawa ····· 1,470,000 ÷ 2,281 = ☐

We can compare crowdedness if we **use population in 1 km² as a per unit quantity.**

Misaki

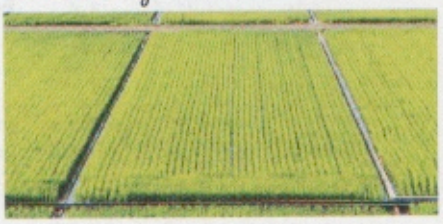
1 Investigate the population density of your town, city, or prefecture.

Haruto: Some parts of the area are not populated, such as in the mountains and on lake surfaces, but these parts are included in the area, so population is considered evened out.

A 570 kg



B 680 kg



Rice fields A and B grow the same type of rice.

2 Which of the two rice fields had a better harvest, A or B?



Kota: We can't simply compare the amounts of rice harvested.

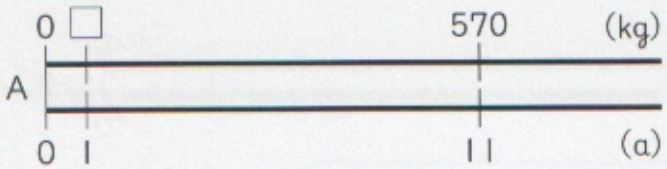
Let's think about how to compare.

1 To compare harvests, what quantities do we need to know?

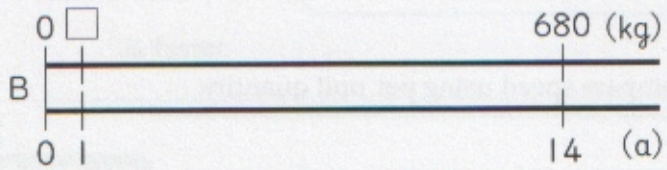
2 Look at the table on the right and find the amount of rice harvested in 1 a of each field, A and B.

Area and Harvest of Rice Fields

	Area (a)	Harvest (kg)
A	11	570
B	14	680



$570 \div 11 = \square$ (kg)



$680 \div 14 = \square$ (kg)

Round the answers to the second highest place.

We can compare the rice fields in harvest if we **use the amount of harvest in 1 a as a per unit quantity.**

Kota

2 Car C can travel 630 km on 45 L of gasoline. Car D can travel 480 km on 30 L of gasoline.
Which car can travel farther on the same amount of gasoline, C or D?

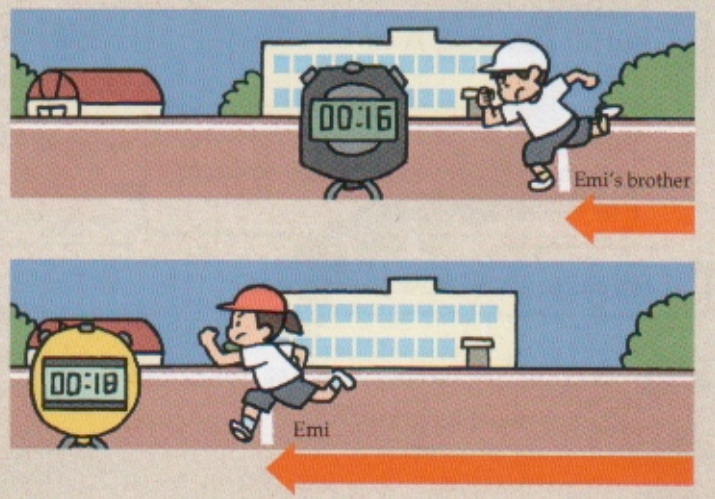
Additional Problems
→Page 137 G

Shiho: It is important to focus on two quantities when using the per unit quantity.

3 Speed

Emi and her brother ran sprints. Their times are as shown below.

	Time (sec)
Emi's brother	16
Emi	18



1 Who ran faster in the sprints, Emi or her brother?


Shiho

We can't simply compare them using time.

Let's think about how to compare speed using per unit quantity.

1 To compare speed, what quantities do we need to know?

2 The table on the right shows Emi's time, her brother's time, and the distances they each ran. Look at this table and find out which one ran faster.

Time and Distance

	Time (sec)	Distance (m)
Emi's brother	16	80
Emi	18	100

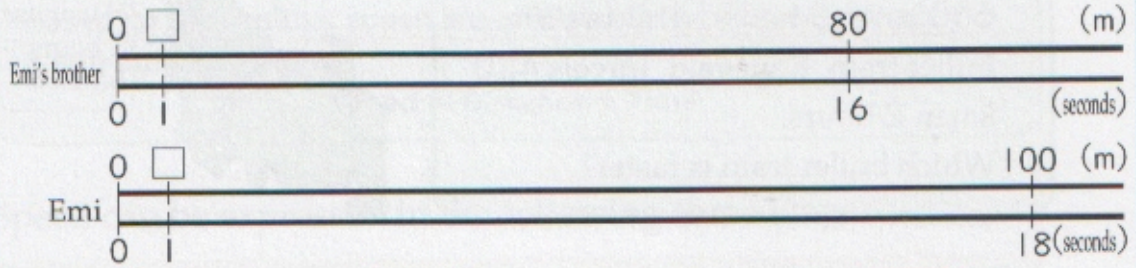
You can't run at a constant speed from start to finish, but if we assume they kept running at the average speed...


Kota



Misaki

I compared how far they ran in 1 second.



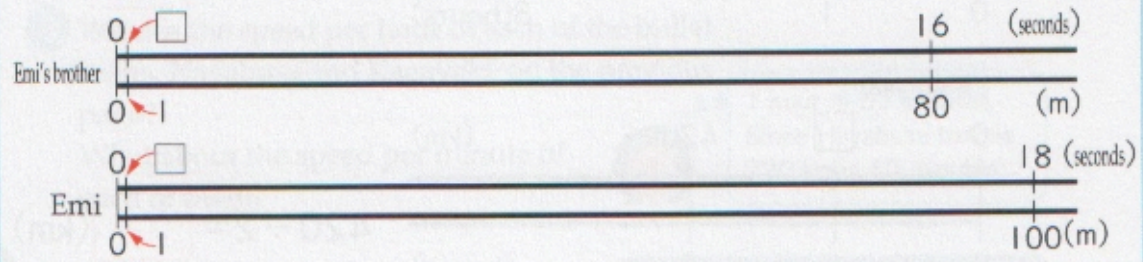
Emi's brother···80 ÷ 16 = ☐ (m) Emi···100 ÷ 18 = ☐ (m)

☐ is faster.



Haruto

I compared how long it took to run 1 m.



Emi's brother···16 ÷ 80 = ☐ (seconds) Emi···18 ÷ 100 = ☐ (seconds)

☐ is faster.

Summary

We can compare speed if we **use a per unit quantity, for example, the average distance run in 1 second or the average time it took to run 1 m.**

Crowdedness ··· We evened out the population and assumed it was evenly distributed.
Speed ··· We assumed they ran at a constant speed from start to finish.
In both cases, we evened out the amounts and made a comparison based on a per unit quantity.

3 Which student's method gives a greater number for a faster runner, Misaki's or Haruto's?

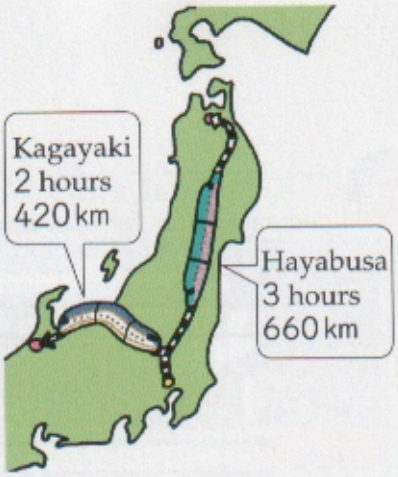


Ami

We can compare speed if we use two quantities, time and distance, to find a per unit quantity.

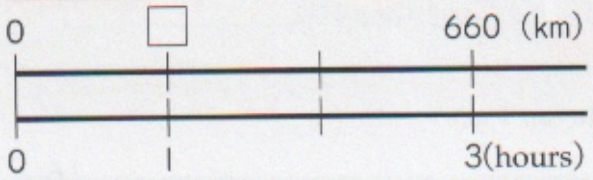
2

A bullet train, Hayabusa, travels 660 km in 3 hours, while a different bullet train, Kagayaki, travels 420 km in 2 hours.
Which bullet train is faster?



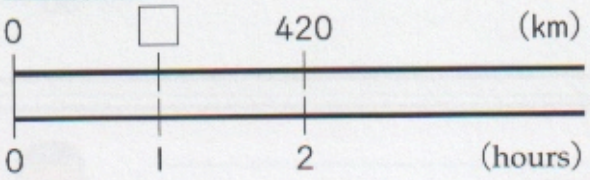
1 Let's compare the distances traveled in 1 hour.

Hayabusa



$660 \div 3 = \square$ (km)

Kagayaki



$420 \div 2 = \square$ (km)


Riku

Since ☐ travels further in 1 hour, ☐ is faster, isn't it?

Speed is expressed using the distance traveled in a unit amount of time.

I wonder if we can make a math sentence for calculating speed.


Shiho

Let's make a math sentence for calculating speed.

2 When we calculated speed in 1, what did we divide by what?


Haruto

We divided ☐ by ☐.

Summary

We can make a math sentence for calculating speed as follows **based on the fact that speed is expressed as the distance traveled in a unit amount of time.**

Speed = Distance ÷ Time

Speed can be expressed in the following three ways.

speed per hour ····· the speed expressed as the distance traveled **in 1 hour**

speed per minute ····· the speed expressed as the distance traveled **in 1 minute**

speed per second ····· the speed expressed as the distance traveled **in 1 second**

3 What is the speed per hour of each of the bullet trains, Hayabusa and Kagayaki, on the previous page?
What about the speed per minute of each of them?


Ami

1 hour = 60 minutes.
Since Hayabusa travels 220 km in 60 minutes...

1 The sailfish is the fastest fish in the sea. Calculate the speed of a sailfish that can swim 360 km in 4 hours using hours as the unit of time. Also, find the speed per minute and per second.



1 hour = 60 minutes
1 minute = 60 seconds

Additional Problems
→Page 137 H


Kota

When you calculate speed, it is important to think about which unit amount of time to use.



How to Express Speed

Speed per hour, minute, or second can also be expressed as follows.

(Example) 220 km per hour ⟷ 220 km an hour
220 m per minute ⟷ 220 m a minute
220 mm per second ⟷ 220 cm a second

3

A swallow can fly at a speed of 70 km per hour. Determine the distance a swallow can fly in 3 hours.

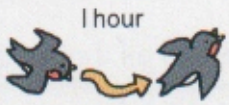


Let's think about how to determine the distance traveled from the speed and the time.



Haruto

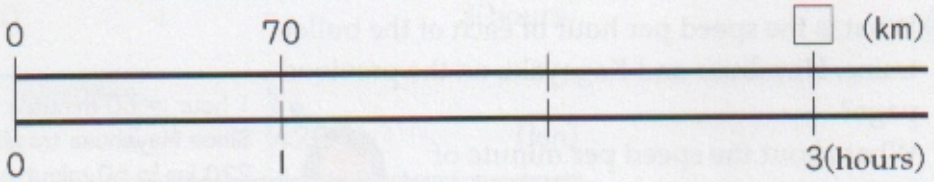
Since the speed is 70 km per hour...





Ami

A swallow travels 70 km in 1 hour. When the time increases to 3 times, the distance increases to...



1 Write a math sentence to find the distance, and then find the answer.

Math Sentence ▸ _____ Answer □ km

2 Apply the above swallow's speed, the time, and the distance you found into the math sentence, Speed = Distance ÷ Time, and check if the math sentence is true.

Summary

Distance may be calculated using the following formula.

Distance = Speed × Time

This formula expresses the same relationship as Speed = Distance ÷ Time.

2 How far can a seagull that can fly at the speed of 800 m per minute fly in 5 minutes?

Additional Problems → Page 137

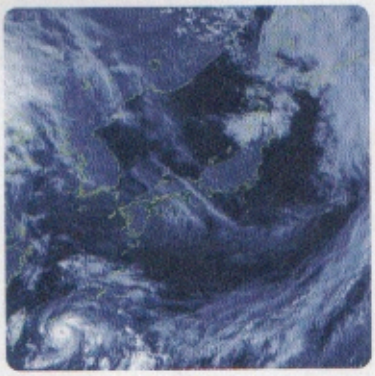


Misaki

To calculate the time...

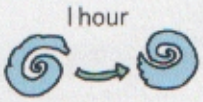
4

A typhoon is moving at a speed of 25 km per hour. How long will it take for this typhoon to travel 400 km, the distance between Ishigaki Island and Naha City?




Riku

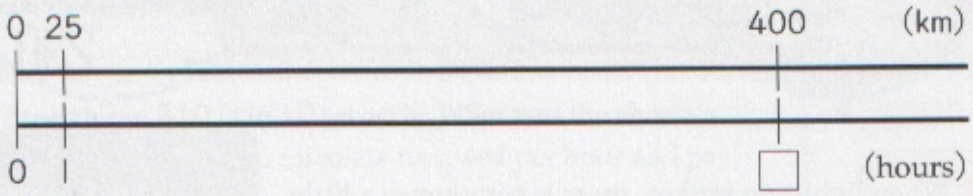
Since it is moving at 25 km per hour




Shiho

Since it travels 25 km in 1 hour, it travels 400 km in ☐ hours.

Let's think about how to determine the time from the speed and the distance.



1 Write a multiplication sentence by considering the time as ☐ hours. Then, find the number that should go in ☐.

$25 \times \square = 400$

$\square = 400 \div 25$

$= \square$

If we use the formula we learned before...


Kota

Answer ☐ hours

Since the relationship between the three quantities does not change, we can use the formula on the previous page here.


Ami

3 How long will it take for a person who walks at a speed of 65 m per minute to travel 2.6 km?

1 km = 1,000 m
So, 2.6 km is equal to ☐ m.

〈Practice〉

Additional Problems
→Page 137 J

Haruto

If we know two quantities out of three: speed, distance, and time, you can find the unknown third quantity.

Use What You Have Learned

● Let's find various situations around you where per unit quantities are used.



● When lightning strikes, there is sometimes a little time gap between a flash and a crash or roar of thunder.
This gap has something to do with the traveling speed of sound.



① The speed of sound in air is known to be approximately 0.34 km per second. You heard thunder 10 seconds after you saw a flash of lightening. About how many km away were you from the lightening strike?

In both ① and ②, assume you saw the lightening strike at the exact moment of the lightening flash.

② If you were within 1 km from the lightning strike, within how many seconds would you hear the thunder after seeing the lightning flash? Round the answer to a whole number.

If you hear thunder, evacuate to a safe place, however small the sound may be.

Check Your Understanding

1 There are 740 students at Midori's school. There is 21 m^2 of playground area for each student.

① What is the area of the playground?

② Next year, it is anticipated that the number of students will decrease by 20.
What will be the approximate amount of playground area for each student next year?

Round your answer to the second highest place.

Can you find area by using a per unit quantity?
Page 32 1

2 Which pencils are more expensive, one that sells for 600 yen for a dozen, or one that sells for 450 yen for 10?

1 dozen means 12.



Can you make a comparison using a per unit quantity?
Page 33 2

3 A cheetah ran 310 m in 10 seconds. What was the cheeta's speed per second? Also calculate its speed per hour and per minute.

Can you tell what speed is?
Page 36 2

4 An express train travels at a speed of 96 km per hour.
How far can this express train travel in 2 hours?

Can you calculate distance?
Page 38 3

5 A bicycle is traveling at 3 m per second. How many m does it travel in 30 minutes?

Can you calculate distance?
Page 38 3

Misaki: 60 seconds = 1 minute.
So, if I change 3 m per second into the speed per minute...

Kota: Since 1 minute = 60 seconds,
30 minutes will be ☐ seconds.

6 The evacuation center is 780 m from your house.
If you walk at 65 m per minute, how long does it take for you to get there?

Can you calculate time?
Page 39 4

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Relationship between Two Quantities and Think about How to Make a Comparison Using Per Unit Quantity

What do Riku's and Ami's ideas have in common? Fill in the ☐ with the appropriate words.



Riku

Ⓐ  4 m² 6 people

Ⓑ  5 m² 10 people

We are comparing the crowdedness in two elevators.

Ⓐ $6 \div 4 = 1.5$(people)

Ⓑ is more crowded.

Ⓑ $10 \div 5 = 2$(people)



Ami

Ⓒ  Travels 120 km in 3 hours

Ⓓ  Travels 60 km in 2 hours

We are comparing two cars in speed.

Ⓒ $120 \div 3 = 40$(km)

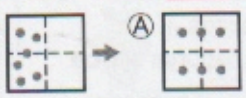
Ⓒ travels faster.

Ⓓ $60 \div 2 = 30$(km)



Shiho

For comparison of both crowdedness and speed, we are assuming the amounts are ☐.

Ⓐ  Ⓐ



Haruto

We are making a comparison using per unit quantity based on a combination of two quantities: area and ☐ for comparison in crowdedness; and distance and ☐ for comparison in speed.

Look back on what you have learned in "Let's Think about How to Compare (1)" and discuss.



Misaki

We used per unit quantity for comparing both crowdedness and speed.



Kota

I want to find more situations where per unit quantity is used.

Do You Remember?

Answers→ Page 149

1 ① $\frac{5}{6}+\frac{3}{4}$ ② $\frac{1}{15}+\frac{1}{3}$ ③ $1\frac{2}{7}+2\frac{2}{5}$ ④ $0.8+\frac{4}{9}$

⑤ $\frac{2}{3}-\frac{2}{9}$ ⑥ $\frac{5}{6}-\frac{1}{2}$ ⑦ $3\frac{7}{8}-\frac{5}{12}$ ⑧ $\frac{3}{4}-0.4$

2 Use four out of the cards on the right to make the following numbers.

① The largest number

② The smallest number

1	2	6
0	0	.

(Decimal point)

3 There are three jogging trails: A, B, and C. Trail A is 2.8 km long. Trail B is 1.2 times as long as trail A. Trail C is 0.8 times as long as trail A. How many km are trails B and C?



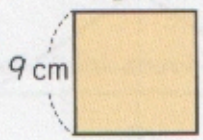
Times As Much
Page 150 ①

Warm-up

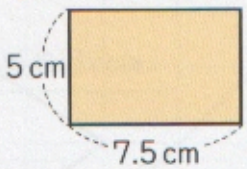
4 Calculate the area of the following figures.

How to Express Area and Formulas for Area
Page 151 ⑥

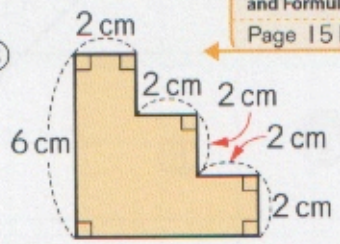
① Square



② Rectangle



③



Playing with Numbers and Calculations

Addition Puzzle

Fill in the blanks with numbers so that the sum of the three numbers in any column, row, or diagonal is equal.

①

1.3	Ⓐ	0.5
Ⓑ	1	Ⓒ
1.5	Ⓓ	Ⓔ

Ⓐ – Ⓔ will have decimal numbers in them.

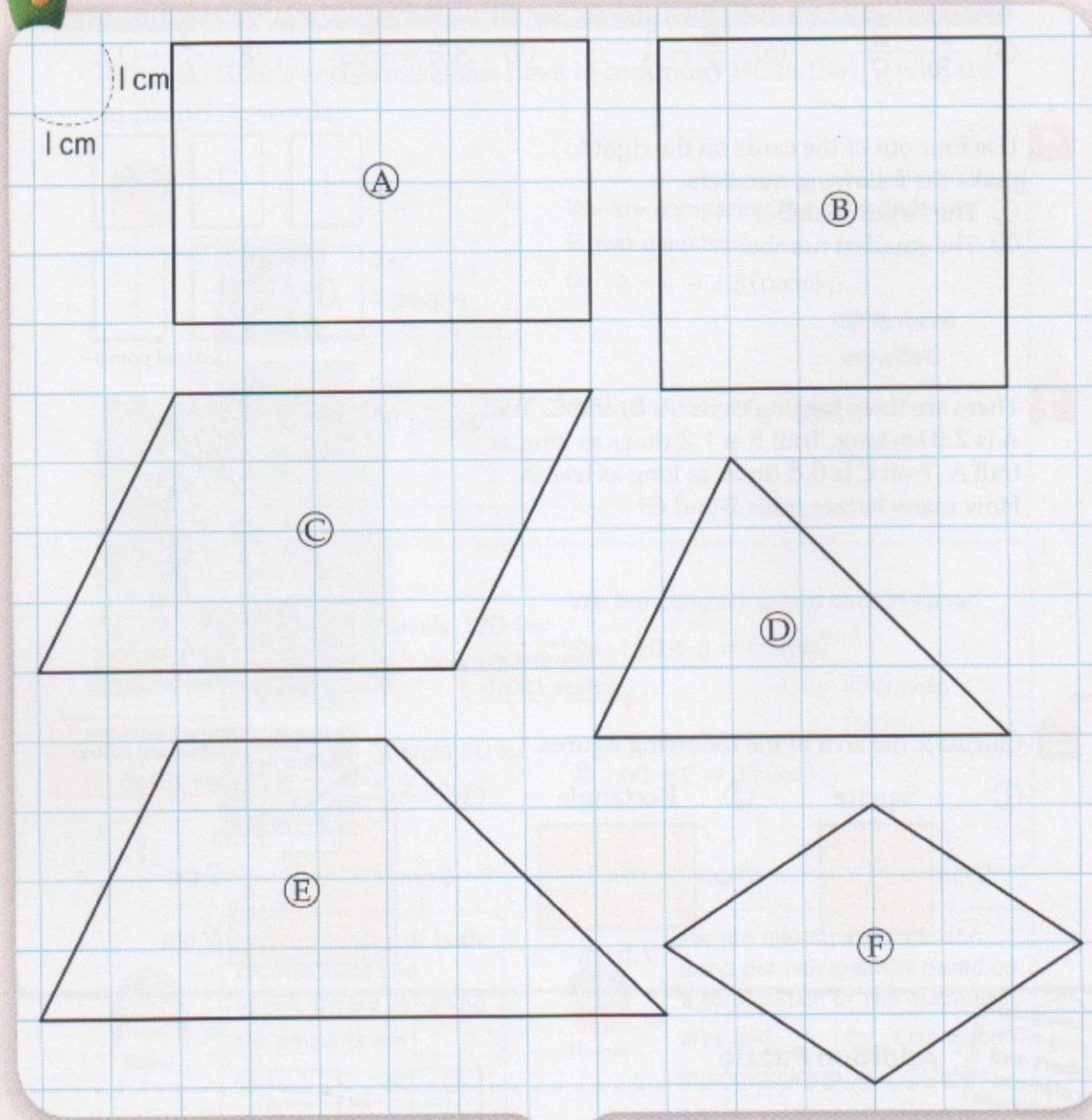
②

$\frac{5}{12}$	Ⓕ	Ⓖ
Ⓗ	$\frac{1}{3}$	$\frac{1}{5}$
Ⓘ	Ⓙ	$\frac{1}{4}$

Ⓕ – Ⓙ will have fractions in them.

Ojo

What have you learned about quadrilaterals and triangles?



What are the things you have or have not learned about the quadrilaterals and triangles above? Discuss with your classmates.

Misaki: I know the names of all these figures.

Riku: We examined these shapes by focusing on lengths of sides, measures of angles, and perpendicular and parallel sides.

Shiho: We haven't learned how to find the area of Ⓒ, Ⓓ, Ⓔ, or Ⓕ.

13 Let's Think about How to Find Area



Tsuyoshi

Since Ⓐ is a rectangle and Ⓑ is a square, we can find their areas using a formula can't we?

Area of rectangle = ☐ × ☐

Area of square = ☐ × ☐

How to Express Area and Formulas for Areas
Page 151 ⑥



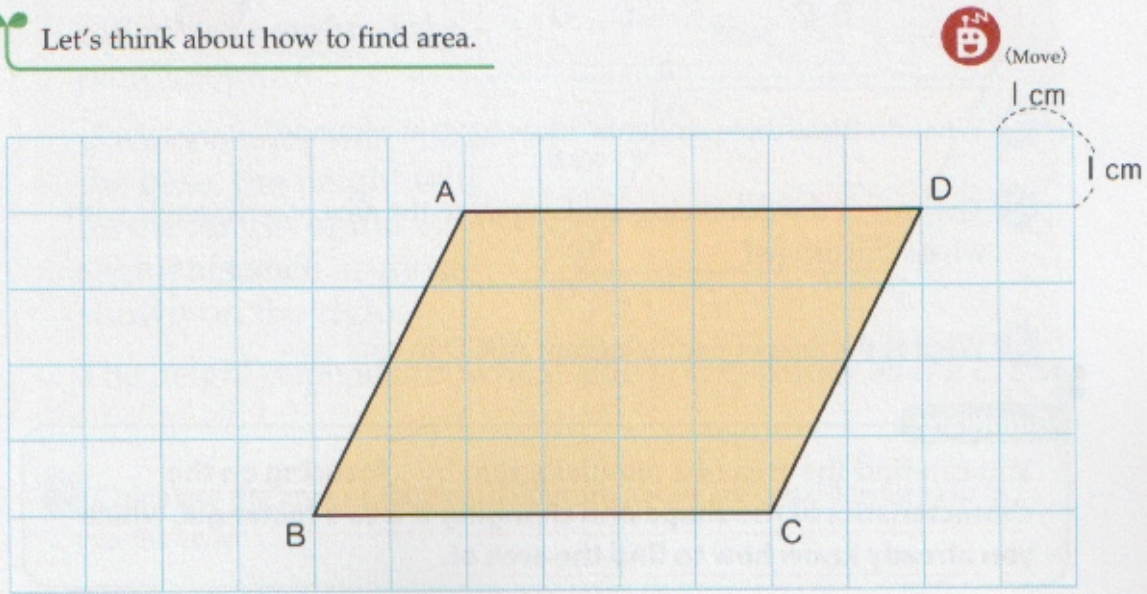
Yukina

Parallelograms have slanted sides. I wonder if we can find the area of a parallelogram by multiplying the number representing the lengths of adjacent sides like we did with rectangles and squares.

1 How to Find the Area of Parallelograms

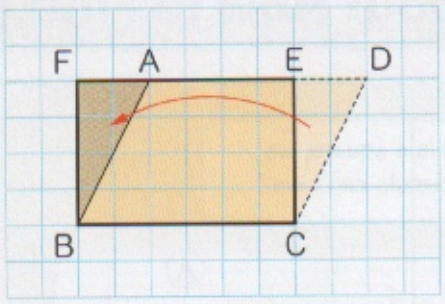
1 What is the area of parallelogram ABCD?

Let's think about how to find area.



1 Explain the three students' ideas below.

Shiho



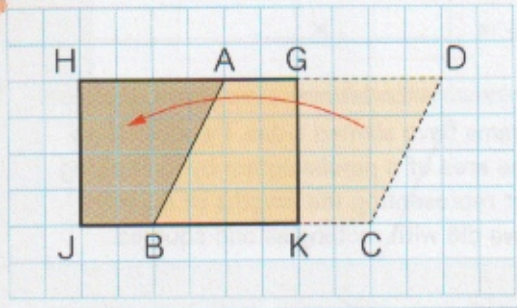
Shiho moved triangle ECD to change parallelogram ABCD into rectangle FBCE to find the area.

Haruto

Why did Shiho change the shape into a rectangle?

Riku

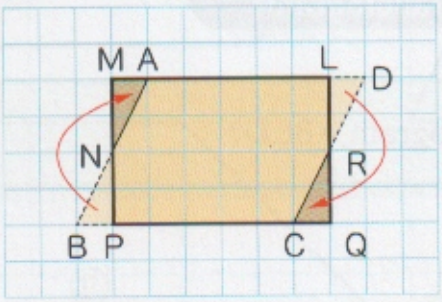
Kota



Kota moved quadrilateral GKCD to change parallelogram ABCD into ...

Ami

Misaki



Misaki moved triangles NBP and LRD...

Riku

2 What do these three students' ideas have in common?

3 The area of this parallelogram is equal to the area of a rectangle with what dimensions?

4 What is the area of parallelogram ABCD in cm^2?

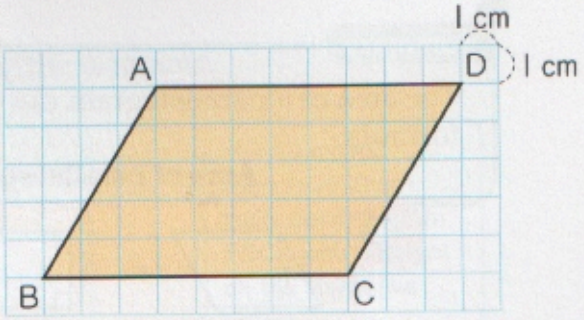
Summary

You can find the area of a parallelogram by **focusing on the characteristics of the shape and changing it into a rectangle, which you already know how to find the area of.**

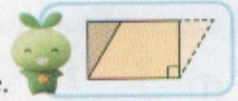
Ami

I wonder if we can make a formula for calculating the area of a parallelogram, like those for a rectangle and a square.

2 Calculate the area of parallelogram ABCD on the right.



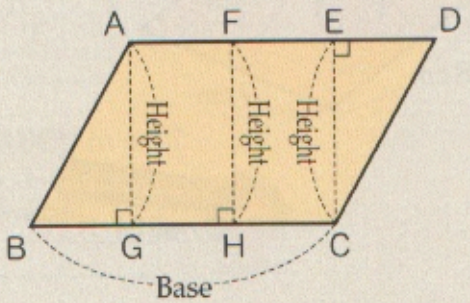
Let's come up with a formula to find the area of a parallelogram.



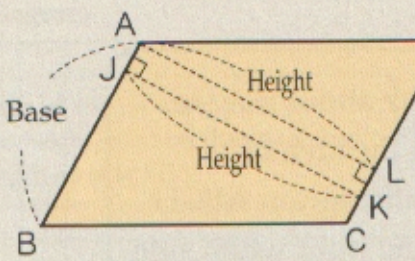
1 We are going to use Shiho's idea on the previous page.
The area of the parallelogram above is equal to the area of a rectangle with what dimensions?
Which parts of the parallelogram are equal to the length and the width of this rectangle?

Draw the part that is the same length as the vertical side of the rectangle in the figure above.

In the parallelogram on the right, if we make side BC as the **base**, the length of the segments that are perpendicular to the base such as segment EC is called the **height** of the parallelogram.



If we consider side AB as the base, the height will be the length of the segments such as those shown on the right.



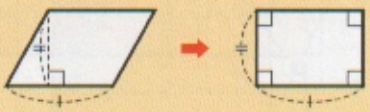
The height depends on which side you consider as the base.

2 Calculate the area of the parallelogram above by considering side BC as the base.

Summary

The area of a parallelogram can be calculated using the following formula.

Area of parallelogram = Base × Height



Base of a parallelogram
= Length of a rectangle
Height of a parallelogram
= Width of a rectangle

1 Calculate the area of the parallelograms below.

① 6 cm, 10 cm

② 8 cm, 15 cm, 10 cm

③ 7 cm, 8 cm, 9 cm

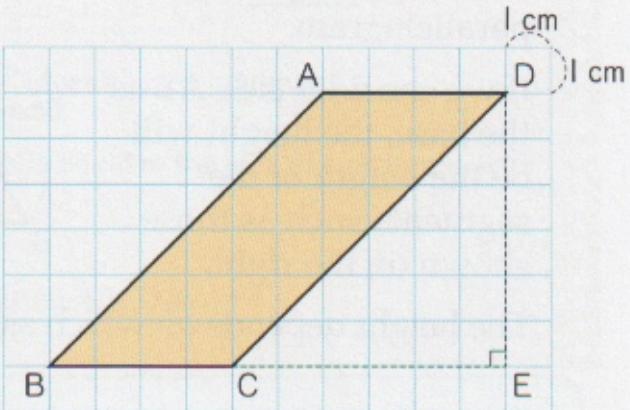
④ 1 cm, 1 cm

Additional Problems
→Page 138 K



Riku: Choosing the base automatically determines the height.

3 Think about how to find the area of parallelogram ABCD on the right when side BC is considered as the base.



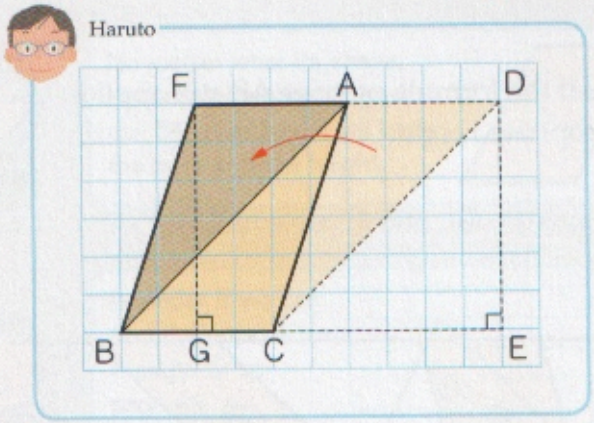


Misaki: The height is ...



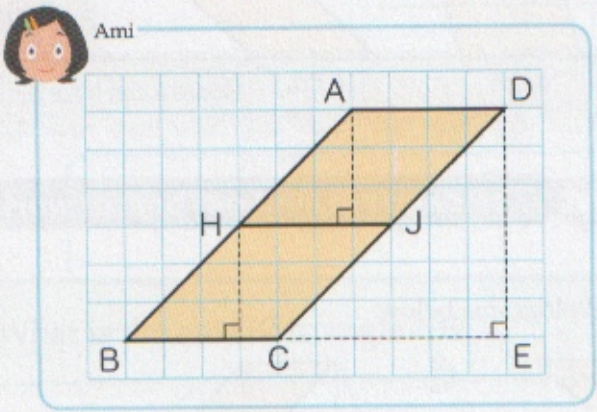
Kota: To use the formula ...

1 Explain the two students' ideas using the diagrams.



Haruto moved triangle ACD to make parallelogram FBCA with a height on the inside like before.


Shiho



Ami split parallelogram ABCD into two parallelograms AHJD and HBCJ with a height on the inside like before.


Riku

2 What is the area of parallelogram ABCD in cm^2?

If the height can't be found inside, I wonder where I can find it.


Misaki

Let's think about the height of a parallelogram.

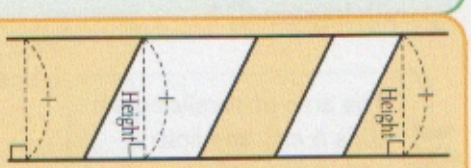
Based on Haruto's idea, changing the shape of the parallelogram does not change the length of the base. So, the height must be the same, too. I wonder which segment has the same length as side FG ...


Kota

Summary

The height of a parallelogram can be considered as the distance between the lines obtained by extending the base and the opposite side.

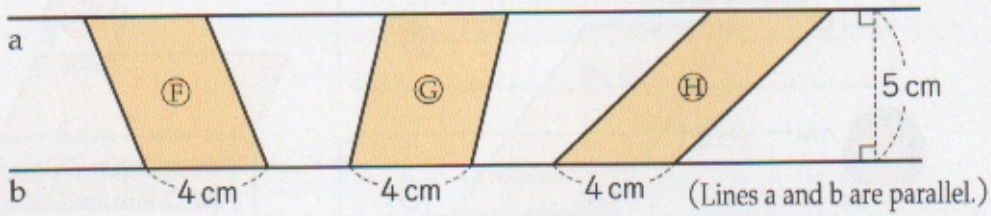
Regardless of whether the height is inside or outside, we can think about the height in the same way.



3 If side BC is the base of parallelogram ABCD in 3 on page 48, what is the height in cm?

4 Calculate the area to see if the formula on page 48 still applies. Use BC as the base and the appropriate height.

5 The areas of parallelograms Ⓕ, Ⓖ, and Ⓗ are equal. Explain the reason why.

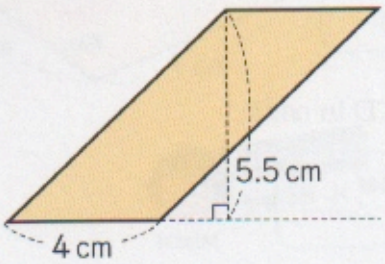



Ami

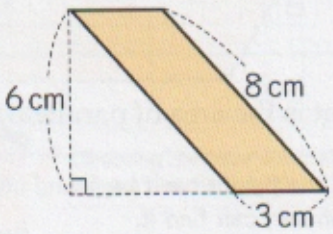
Area of parallelogram = Base × Height. So, as long as the base and the height do not change in length, parallelograms of any shape...

2 Calculate the area of the parallelograms below.

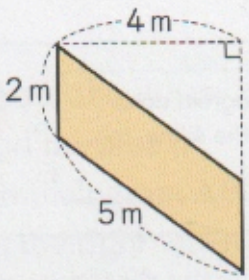
①



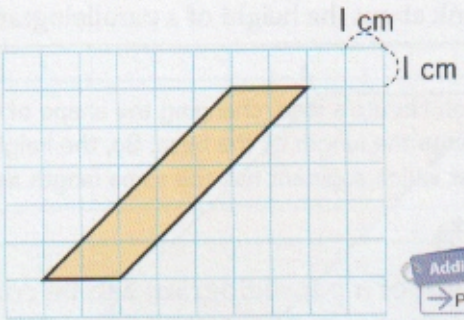
②



③



④

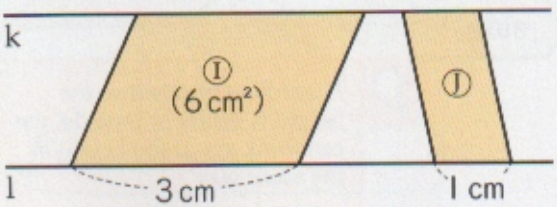


Additional Problems
→Page 138

3 What is the area of parallelogram Ⓘ?



The area of parallelogram Ⓘ is 6 cm², and lines k and l are parallel.




Haruto

I want to find the area of other geometric figures.

Three students are reviewing what they have studied about area so far.



Yukina

No matter what its shape, the area of a parallelogram can be calculated if we know the base and the height.





Makoto

I wonder if we can find the area of triangle Ⓓ on page 44.



We should change the triangle into a shape we already know how to find the area of...



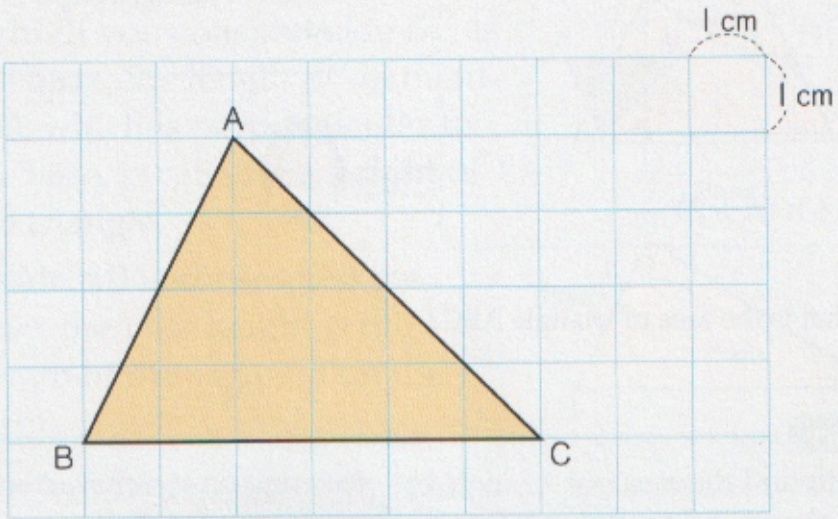
Tsuyoshi

2 How to Find the Area of Triangles

1 What is the area of triangle ABC?

Let's think about how to find the area.

〈Move〉

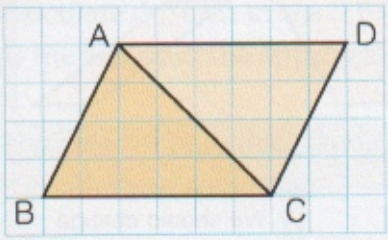


When you come up with an idea, express it using a diagram and a math sentence so that your classmates can understand your idea.

1 Explain the three students' ideas below using diagrams and math sentences.



Haruto



$6 \times 4 \div 2$

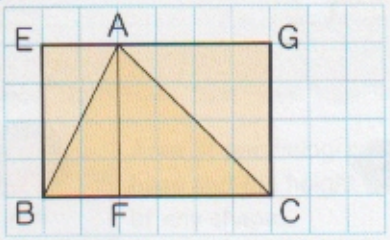


Misaki

Haruto used the fact that you can make a parallelogram using two copies of triangle ABC, and ...



Ami



$4 \times 6 \div 2$

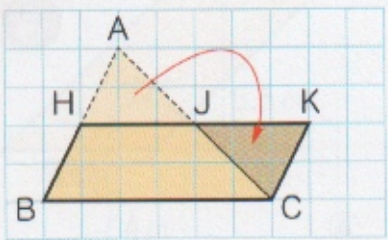


Kota

Ami made rectangle EBCG using 2 copies each of triangle ABF and triangle AFC, and ...



Riku



$6 \times (4 \div 2)$



Shiho

Riku used the fact that you can make parallelogram HBCK by moving triangle AHJ ...

2 What is the area of triangle ABC?

Summary

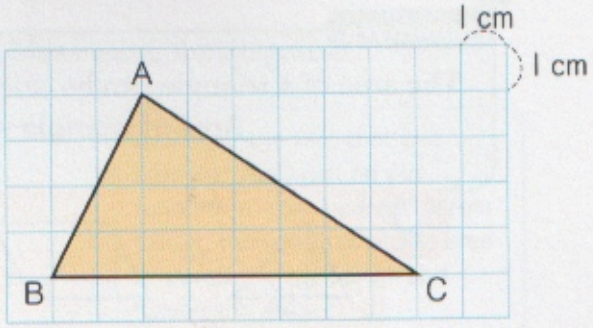
You can find the area of a triangle by **focusing on the characteristics of the shape and changing it into a rectangle or a parallelogram, which you already know how to find the area of.**



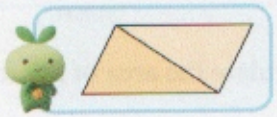
Misaki

I wonder if we can make a formula for calculating the area of a triangle, too.

2 Calculate the area of triangle ABC on the right.



Let's come up with a formula for calculating the area of a triangle.



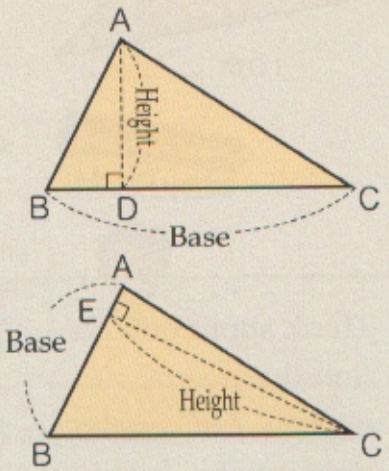
1 We are going to use Haruto's idea on the previous page.
The area of the triangle above is a half the area of a parallelogram. What is the base and the height of the parallelogram?
Which parts of the triangle correspond to the base and the height of the triangle?



Draw the segment that will be equal to the height of the parallelogram in the diagram above.

Look at triangle ABC on the right. If you consider side BC as the **base**, the length of segment AD, which is perpendicular to the base, is called the **height** of the triangle.

If side AB is considered the base, then the height is the length of the segment on the right.



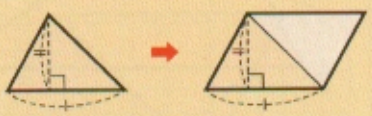
The height depends on which side you consider as the base.

2 Calculate the area of the triangle above using BC as the base.

Summary

The area of a triangle can be calculated using the following formula:

Area of triangle = Base × Height ÷ 2

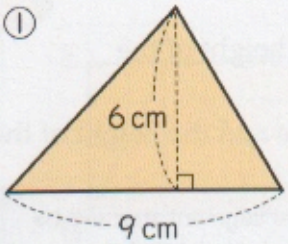


If we use Haruto's idea, it means that Area of Triangle = Area of Parallelogram ÷ 2, doesn't it?

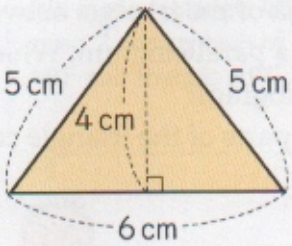
I wonder if we can make this formula using Ami's or Riku's ideas, too?

1 Calculate the area of the following triangles.

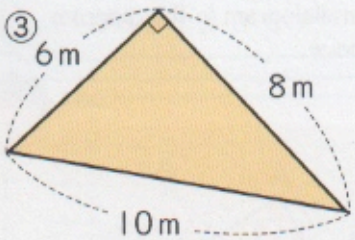
①



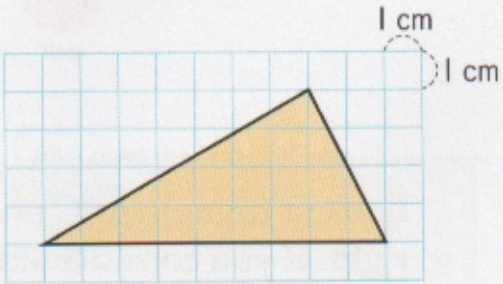
②



③



④

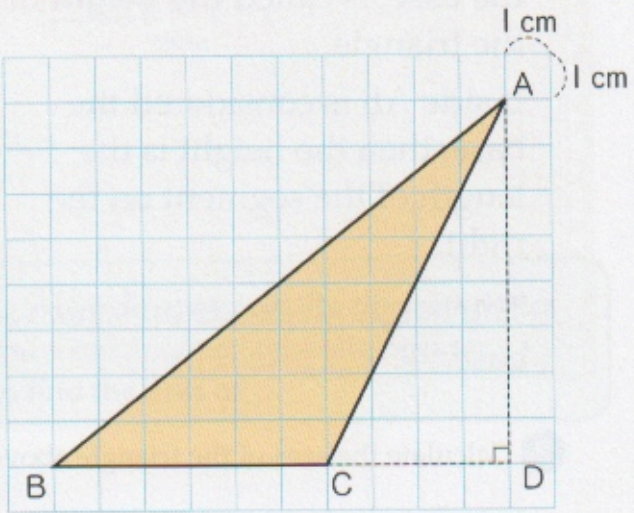


Additional Problems
→Page 139 M



Shiho: The fact that choosing the base automatically determines the height applies to triangles, too.

3 Think about how to find the area of triangle ABC on the right when we consider BC as the base.



The height is ...

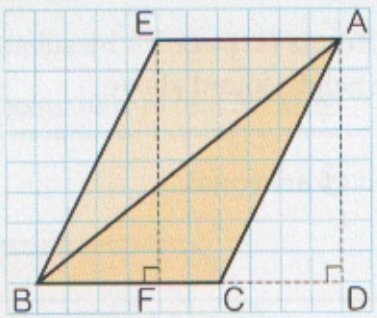


Riku

1 Explain the following two students' ideas using the diagrams.



Ami



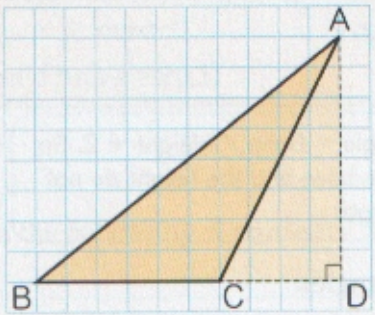
Ami used the fact that two triangles ABC can be put together to make parallelogram EBCA, then she found the area of parallelogram EBCA and halved the area ...



Haruto



Kota



Kota subtracted the area of triangle ACD from the area of triangle ABD to ...



Misaki



Why did Kota think about using triangles ABD?

2 What is the area of triangle ABC?

Let's think about the height of a triangle.

Even if the height can't be found inside the triangle, I wonder if we can think about the height just like we did with a parallelogram.

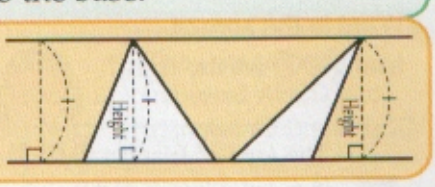


Riku

Summary

The height of a triangle can be considered as the distance between the extended line of the base and the line that is parallel to the base and passes through the vertex opposite to the base.

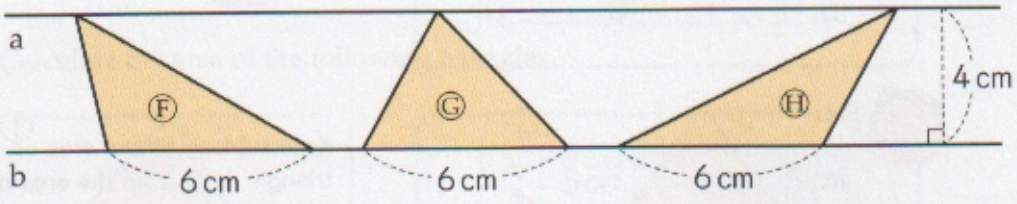
We can think about the height of a triangle just like we did with a parallelogram.



3 What is the height of triangle ABC in 3 on page 54 if you consider side BC as the base?

4 Check to see if we can use the formula on page 54 by substituting the length of BC as the base and the height found above.

5 The areas of triangles Ⓕ, Ⓖ, and Ⓗ are equal.
Explain the reason why.



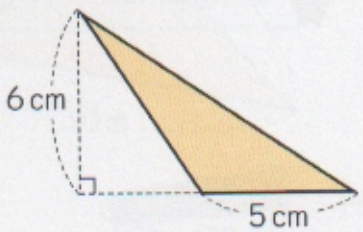
(Lines a and b are parallel.)

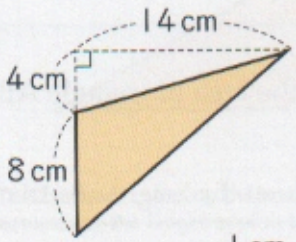


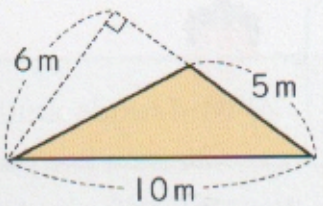
Shiho

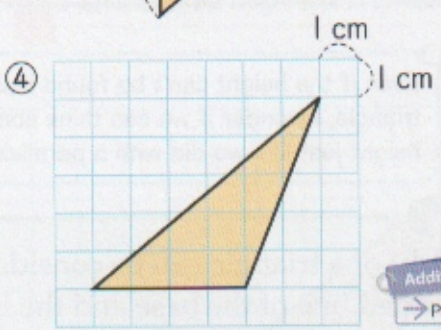
Area of triangle = Base × Height ÷ 2. So, as long as the base and the height do not change in length ...

2 Calculate the area of the following triangles.

① 

② 

③ 

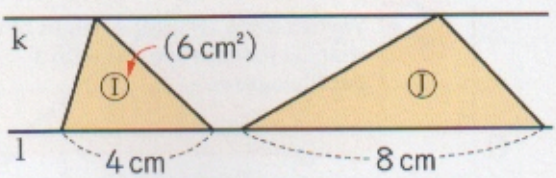
④ 

Additional Problems
→Page 139 N

3 What is the area of triangle Ⓙ on the right?



The area of triangle Ⓘ is 6 cm². Lines k and l are parallel.





Kota

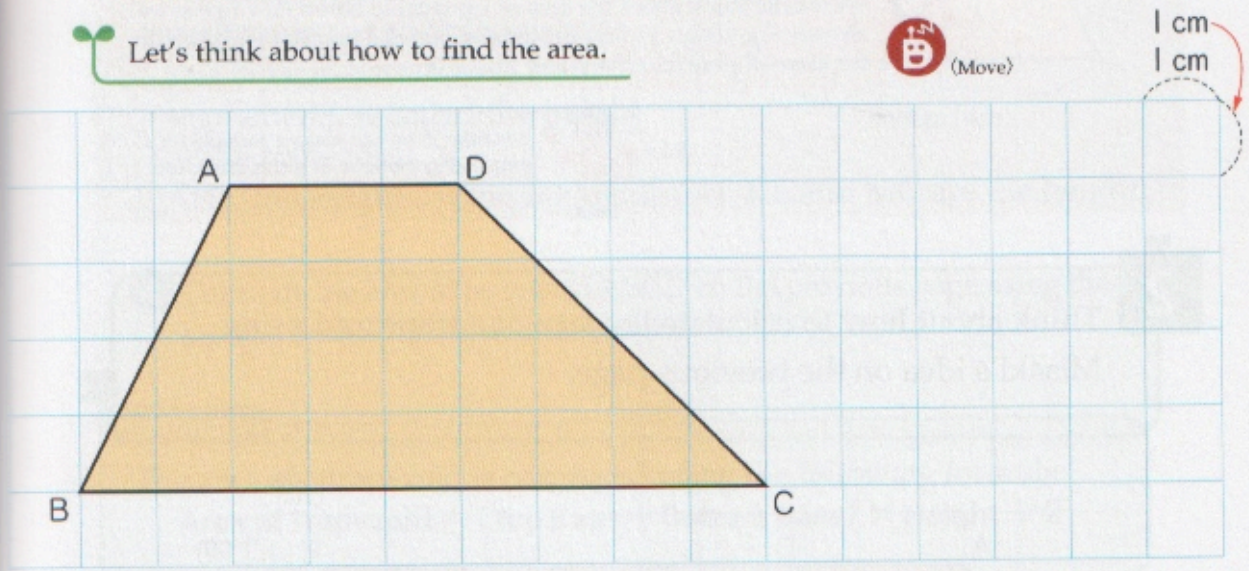
I want to find the area of trapezoids, too.

3 How to Find the Area of Various Quadrilaterals

1 What is the area of trapezoid ABCD?

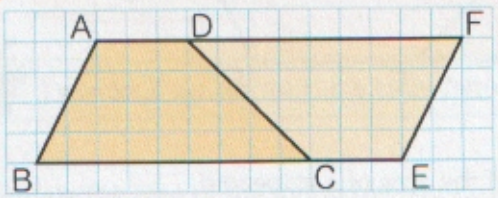
Express your idea using a diagram and a math sentence.

Let's think about how to find the area.



(1) Explain the following 3 students' ideas using diagrams and math sentences.

Misaki

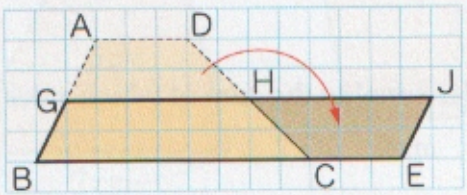


$(9+3)\times4\div2$

Kota

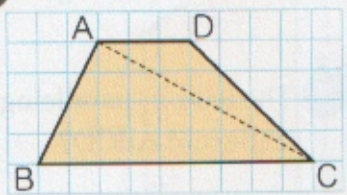
We can form parallelogram ABEF by combining two of trapezoid ABCD. $(9+3)$ is the length of base BE of parallelogram ABEF and ...

Haruto



$(9+3)\times(4\div2)$

Ami



$(9\times4\div2)+(3\times4\div2)$

Riku

Since we can make parallelogram GBEJ by moving trapezoid AGHD ...

Shiho

Since we can split trapezoid ABCD into triangles ABC and ACD using a diagonal ...

(2) What is the area of trapezoid ABCD?

Summary

You can find the area of a trapezoid by **focusing on the characteristics of the shape and changing it into a parallelogram or dividing it into triangles**.

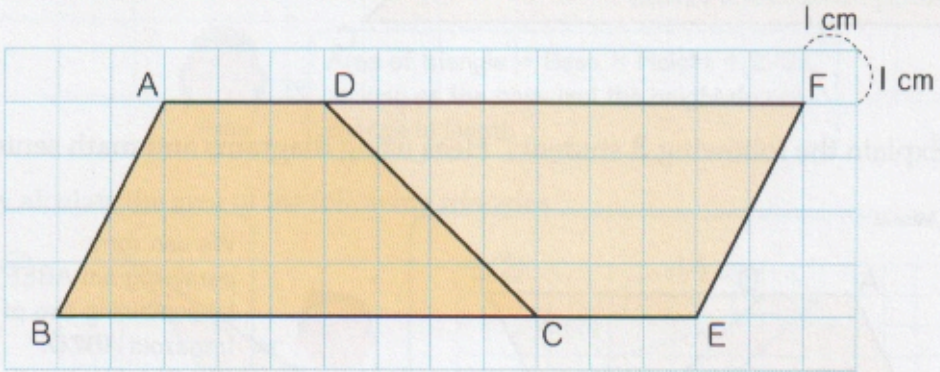
We should think about the area of a trapezoid based on a figure we already know how to find the area of, just like the way we did with the areas of parallelograms and triangles.



Riku: I wonder if we can make a formula for calculating the area of a trapezoid, too.

2

Think about how to calculate the area of a trapezoid using Misaki's idea on the previous page.



Let's come up with a formula to find the area of a trapezoid.

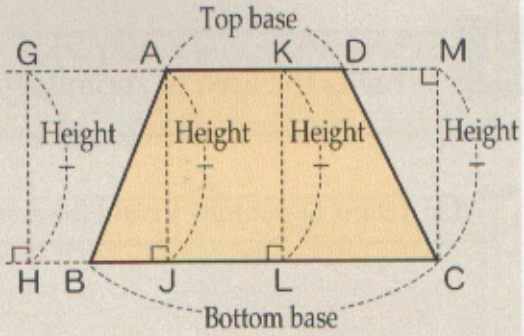
1. The area of the trapezoid above is a half of the area of a parallelogram. What are the lengths of the base and height of the parallelogram?

2. The base in parallelogram ABEF is equal to the sum of which sides of trapezoid ABCD?

Length of base of parallelogram ABEF	= Length of side ☐ + Length of side ☐

3. Which part of trapezoid ABCD is the same length as the height of paralalelogram ABEF? Draw a segment in the diagram above.

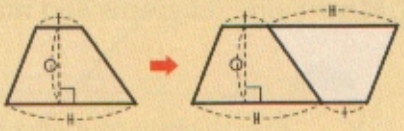
In the trapezoid on the right, the two parallel sides, AD and BC, are called the **top base** and the **bottom base**, respectively. The length of a segment that is perpendicular to both the top and bottom bases, such as segment AJ, is called the **height** of the trapezoid.
Also, the lengths of the segments GH, KL and MC are the height.



4 Calculate the area of trapezoid ABCD on the previous page using the top and bottom bases, and the height.

Summary

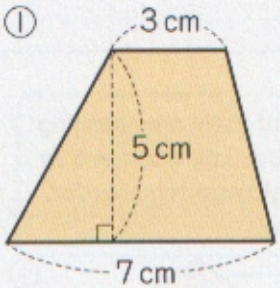
The area of a trapezoid is calculated using the following formula:
Area of Trapezoid = (Top Base + Bottom Base) × Height ÷ 2

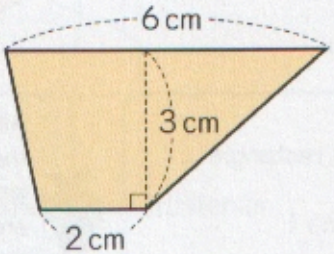


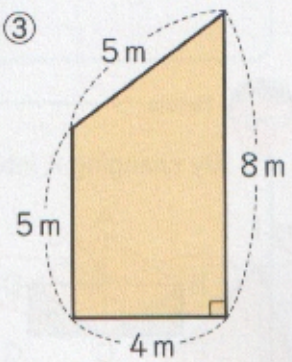
If you think based on Misaki's idea, it means that the Area of Trapezoid = Area of Parallelogram ÷ 2, doesn't it?

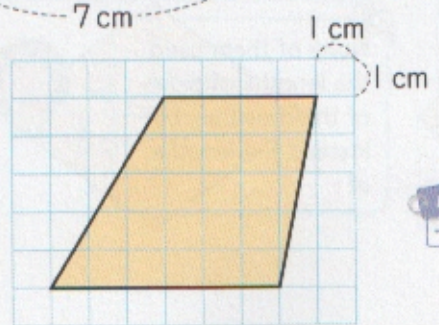
I wonder if we can make the same formula using Haruto's and Ami's ideas.

1 Calculate the area of the following trapezoids.

① 

② 

③ 

④ 

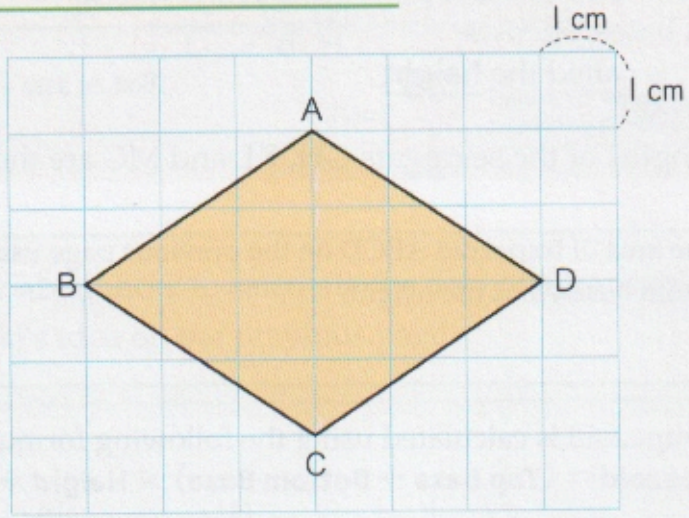
Additional Problems → Page 139



Misaki: I wonder if we can find the area of a rhombus, too.

3 What is the area of rhombus ABCD?

Let's think about how to find the area.

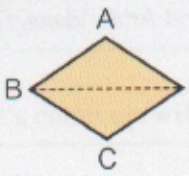


1 Explain the following 3 students' ideas using diagrams and math sentences.



Kota

By splitting it into 2 triangles ...

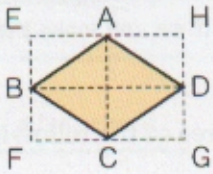


$(6 \times 2 \div 2) \times 2$



Shiho

By thinking of it as a half of a rectangle ...

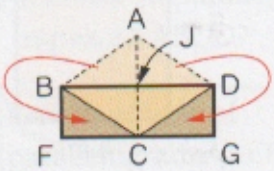


$(4 \times 6) \div 2$



Haruto

By changing it into a rectangle ...



$(4 \div 2) \times 6$



All three of them are changing the rhombus into a shape that we can calculate the area for, aren't they?

None of them used the lengths of sides of the rhombus, but instead the lengths of ☐ ...



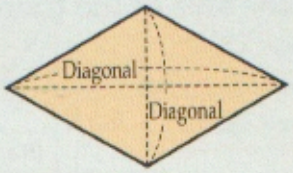
Ami

2 What is the area of rhombus ABCD?

3 Using Shiho's idea, think about how to calculate the area of a rhombus.

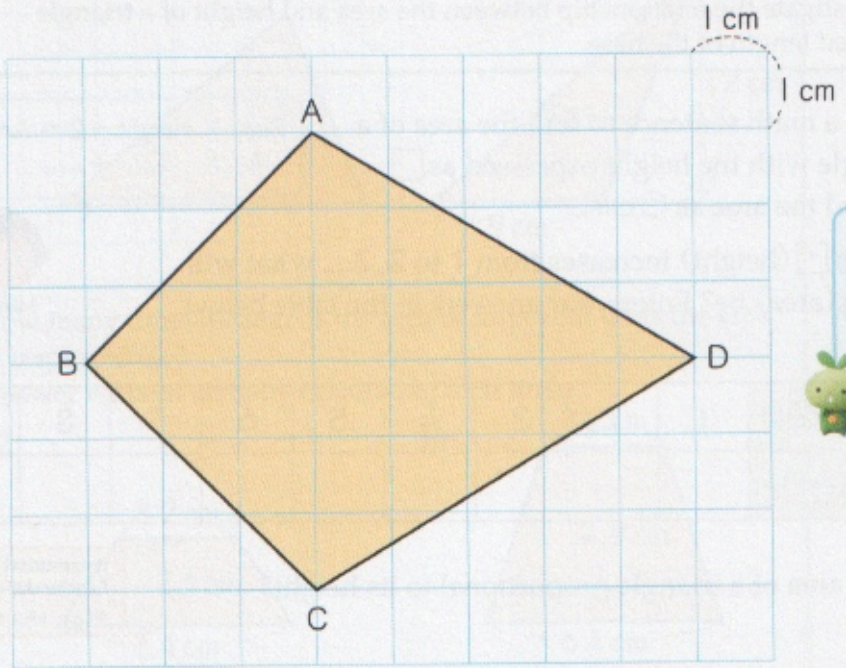
Summary

The area of a rhombus is calculated with the following formula that uses **the lengths of two diagonals**.



Area of Rhombus = Diagonal × Other Diagonal ÷ 2

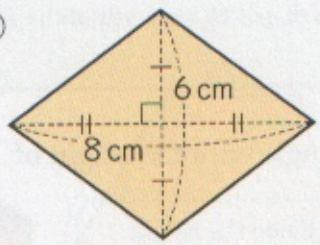
4 Think about how to find the area of quadrilateral ABCD below.

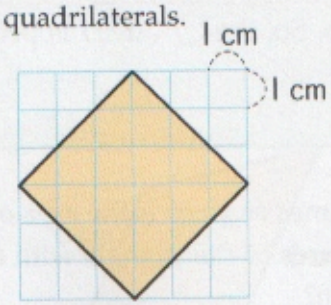


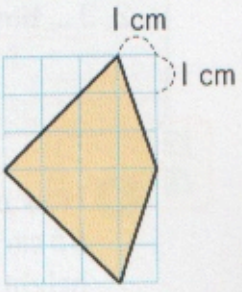


I wonder if we can think just like we did about the area of a rhombus.

2 Calculate the area of the following quadrilaterals.

① 

② 

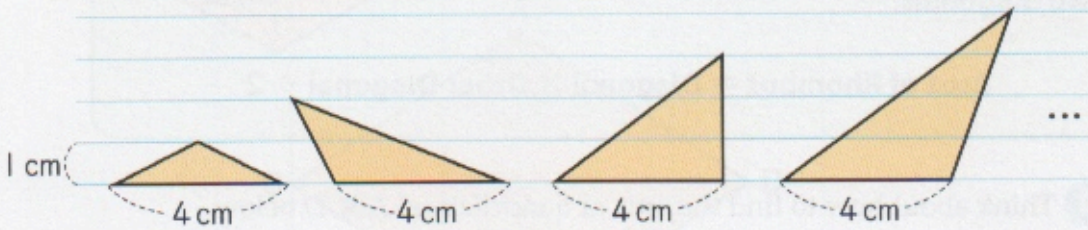
③ 



Riku: We have made formulas for calculating the areas of many different figures.

4 The Relationship between the Height and Area of a Triangle

1 As we increase the height of a triangle from 1 cm to 2 cm, 3 cm... without changing the length of the base, 4 cm, how does the area of the triangle change?



Let's investigate the relationship between the area and height of a triangle with a fixed length of the base.

1 Write a math sentence to find the area of a triangle with the height expressed as □ cm and the area as ○ cm^2.

Base × Height ÷ 2 = Area
4 × □ ÷...


Misaki

2 As the □ (height) increases from 1 to 2, 3..., what will the ○ (area) be? Enter your answers in the table below.

Height □(cm)	1	2	3	4	5	6	7	8	
Area ○(cm^2)									

3 Is the area of a triangle proportional to its height?

Proportional Relationships
Page 151 ⑩

Summary

As the □ (height) grows 2, 3 ... times, the ○ (area) also becomes 2, 3 ... times, too. So, the ○ (area) is proportional to the □ (height).

4 How many times as large is the area of a triangle with the height of 45 cm as the area of the triangle with the height of 5 cm?

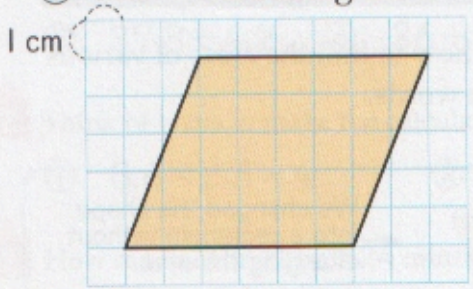
Since the height increased to 9 times, the area also...


Riku

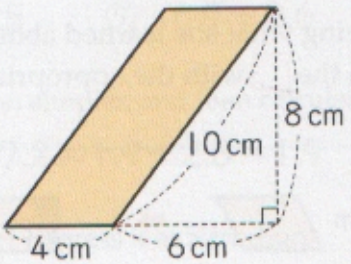
Check Your Understanding

1 Calculate the area of the following figures.

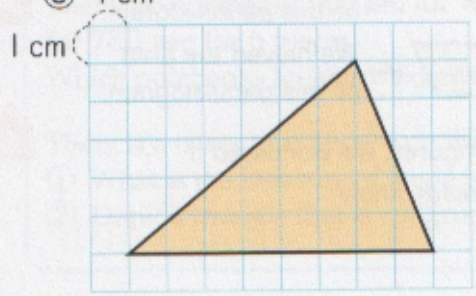
① 1 cm Parallelogram



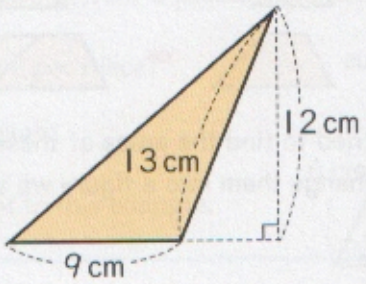
② Parallelogram



③ 1 cm



④



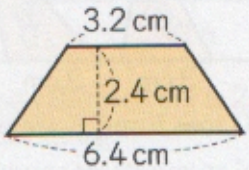
Can you find the areas of parallelograms and triangles?

① Page 47 2
② Page 48 3
③ Page 53 2
④ Page 54 3

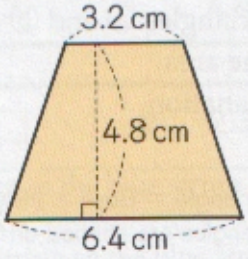
2 How many times as large is the area of trapezoid Ⓑ as the area of trapezoid Ⓐ ?

Answer without actually calculating their areas.

Ⓐ



Ⓑ



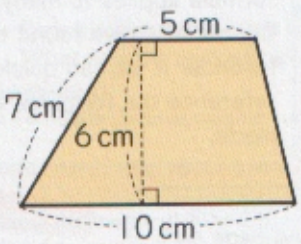
If we compare the math sentences for calculating their areas ...

Can you use the formula for finding the area of a trapezoid and compare the areas of trapezoids?

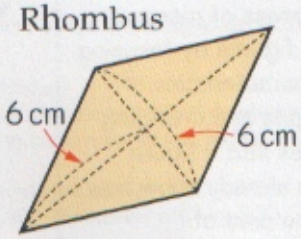
Page 58 2

3 Calculate the area of the following figures.

①



② Rhombus



Can you find the areas of trapezoids and rhombuses?

① Page 58 2
② Page 60 3

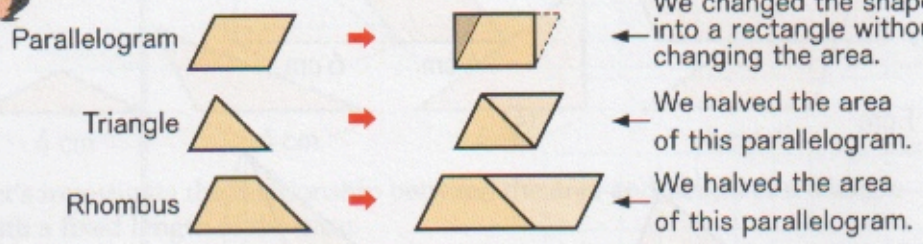
Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

1 Focus on the Characteristics of Shapes and Think about How to Calculate Their Areas

Ami is reviewing what she learned about how to find the areas of various figures. Fill in the ☐ with the appropriate words.



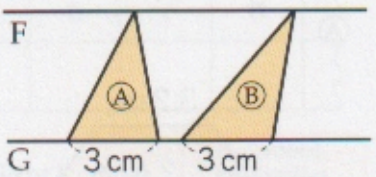
Ami



When we tried to find the areas of these figures, we wondered if we could change them into a figure we already know ☐.

2 Focus on the Formula You Have Made and Think about the Lengths that Determine the Area of Figures

Haruto says that triangles Ⓐ and Ⓑ on the right have the same area.
Complete his explanation.



(Lines F and G are parallel.)



Haruto

Area of triangle = Base × Height ÷ 2
These triangles are shaped differently, but their bases and heights are...

Look back on what you have learned in "Let's Think about How to Find Area" and discuss.



Kota

Now we know that we can find the areas of many different figures by focusing on the characteristics of their shapes and changing the figures into a figure which we already know how to find the area of.



Shiho

I was surprised that a single formula applies to many different figures when we found their areas. I wonder if we can quickly reference the formulas we have made.



See page 145.

Challenge Yourself
→Page 145

Do You Remember?

Answers→ Page 149

1 Calculate. For division problems, divide completely.

① 6.92×4.3 ② 6.05×3.8 ③ 3.46×5.2 ④ 0.25×0.6

⑤ $37.8 \div 4.2$ ⑥ $36 \div 7.5$ ⑦ $6.9 \div 0.6$ ⑧ $2.34 \div 3.6$

2 Think of ways to make the calculation simpler, and then calculate.

Order and Properties of Operations Page 150 ③

① $3.6 \times 2.5 \times 4$ ② $7.2 \times 1.9 + 2.8 \times 1.9$

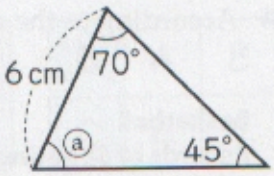
3 How much of an hour is 24 minutes? Express it as a fraction.

4 There are notebooks that sell for 1,000 yen for 8 pieces and notebooks that sell for 780 yen for 6 pieces.
Which notebooks are more expensive per piece?

5 There is a triangle as shown on the right.

① What is the measure of angle ⓐ.

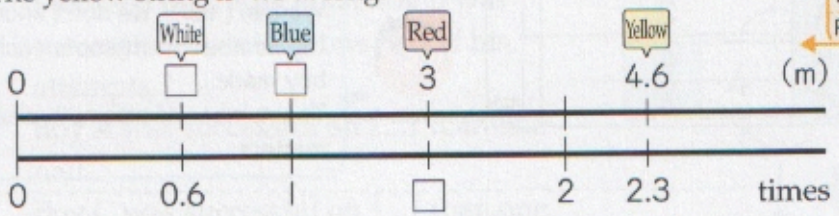
② Draw a triangle that is congruent to this triangle.



Warm-up

6 There are four colored strings, red, blue, white, and yellow. The red string is 3 m long. The yellow string is 4.6 m long.

Times As Much Page 150 ①



① The yellow string is 2.3 times as long as the blue string. How long is the blue string?

② How many times as long is the red string as the blue string?

③ The white string is 0.6 times as long as the blue string. How long is the white string?

Playing with Numbers and Calculations

Strange Multiplication and Division

Pick three numbers you like and follow steps Ⓐ to Ⓓ to do multiplication and division. What will the answer be?

①(Multiplication)
Ⓐ Pick a whole or decimal number you like.
Ⓑ Multiply the number you picked in Ⓐ by 1.6.
Ⓒ Multiply the answer of Ⓑ by 2.5.
Ⓓ Multiply the answer of Ⓒ by 0.25.

②(Division)
Ⓐ Pick a whole or decimal number you like.
Ⓑ Divide the number you picked in Ⓐ by 1.6.
Ⓒ Divide the answer of Ⓑ by 2.5.
Ⓓ Divide the answer of Ⓒ by 0.25.

Pick numbers other than 0.



Who was the most successful?

For a basketball game, boys are having shooting practice every day.
Boys A, B, and C are reviewing their shooting records from yesterday.



- According to the records below, who can we say was the most successful?

Basketball
Records of Shooting Practice on January 10 (Yesterday)

	Baskets
Boy A	3
Boy B	4
Boy C	4

Boy A made 3 baskets. Boys B and C made 4 baskets each, so...

Ami

We can't tell if we don't know the number of attempts each boy made...
The number of attempts also matters...

Kota

- According to the records below, who can we say was the most successful?

Basketball
Records of Shooting Practice on January 10 (Yesterday)

	Baskets	Total attempts
Boy A	3	8
Boy B	4	8
Boy C	4	7

The number of attempts was different among the three boys.

Boys A and B made a different number of baskets, but they made the same number of attempts.

Haruto

Boy B was successful on exactly one half of his attempts...

Kota

Ami, Haruto, and Kota thought about the problem in the following ways.


Ami

I checked the difference between the number of attempts and the number of baskets.

Boy A $8-3=5$ (attempts)
Boy B $8-4=4$ (attempts)
Boy C $7-4=3$ (attempts)

These are numbers of missed shots.

The one with the fewest number of missed shots...


Haruto

For Boys A and B, we can compare the number of baskets because they made the same number of attempts.
Boys B and C made the same number of

[ ],

so...

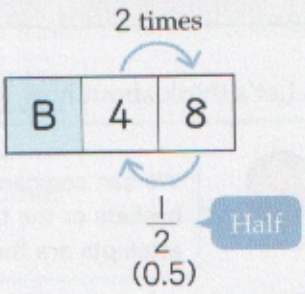
Same

A	3	8
B	4	8

B	4	8
C	4	7


Kota

I focused on the fact that boy B was successful on exactly one half of his attempts.
Boy A was successful on [...] than one half...
Boy C was successful on [...] than one half ... So...



Discuss the three students' ideas.

Which way of comparison applies to all cases?


Misaki

Based on Ami's idea, a boy who made 97 baskets out of 100 attempts is considered as successful as boy C, because the difference is 3 for both of them.


Riku

Haruto's idea applies only if one of the two quantities is common among the boys.


Shiho

Based on Kota's idea, how can we compare the boys who were successful on more than one half of their attempts, for example.


Ami

I wonder if there is any way of comparing boys who are different in both the number of attempts made and the baskets made.

Rates

14 Let's Think about How to Compare (2)

Shooting records from today's practice are as follows.

Write the number of baskets and the total number of attempts in the table below.

Basketball

Records of Shooting Practice on January 11 (Today)

	○ : Made ● : Missed												Baskets	Total attempts
Boy A	●	○	●	●	○	○	●	○						
Boy B	○	●	○	○	○	●	●	●	●	●				
Boy C	●	○	○	●	○	○	○	○	○	○				
Boy D	○	●	●	○	●	○	○	○	○	○	○	○		

1 Rates

1 Which of the four boys can we say was the most successful, A, B, C, or D?

Let's think about how to compare.

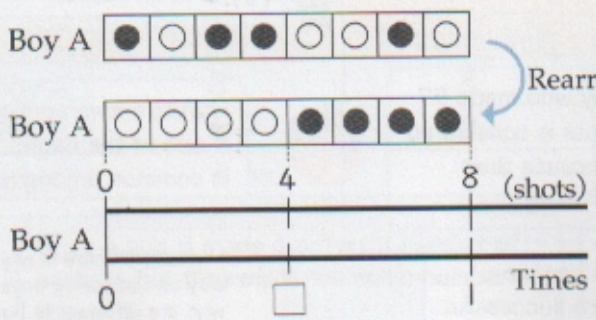

Haruto

We can compare if the number of baskets or the total number of attempts are the same, but...

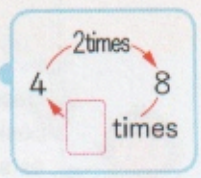

Ami

Boy A was "successful on exactly one half of his attempts."

(1) Think what "successful on one half of his attempts" mean. If a boy was "successful on one half of his attempts," how many times as many is the number of baskets as the number of attempts?




Riku



$4 \div 8 =$ ☐ times

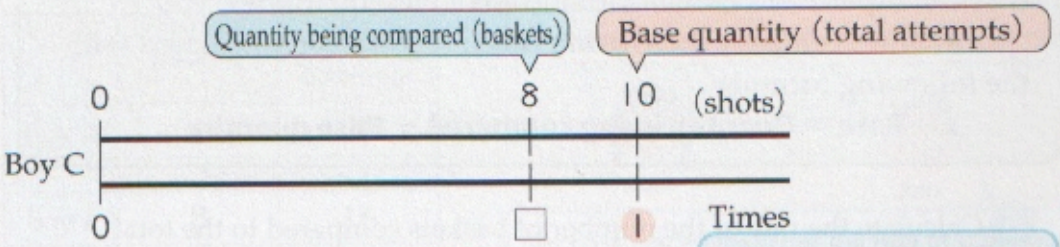
(2) Who were successful on more than one half of their attempts?

3 Think about how to compare who was more successful, C or D.

4 How many times as many is the number of baskets boy C made as the number of attempts he made? What about boy D?

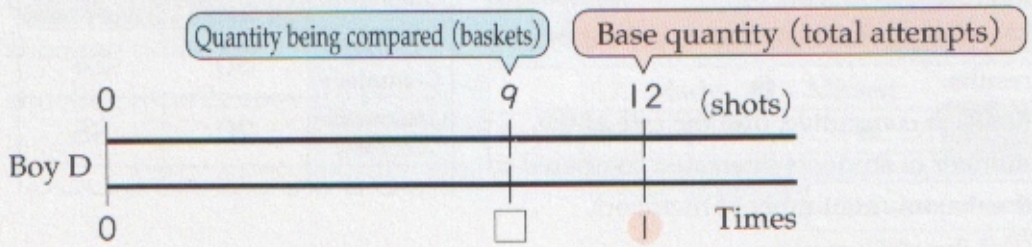
Records of Shooting Practice

	Baskets	Total attempts
Boy C	8	10
Boy D	9	12



$8 \div 10 =$ ☐ times

The double number line diagrams above and below are arranged so that the base quantity is aligned with 1 time.



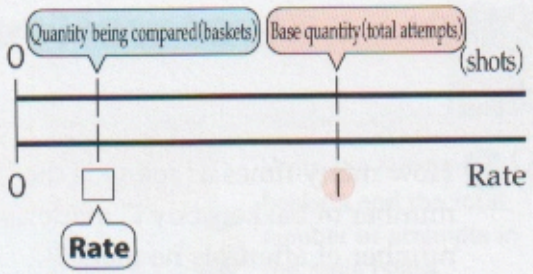
$9 \div 12 =$ ☐ times

If we think of boy C's 10 attempts as 1, the number of baskets, 8, can be considered as 0.8.
If we think of boy D's 12 attempts as 1, the number of baskets, 9, can be considered as 0.75.

If we think of boy A's 8 attempts as 1, the number of 4 baskets can be considered as 0.5.

5 Which boy can we say was more successful in making baskets, C or D?

If we consider the base quantity (the number of attempts) as 1, the number that corresponds to the quantity being compared (the number of baskets) is called the **rate.**



Summary

If you compare two things with different base quantities, you **sometimes compare them using rates.** Rate can be calculated with the following formula.

Rate = Quantity being compared ÷ Base quantity

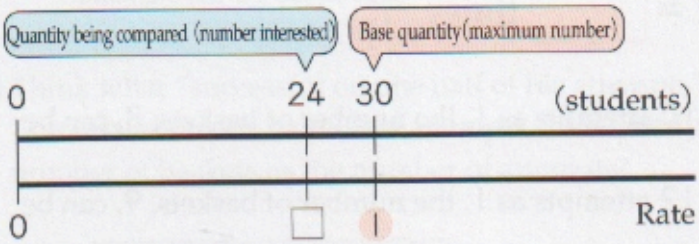
6 Calculate the rate of the number of baskets compared to the total number of attempts for Boy B.

1 Tsuyoshi's school took a survey to see which committee each student was interested in serving on.
The table on the right shows part of the results.
For each committee, find the rate of the number of students interested compared to the maximum number of members.

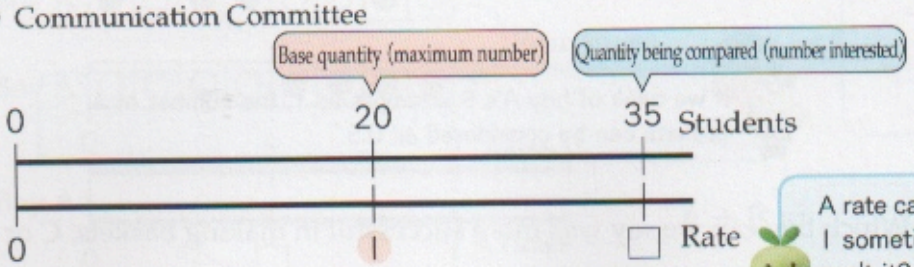
Survey on Student Interest in Committees

Committee	Maximum number of members	Number of students interested
Library Committee	30	24
Communication Committee	20	35

① Library Committee



② Communication Committee




A rate can go over 1 sometimes, can't it?


Kota: The base quantity is what we consider as 1.

Shooting Skills to Make Baskets at the Rate of 0.5

Advanced 6th Grade

In 1 on page 68, it was found that we could say that boy A made baskets at the rate of 0.5.

Records of the shooting practice for boys E, F, and G are as follows.
Which boy made baskets at the rate of 0.5 as boy A did?

Records of Shooting Practice

	Baskets	Total attempts
Boy A	4	8
Boy E	8	16
Boy F	7	12
Boy G	2	4

We can find the answer if we express the relationship between the number of baskets and the number of total attempts made by each of the boys as a rate.


Shiho

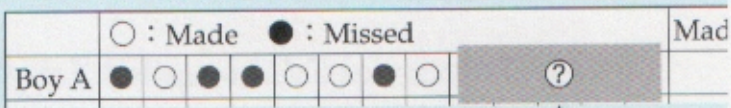

Kota

If two boys made baskets at the rate of 0.5, I wonder if we can say that the boys are at the same level in terms of shooting skills, although the two were different in the numbers of baskets and total attempts.

Can we compare the boys in shooting skills based on the shooting records above?

Basketball

Records of Shooting Practice on January 11 (Today)



If they keep shooting...

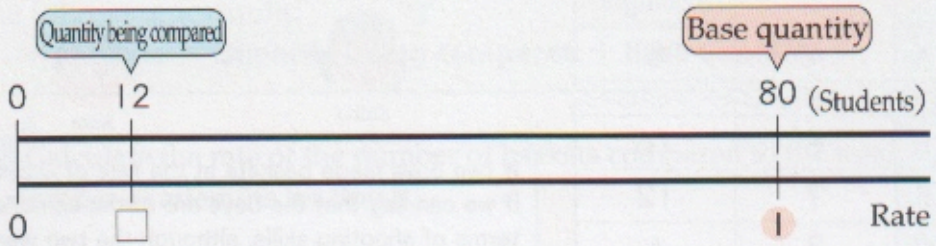
Assume that the number of baskets is proportional to the number of total attempts. To be specific, assume that boy A, who made 8 attempts and 4 baskets, will make 8 baskets if he makes 16 attempts, make 12 baskets if he makes 24 attempts, and so on. You can't tell exactly how many baskets he can make unless he makes actual attempts, but if you assume that the number of baskets is proportional to the number of total attempts, you can compare the boys in their level of shooting skills.

Baskets	Total attempts
4	8
8	16
12	24

How to Express Rates

2 At Aoi's school, there are 80 fifth graders, and 12 of them belong to the soccer club.
Find the rate of fifth graders who belong to the soccer club compared to the total number of fifth graders.



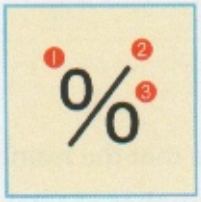


1 Write the math sentence.

Math Sentence ▸ _____

2 Find the rate of fifth graders who belong to the soccer club.

The rate of 0.01 is called **1 percent**, and it is written as 1 %.
A rate that is expressed with percent is called a **percentage.**



Let's express rates as percentages.

3 Express the rate of fifth graders who belong to the soccer club as a percentage.



4 As what % is the rate of 1 expressed?

The rate of 0.15 is expressed as 15%. In percentage, we can express it as a whole number.



Misaki

I have seen the % sign. I didn't know **this is a way of expressing rates.**



Riku

2 The floor of the gym at Aoi's school has an area of 1,040 m². The area of a basketball court is 364 m².
Find the rate of the area of the basketball court compared to the floor area of the gym and express it as a percentage.

3 Write the following rates that are expressed as decimal numbers or whole numbers as percentages.

Additional Problems → Page 140 P

① 0.07 ② 0.54 ③ 1.48 ④ 0.604 ⑤ 2

4 Write the following rates expressed as percentages as decimal numbers.

Additional Problems → Page 140 Q

① 8% ② 90% ③ 37.6% ④ 120% ⑤ 0.6%



Shiho

There is more than one way of expressing rates.

Mathlin Newsletter

What Is 3 Wari?

In professional baseball (in Japan), one of the expressions that describe excellent batters is "3-wari batters." What does "3-wari" mean?
The batting average (BA) is the rate of number of hits to total number of at bats (AB).
BA = Hits ÷ AB
In Japan, BA is sometimes expressed by considering the rate of 0.1 as **1 wari**, the rate of 0.01 as **1 bu**, and the rate of 0.001 as **1 rin**. The way of expressing rates this way is called bu-a-i.

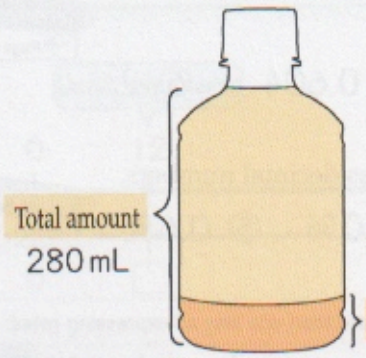
Rates	1	0.1	0.01	0.001
Percents	100%	10%	1%	0.1%
bu-a-i	10wari	1wari	1bu	1rin

"3-wari" in "3-wari batters" is a batting average expressed in bu-a-i.
If a player had 13 hits at bats 40, his batting average is 13 ÷ 40 = 0.325, which can be expressed as 3 wari 2 bu 5 rin in bu-a-i.

2 Percentage Problems

1 The bottle on the right contains 280 mL of juice drink. Of the total content, 20% is fruit juice. How many mL of fruit juice are in the bottle?





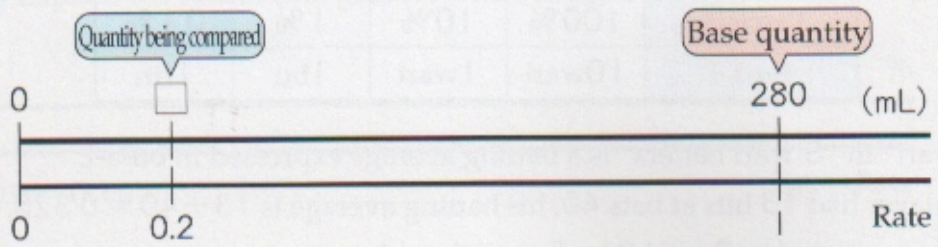
20% of the total amount

(1) What is the base quantity?
What quantity is being compared?

Let's think about how to find the quantity being compared given the rate and the base quantity.

(2) Express 20% using a decimal number.

(3) Write a math sentence to calculate the quantity being compared.



Since 20% of 280 mL is 0.2 times as much of 280 mL,

$280 \times 0.2 =$ ☐

Answer ☐ mL

We should express the percentage as a decimal number to calculate.

Summary

The quantity being compared can be calculated using the following formula.

Quantity being compared = Base quantity × Rate

Quantity being compared = Base quantity × Rate

56 = 280 × 0.2

0.2 = 56 ÷ 280

Rate = Quantity being compared ÷ Base quantity

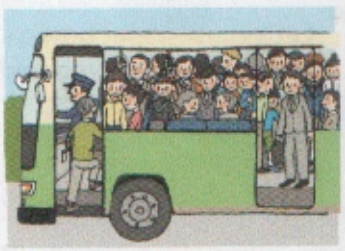
The relationship between the quantities expressed here remains the same.

1 The bottle on the right contains 470 mL of the same juice drink as in problem 1.
How many mL of fruit juice are in this bottle?



The rate of fruit juice was 20%, wasn't it?

2 The number of passengers on a bus is 120% of its 70-person capacity.
How many passengers are on the bus?



120% means that the number of passengers is more than its capacity, doesn't it?

3 Yuto bought a 3,000 yen sweater at the sale price that was 90% of the original price.
What was the sale price?
How much lower was the sale price than the original price?

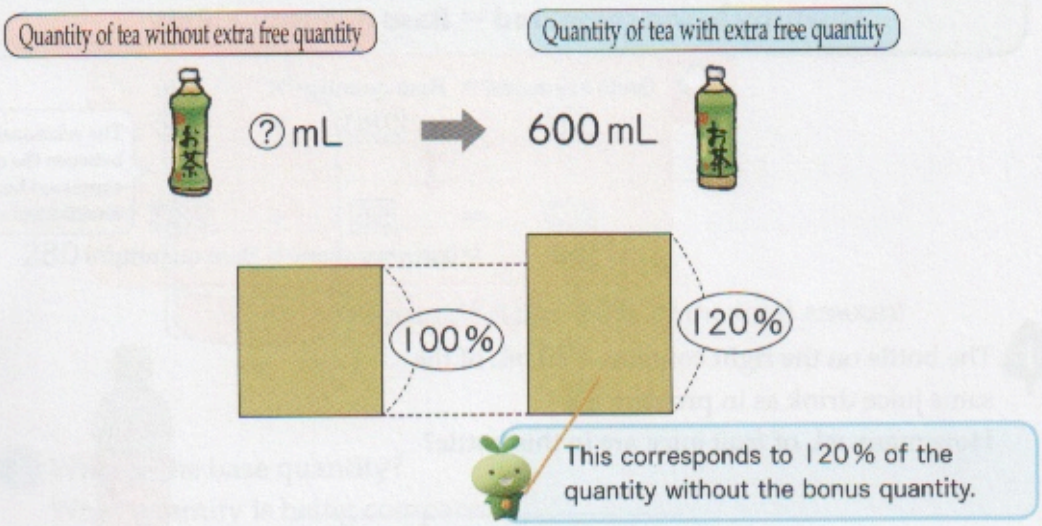


Haruto: Today, we found the quantities being compared. Next time...

2

Bottles of tea with an added bonus quantity are being sold. The quantity of tea including the bonus quantity is 600 mL, which corresponds to 120% of the quantity without the bonus quantity.
How many mL was the quantity of tea without the bonus?



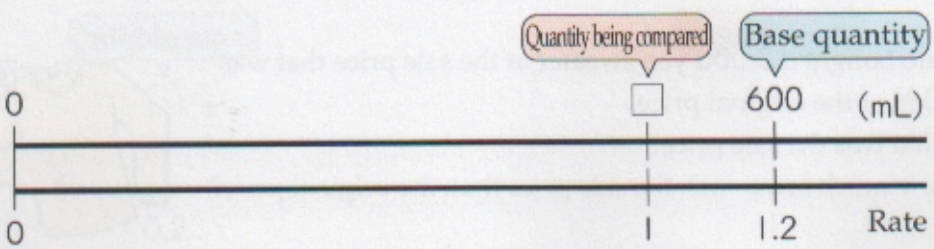


1 What is the base quantity?
What quantity is being compared?

Let's think about how to find the base quantity given the rate and the quantity being compared.

2 Express 120% as a decimal number.

3 Write a multiplication sentence using a □ as the base quantity.



We can express the relationship that 1.2 times as much of □ is 600 mL as,

□ × 1.2 = □

120% is expressed as a decimal number, 1.2, to make a math sentence.

4 Find the number that goes into the □.

□ × 1.2 = 600

□ = 600 ÷ 1.2

= □

Answer □ mL

Summary

When finding the base quantity, it may be helpful to write a multiplication sentence to find the quantity being compared using a □ for the base quantity first.

We calculated using the following relationship: Quantity being compared = Base quantity × Rate.

4 At a store, a carton of milk is being sold for 180 yen. This price is 90% of yesterday's price for the same carton of milk.
What was the price of a carton of milk yesterday?



5 Shirakami Sanchi, a mountain range extending over Aomori and Akita Prefectures, has been registered as a world natural heritage site.
About 26% of the registered site is in Akita Prefecture. The area of this part of the site is 4,344 ha.
About how many ha is the area of the whole registered site?
Round the answer to the second highest place.



B 〈Practice〉

Ami

If we know two quantities out of three: the quantity being compared, the base quantity, and the rate, you can find the other one.

3 Practice

1 Write the following rates as percentages.

① 0.96 ② 0.814 ③ 1.3 ④ 3

2 Write the following percentages as decimal numbers.

① 40% ② 7% ③ 3.5% ④ 145%

3 Answer the following questions.

① What is the rate of 8 g to 12.5 g?
② What % of 20 m is 11 m?
③ How many L is 62% of 250 L?
④ How many students is 120% of 150 students?
⑤ What % of 3.5 m^2 is 8.4 m^2?
⑥ 30% of a flower garden is 9 m^2. What is the area of the flower garden?

4 Japan's land area is about 380,000 km^2, including about 250,000 km^2 of forest land. About what % of land is the forest land? Round your answer to the second highest place.



Forest is essential to the Earth.

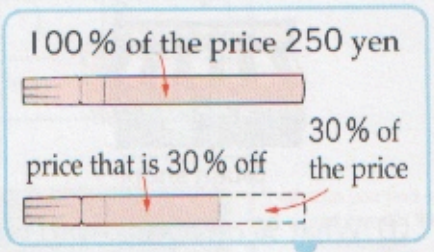
5 Carnations are grown in a green house. 48% of the area of the green house floor is used to grow carnations. If the area of green house used to grow carnations is 132 m^2, what is the total area of the green house?



4 Problems of Discounts and Extras

1 Mika bought a marker for 30% off the regular price of 250 yen. What was the sale price?





The difference between 30% of the price and the price that is 30% off is ...

Let's think about how to find the price that is 30% off.

Kota

(1) Explain the thinking of the following two students.

Misaki

$250 \times 0.3 = 75$

$250 - 75 = 175$

Answer 175 yen

Riku

$250 \times (1 - 0.3) = 250 \times 0.7$

$= 175$

Answer 175 yen

What is the meaning of the math sentence, $1 - 0.3$?

Summary

You can find the price that is 30% off in either of the following ways:

- Find 30% of the regular price and subtract it from the regular price.
- Since 30% subtracted from 100% is 70%, find 70% of the regular price.

To understand the relationship between quantities, we should pay extra attention to what 30% off means.

1 Ken bought a game that cost 3,600 yen at the price that is 25% off the regular price. What was the sale price?

2 The store purchased the pencil case on the right at 600 yen. The store is going to add 30% as the profit. How much will the pencil case cost?



Additional Problems → Page 140 R

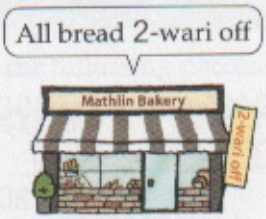
Shiho



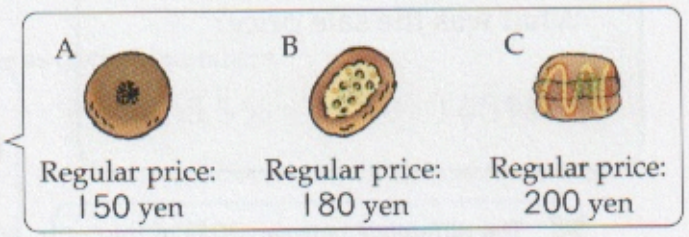
We can see "● % off" signs around us.

Use What You Have Learned

Today is a bargain day at our bakeries. The main store is doing a special sale as follows.



Main shop



① What % off does 2-wari off mean?

② What are the discounted prices of bread A, B, and C per piece?



Pay attention to the difference between "●-wari off" and "● wari."



Shiho

The three types of bread are all 2-wari off, but their regular prices are different, so their discounted prices are also...

③ How much are two pieces of bread A? What about three pieces of bread A?



Ami

I'll find the total cost before the discount and then make it 2-wari off...

I'll find the discounted prices per piece...



Haruto

The station-front branch, too, is doing a special sale today, and selling the same types of bread as the main store.



Store in front of the station

Types of bread that are usually less than 200 yen are 30 yen off.
Types of bread that are usually 200 yen or more are 50 yen off.

④ At the station-front branch, how much are the discounted prices of bread A, B, and C per piece?



Kota

I wonder how many yen off is the type of bread that is usually 200 yen.

⑤ On a bargain day, at which store can you buy bread A, B, and C at lower prices, the main store or the station-front branch, or doesn't it make any difference?

Check Your Understanding

1 Fill the blanks in the table below with the appropriate numbers.

Do you understand how to express rates?
Page 72 2

Decimal or whole number that expresses a rate	Ⓐ	0.35	1	Ⓑ
Percentage	20%	Ⓒ	Ⓓ	143%

2 City A protects sea turtle eggs. This year, 646 of the 680 eggs laid hatched. What % of total eggs hatched?



Can you find rates and express them in percentage?
Page 72 2

3 Write the appropriate number or word in each ☐.

① Quantity being compared = ☐ × Rate

② 30% of 20 L is ☐ L.

③ 6 L is ☐ % of 20 L.

④ 30% of ☐ L of water is 6 L.

Do you understand how to find quantities being compared, base quantities, and rates?
①② Page 74 1
③ Page 72 2
④ Page 76 2

4 Misaki is explaining how to find the answer for the problem on the right. Write the appropriate number in each ☐ to complete Misaki's idea.

Problem
A customer bought a piece of clothing for 40% off the regular price of 2,000 yen. What was the sale price?

Misaki

Since 1 − Ⓕ = Ⓖ, 40% off the regular price is Ⓖ of 2,000 yen.
2,000 × Ⓖ = Ⓗ, so the sale price was Ⓗ yen.

Ⓕ ☐
Ⓖ ☐
Ⓗ ☐

Do you understand what a discount means?
Page 79 1

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Relationship between Two Quantities and Think about How to Make a Comparison Using Rates

The table on the right shows records of basketball shots by players A and B. Think which player we can say was more successful in making baskets.

Record of shots

	Baskets	Total attempts
Player A	6	10
Player B	8	16

① To determine which player we can say was more successful in making baskets, which of the following ways do you use, Ⓐ or Ⓑ?

Ⓐ Compare each of the players by their number of baskets.
Ⓑ Express the relationship between the number of baskets and total attempts by player A as a rate. Express the relationship between the number of baskets and total attempts by player B as a rate. Then, compare the rates.

② Records of players C and D are as follows. Can either of these two players be considered to be at the same level of shooting skills as player A?

Record of shots

	Baskets	Total attempts
Player C	8	12
Player D	12	20


Kota

The number of baskets made and the total attempts are all different for these players but we can use rates to...

Look back on what you have learned in "Let's Think about How to Compare (2)" and discuss.


Shiho

Now I know that we can use rates to compare the relationship between two quantities even when the numbers are all different.


Riku

Rates can be expressed in many ways, such as percentage and bu-a-i.
I wonder if there is any other way.


You'll study it in 6th grade.

Do You Remember?

Answers→ Page 149

1 Calculate the following.

Order and Properties of Operations Page 150 ③

① $15-(6.8+3.2)$ ② $(10-8.75)\times 8$

③ $12-4.5\div 0.5$ ④ $3.9+(2-1.1)\div 9$

⑤ $7.2\times 8-6.4\div 0.2$ ⑥ $5.4\times 0.7+5.4\times 1.3$

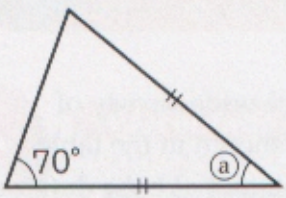
2 The numbers below are the runs Shin's baseball team scored in the last five games.

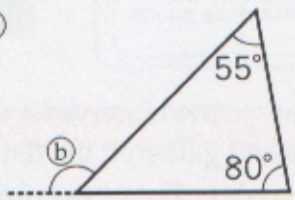
4, 6, 0, 2, 1

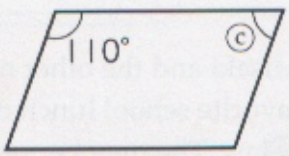
How many runs does the team have to make in the 6th game to make the average runs of 3?

3 What are the measures of angles ⓐ, ⓑ, and ⓒ?
Calculate their measures.

Angles of Triangles and Quadrilaterals Page 151 ⑦

① 

② 

③ 

Parallelogram

4 Look at the table below and answer the following questions.

Survey of Favorite and Least Favorite Vegetables (students)

		Bell peppers		Total
		Favorite	Least favorite	
Carrots	Favorite	8	ⓐ	18
	Least favorite	ⓑ	6	15
Total		ⓒ	16	33

① How many students like both bell peppers and carrots?
② What type of students does ⓐ in the table represent?
③ Fill in ⓐ, ⓑ, and ⓒ with appropriate numbers.

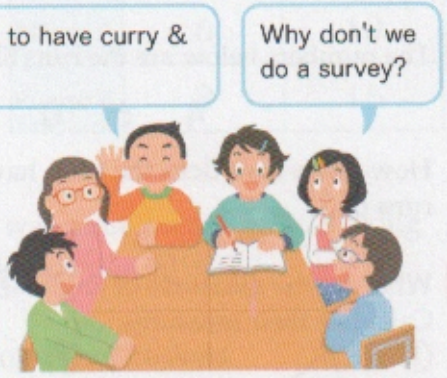
Warm-up

5 What kind of graph is suitable to represent the following?
① A survey of sports your classmates like
② Your height measured in April every year

What are students' favorite school lunch meals?

Misaki's school has a school lunch request week in March. During the five days of the school lunch request week, the school serves students' favorite lunches.

Misaki and the other members of the School Lunch Committee are planning how to decide school lunch menus.




Misaki

Three out of six members of the School Lunch Committee liked curry & rice. That's half of the total number, but...

Misaki and the other committee members did a school-wide survey of favorite school lunch meals and gathered the data as shown in the table below. The members expressed the data in a bar graph to make the data easy to grasp visually.


Haruto

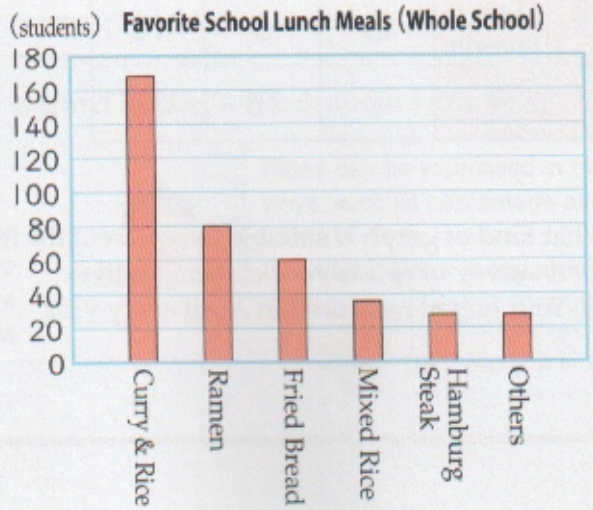
The survey asked the students to choose one out of the 10 meals selected from the school lunch meals.

In the bar graph, we put together the meals not included in the top 5 under "others."


Ami

Favorite School Lunch Meals (Whole School)

Meals	Students
Curry & Rice	168
Ramen	80
Fried Bread	60
Mixed Rice	36
Hamburg Steak	28
Stew	13
Chow Mein	7
Spaghetti	4
Fried Rice	3
Gratin	1
Total	400



What observations can you make based on the table and the bar graph?

What are the trends in the popularity of lunch meals at Misaki's school as a whole?


Kota

As expected, curry & rice was the most popular meal. In a bar graph, you can recognize the numbers of students at a glance.


Shiho

It looks like we can select the five most popular meals.


Ami

I wonder if these trends are the same across grades. We should respect opinions from 1st and 2nd graders, too.


Riku

I wonder if more than half of the students like curry & rice.


Misaki

I'm not sure about that...
From the bar graph on page 84, we don't know exactly the rate of the number of students who chose each meal to the total number of students.

15 Let's Investigate Rates by Representing Them in Graphs

Percentage Bar Graphs and Pie Charts



1 The table on the right shows the results of the school-wide survey of favorite school lunch meals.
Investigate the graphs that show rates.

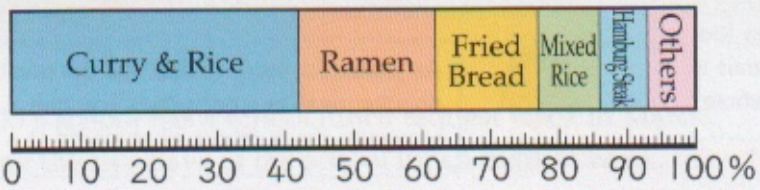
Favorite School Lunch Meals (Whole School)

Meals	Students	Rate (%)
Curry & Rice	168	42
Ramen	80	20
Fried Bread	60	15
Mixed Rice	36	9
Hamburg Steak	28	7
Others	28	7
Total	400	100

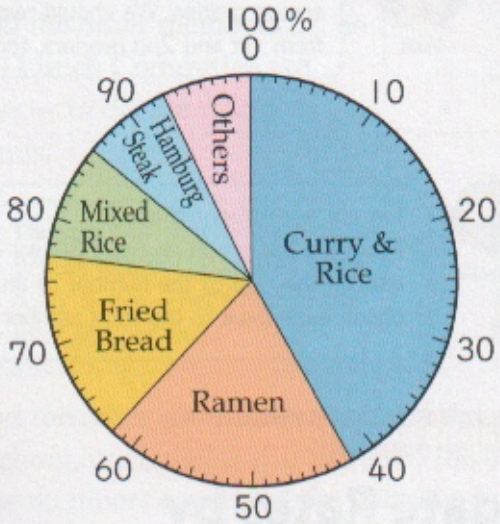
(1) What kind of graph can make it easier to see the rate of the number of students who chose each meal to the total number of students?

To make it easier to see rates, graphs like the ones shown below are used.

Ⓐ **Favorite School Lunch Meals (Whole School)**



Ⓑ **Favorite School Lunch Meals (Whole School)**



Curry & rice accounts for 42%. The section ranges from tick mark 0 to 42.



Haruto

Since the section for ramen ranges from tick mark 42 to 62, ramen accounts for 20%.



Ami

A graph like Graph Ⓐ is called a **percentage bar graph.** It uses a rectangle to represent the whole and is divided into different sized sections by straight lines based on their rates. Graph Ⓑ is called a **pie chart.** A pie chart uses a circle to represent the whole, and then it is divided by radii to represent the rates of various parts.

Let's explore the properties of percentage bar graphs and pie charts.

2 Look at the percentage bar graph Ⓐ and the pie chart Ⓑ and investigate the following.

(1) Is the number of students who chose curry & rice more than half or less than half of the total number of students?

(2) What is the rate of the numbers of students who chose curry & rice, ramen, or fried bread combined to the total number of students? Express the rate as a fraction.

(3) About how many times as much is the number of students who chose curry & rice as the number of students who chose ramen?



Did you notice anything else?

Summary

Percentage bar graphs and pie charts are useful to show the rates of different parts to the whole and to compare the rates of different parts.



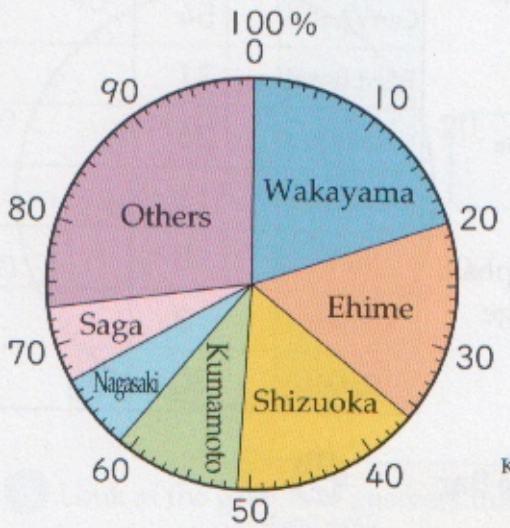
Ami

The rate of the number of students who chose curry & rice was less than 50%.

Although the percentage bar graph represents the whole as a rectangle and the pie chart as a circle, both of them make it easy to see the rates.

1 The pie chart below represents the rates of orange yields from different prefectures. Investigate the following.

Rates of Orange Yields from Different Prefectures (2016) (Developed by Tokyo Shoseki based on Statistics on Production and Shipment of Fruits and Nuts (the Ministry of Agriculture, Forestry and Fisheries, 2017))



① What is the rate of each section in %?

② How many times as much is the yield from Wakayama as that from Kumamoto?

③ What fraction of the total yield is the yields from Wakayama, Ehime, and Shizuoka combined? How many % is the combined yields?

Additional Problems → Page 140 S



Kota

I suspect that the rates of favorite lunch menus differ between school grades.

The table on the next page is a breakdown of data on page 84 by group of grades: lower, middle and upper grades.

2 Using a percentage bar graph or a pie chart, represent the survey results from each group of grades: lower, middle and upper grades.

Let's summarize how to create a percentage bar graph and a pie chart.

Favorite School Lunch Meals (Lower Graders)

Meals	Students	Rate (%)
Curry & Rice	54	45
Ramen	24	
Hamburg Steak	14	
Fried Bread	12	
Mixed Rice	10	
Others	6	
Total	120	

Favorite School Lunch Meals (Middle Graders)

Meals	Students	Rate (%)
Curry & Rice	60	
Ramen	30	
Fried Bread	17	
Mixed Rice	15	
Hamburg Steak	12	
Others	16	
Total	150	

Favorite School Lunch Meals (Upper Graders)

Meals	Students	Rate (%)
Curry & Rice	54	
Fried Bread	31	
Ramen	26	
Mixed Rice	11	
Stew	5	
Others	3	
Total	130	

1 Find the percentage of students who chose each meal to the total number of students in each grade band and complete the tables.



Round the answers to the tenths place.

2 Complete the percentage bar graphs and the pie charts on the next page using the rates from the tables.



How to Create a Percentage Bar Graph and a Pie Chart



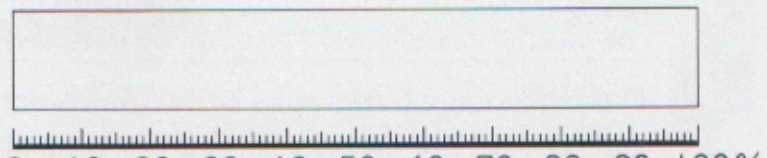
〈Draw Graph〉

1. Calculate the rate for each part and express it as a percentage. If the total does not add up to 100%, adjust the largest category or "others" category by making it bigger or smaller so that the total is 100%.
2. In general, arrange the parts in descending order, starting with the largest part and dividing each region by its percentage.

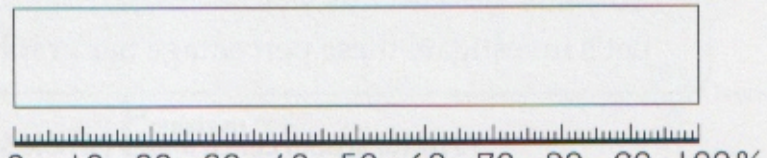


The "others" category is the last one.

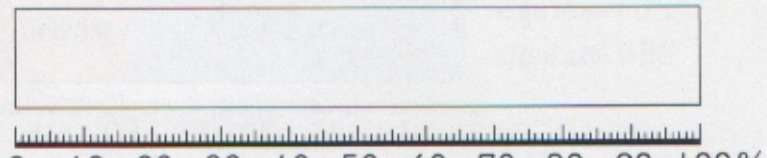
Favorite School Lunch Meals (Lower Graders)



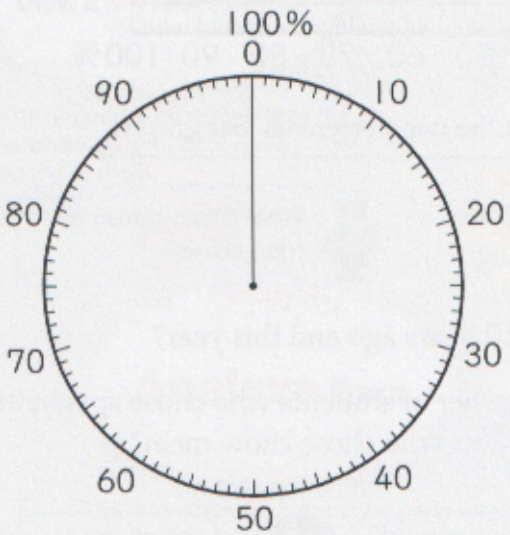
Favorite School Lunch Meals (Middle Graders)



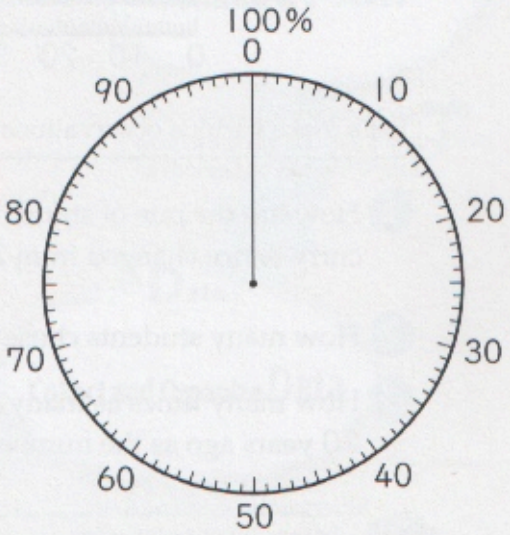
Favorite School Lunch Meals (Upper Graders)



Favorite School Lunch Meals (Lower Graders)

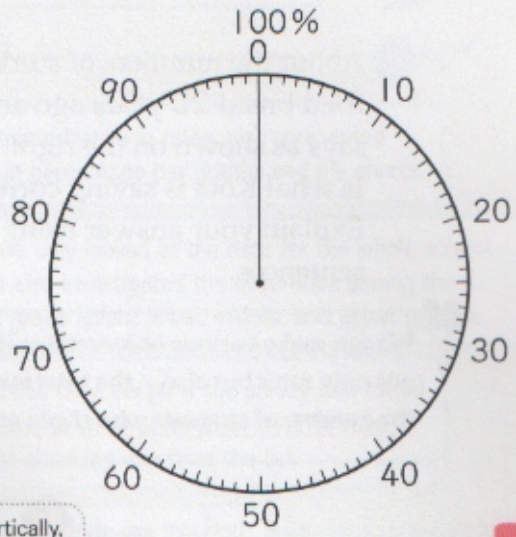


Favorite School Lunch Meals (Middle Graders)



3 Look at the graphs of answers from lower, middle and upper grades and discuss lunch meals to select.

Favorite School Lunch Meals (Upper Graders)





Also, compare these graphs with the graph for the whole school on page 86.



Haruto

Curry & rice ranks first in all groups of grades. Among upper graders, stew, too, is...

Shiho



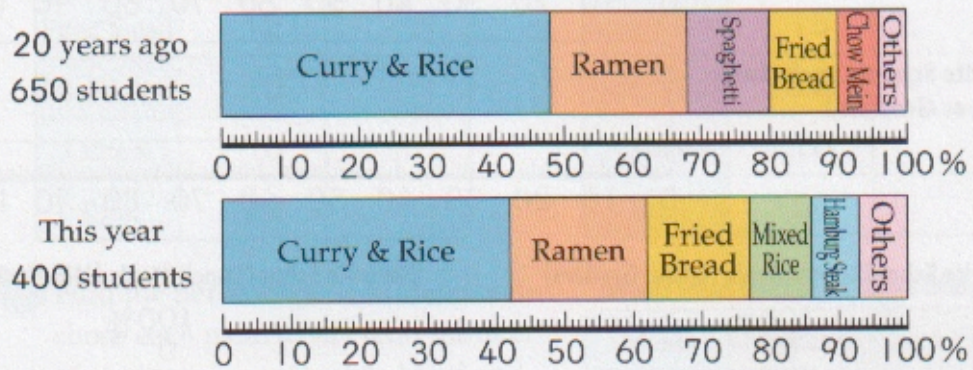
If we arrange the percentage bar graphs vertically, we can compare the rates more easily.

The school did the same survey 20 years ago, and the teacher gave us the results.

3

The percentage bar graphs below represent the results of school-wide surveys of favorite school lunch meals that were conducted 20 years ago and this year.
Let's investigate these percentage bar graphs.

Favorite School Lunch Meals (Whole School)



Let's make various observations based on the two percentage bar graphs.

1. How has the rate of students who chose curry & rice changed from 20 years ago?



What about ramen and fried bread?

2. How many students chose ramen 20 years ago and this year?

3. How many times as many is the number of students who chose spaghetti 20 years ago as the number of students who chose chow mein?



Riku: I may begin calculating the actual numbers of students from the total numbers and the rates...



Ami: If we compare the rates...

4. About the numbers of students who chose fried bread 20 years ago and this year, Kota says as shown on the right.
Is what Kota is saying correct or incorrect?
Explain your answer using words and math sentences.

Since the rate for this year is higher than that for 20 years ago, the number of students for this year is greater than that for 20 years ago.



Kota

We can make various observations if we carefully check not only rates but also **the total number of students and the number of students who chose each meal.**



Shiho



Misaki: When we look at the percentage bar graphs and pie charts around us, we should pay attention to the total numbers.

Let's look back the problem solving methods we have learned so far.


Misaki

's school has a school lunch request week in March.

English

Problem

Pose a Problem

The committee members decided to select school lunch meals for the request week.

Plan

Make a Plan

The committee members decided to do a survey and ask the students to choose one out of 10 meals.

Data

Collect and Organize Data

The committee members did a school-wide survey and collected data.

Analysis

Analyze Data

The committee members organized data into the following groups: whole school; lower, middle, and upper graders. The members represented the data in graphs and found out some characteristics of the data.

Conclusion

Draw a Conclusion

The committee members chose the lunch meals with clear reasons based on the analysis.





Drawing a conclusion is not the end of the process. We should review the process and examine if the conclusion was the best and if there was a better analysis method.


Kota

To investigate trends in rates, we represented the data in percentage bar graphs and pie charts.


Ami

At first, we only looked at the data for the whole school. Then, we also investigated the difference among the different grade levels: lower, middle, and upper grades.


Haruto

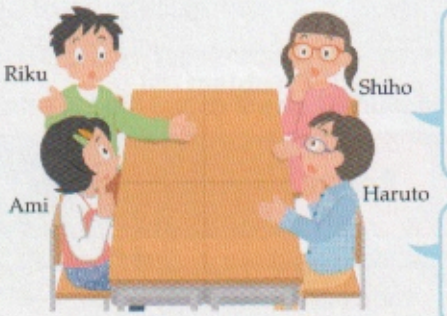
It might have been better if the survey had asked the students to write down their favorite meal instead of choosing one from the list.


Shiho

When problems comes up around us, I'd like to use this kind of method to clear up the matter.

Use What You Have Learned

Riku and his classmates are reviewing their school life.



Riku: The school nurse says the number of injured students is increasing.

Shiho: I went to the school library at recess, and there were many students there. I want to know more about the library usage...

Haruto: There are many students who have a cold. I wonder about how many...

Pick one thing you want to investigate about your class and clear up the matter using what you have studied.

For example, suppose you investigate injuries at school.

① Pose a Problem. What do you want to find out?


Riku

I'll investigate whether or not there are more injuries among us, 5th graders, than students in other grades.

② Make a Plan. What data do you need to gather?


Ami

Maybe we can get records of injured students at the nurse's office.

Data include grades and the types, places, date, and time of injury, but we should focus on ____ ...


Kota

③ Collect and organize Data.

④ Analyze the data. What kind of characteristics can you find out?

I'll represent the number of injured students in each grade in a bar graph...


Shiho

⑤ Draw a Conclusion.

Bars in the bar graph are almost the same in length across grades. So, we don't think there are more injuries in 5th grade than students in other grades.


Riku

⑥ Review the process.


Haruto

Why don't we compare grades using the rates of the number of injuries?

What about the difference between months instead of grades? Changes in the number of injuries...


Ami

Check Your Understanding

1 Taichi's school decided to investigate the usage of its school library. The table below summarizes the numbers and rates of books checked out in January at Taichi's school according to different categories of books.
Answer the following questions.

◀ Can you compare rate and numbers in tables and graphs?

①②
Page 87 2
③ Page 85 1
④ Page 90 3

Numbers and Rates of Checked Out Books (January)

Types	Books	Rates (%)
Stories	120	Ⓐ
Biographies	Ⓑ	25
Sciences	54	18
Encyclopedia	Ⓒ	Ⓓ
Others	30	Ⓔ
Total	300	100

① Fill in the blanks in the table on the right with numbers.

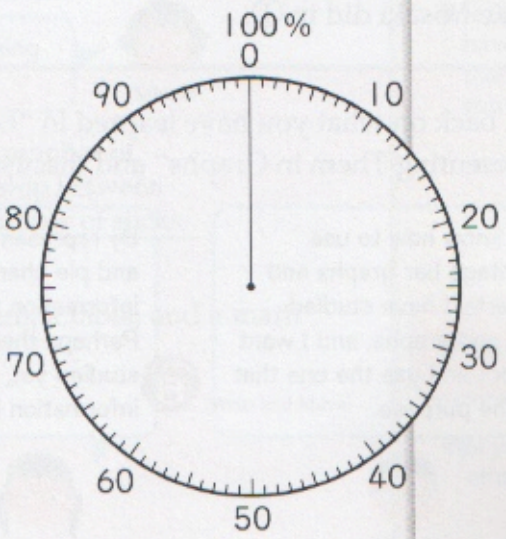
Does the total of the percentages add up to 100%?

② Represent these rates using a pie chart below.

③ What fraction of the books were biographies?

④ A total of 320 books were checked out in December, and biographies accounted for 25%.
Which of the following statements, Ⓕ, Ⓖ and Ⓗ, is correct about the numbers of biographies checked out in December and January? Explain the reason why.

Rates of Borrowed Books (January)



Ⓕ Equal Ⓖ Greater in December Ⓗ Greater in January

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Characteristics of Data and Select a Graph that Suits the Purpose

① Kota is going to investigate the difference in daily temperature changes among fine, cloudy, and rainy weather.



Kota

I'll measure the temperature every hour on fine, cloudy, and rainy days and represent the data in a graph.

What type of graph is good for this situation? Explain the reason why.



Misaki

〈Goal〉
To investigate how temperature changes by the hour in each type of weather
〈Graph selected and reason for selection〉
To represent changes by the hour, a [ ] graph is used.
This is because [ ] graph is characterized by...

② Ami is going to investigate the types and rate of vehicles passing in front of the school in 1 hour and represent the results in a graph.
What type of graph is good for this situation? Explain the reason why like Misaki did in ①.

Look back on what you have learned in "Let's Investigate Rates by Representing Them in Graphs" and discuss.

Now I know how to use percentage bar graphs and pie charts. I have studied tables and graphs, and I want to select and use the one that suits the purpose.



Haruto

By representing rates in percentage bar graphs and pie charts, I was able to find some information not given in tables or bar graphs. Perhaps there are still some graphs we haven't studied yet, and I wonder what type of information is made obvious in these graphs.



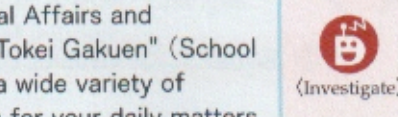
Shiho



You'll study it in 6th grade.



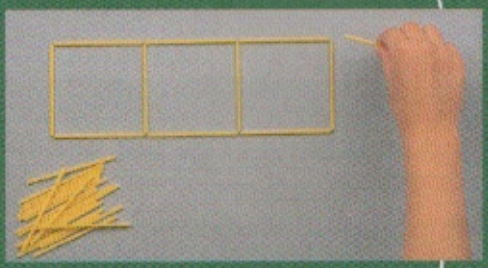
The Statistics Bureau, the Ministry of Internal Affairs and Communications has a webpage "Naruhodo Tokei Gakuen" 〈School for the Study of Statistics〉, which provides a wide variety of statistical data. Use the data to find answers for your daily matters.



〈Investigate〉

16 Let's Investigate How Quantities Change (2)

1 Using sticks of the same length, we will make squares side by side as shown on the right.



To make 30 squares, how many sticks are needed?

Deepen Your Learning

Grasp the problem.

- What problem are we going to work on today?

1 Plan how to find the answer.



Kota: As the number of squares increases, the number of sticks also...



Misaki: We should find a pattern while making fewer squares...

Let's think about how to find the number of sticks by focusing on the relationship between the number of squares and the number of sticks.

- What idea may be useful to solve the problem?
- Is there anything you have learned before that you can use to solve this problem?

2 Put your ideas into a diagram, a table, and a math sentence.

(Write and Move)

Write down your ideas.

- Is your idea clear to others?



Haruto



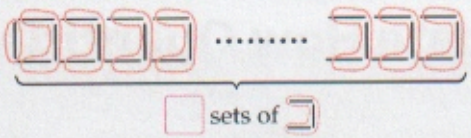


Shiho

Number of squares □	1	2	3	
Number of sticks ○	4	7	10	

Ami and Kota are explaining their classmates' ideas.

Haruto



□ sets of 3

$1+\underbrace{3+3+\cdots\cdots+3}_{\text{30 sets of 3}}=1+3\times 30$

$=\square$ Answer □ sticks



Haruto thought about the stick at the left end and the sets of 3 sticks separately. The number of the sets of 3 sticks to make 30 squares is...

Ami

Learn with your classmates.

- Can you understand your classmates' ideas are based on their diagrams, tables, and math sentences?
- What is common and what is different about your own idea and your classmates' ideas?
- What are the good points in your classmates' ideas?

3 Look at the diagram and math sentence Haruto wrote. Explain his thinking.

4 What do 1, 3, and 30 in Haruto's math sentence represent?

5 Using Haruto's idea, find the number of sticks needed to make 50 squares.

Math Sentence ▸ Answer □ sticks

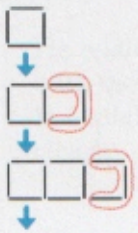
6 Based on Haruto's idea, write a math sentence that expresses the relationship between the number of squares, □, and the number of sticks, ○.



Ami

	1 stick at the left end		3 sticks per set		Number of squares		Number of sticks
30 squares	1	+	3	×	30	=	91
50 squares	1	+	3	×	50	=	151
⋮					⋮		
□ squares	1	+	3	×	□	=	□

Shiho



4

4+3

4+3+3

⋮

$4+\underbrace{3+3+\cdots\cdots+3}_{(30-1)\text{ sets of }3}=4+3\times(30-1)$

Number of squares □	1	2	3	4	5	6
Number of sticks ○	4	7	10	13	16	19

Increases by 3 each time

Answer ☐ sticks



Kota

7 Look at the diagram, table, and math sentence Shiho wrote. Explain her thinking.

8 Is the number of sticks proportional to the number of squares?



Riku

According to the table, as the number of squares increases to 2, 3 ... times, the number of sticks ...

9 Look back and summarize today's lesson.

Look back and summarize today's lesson.

- What did you learn from today's investigation?
- Which way of thinking was useful?
- What do you want to investigate next?

Summary

If you use a diagram and a table to **find the relationship between the two quantities that change in relationship to each other**, you can find answers through calculation even if the number increase.



Misaki

We can find a pattern more easily if we use a diagram and a table.



Haruto

I wonder what happens if we change the squares into another type of figure and arrange the figures.

Additional Problems
→Page 141 T



Let's look back at the ideas you used to solve problems.



Haruto

Haruto focused on the relationship between two quantities: the number of squares and the number of sticks.

When he used a diagram to think about the problem, it became easier for him to find out about the relationship between the two quantities.

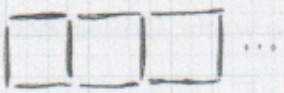
When you work on problems, not only write math sentences and answers but also consider using:

- Diagrams
- Tables
- Graphs

February 5

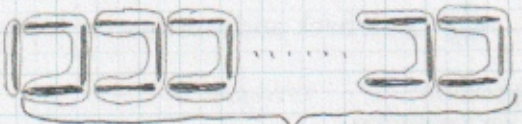
<Problem>

Using sticks of the same length, we will make squares side by side as shown below.
To make 30 squares, how many sticks are needed?



- Think about how to find the number of sticks by focusing on the relationship between the number of squares and the number of sticks.

<My idea>



①

1+3+3+ ····· 3+3 = 1+3×30

30 sets of 3

Answer 91 sticks

Classmates' reflections.



Misaki

I used the idea of finding the relationship between the two quantities in relationship to each other, I was able to calculate the answer at 30 pieces.



She wrote how her learning from before was useful.

Note Taking Tip 1

He relates the diagram with the math sentence to explain his idea clearly.

Note Taking Tip 2

Today's lesson was based on what he had learned. So, he added the date of the lesson so that he can go back to the page of his notebook.

<Shiho's Idea>

Number of squares □(pieces)	1	2	3	4	5	6	7
Number of sticks ○(sticks)	4	7	10	13	16	19	22
		+3	+3	+3	+3	+3	+3

Using a table to find the relationship between the two quantities was done on December 10.

Looking at the table, the nu 2 sticks ○ increased by three.

$4+3+3+\cdots\cdots+3 = 4+3\times(30-1)$

(30−1) sets of 3 = 91

Answer 91 sticks

<Summary>

If you use a diagram and a table to find the relationship between the two quantities that change in relationship to each other, you can find answers through calculation even if the number increase.

<My Reflection>

Both figures and tables are useful for writing math sentences, but when examining how quantities change, I found the tables easier to read.

He used a table to find the relationship between two quantities in relationship to each other.

He learned that he could find answers through calculation even if numbers are large by using the relationship between two quantities.



Riku

I thought that if I made other shapes and laid them sideways, I could think of them in the same way using figures, tables and math sentences.



He wrote about further ideas he would like to investigate.

If you look at umbrellas from directly above?

Look at various umbrellas from directly above.



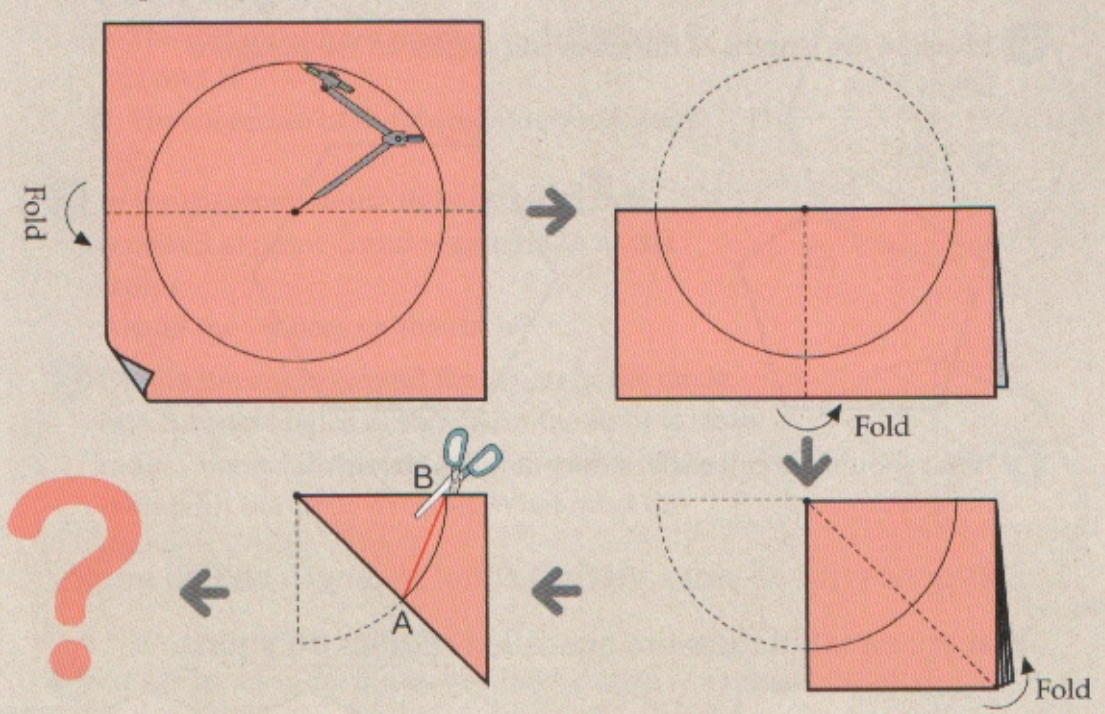
Look at shapes Ⓐ, Ⓑ, and Ⓒ and discuss what you notice.



17 Let's Further Investigate Polygons and Circles

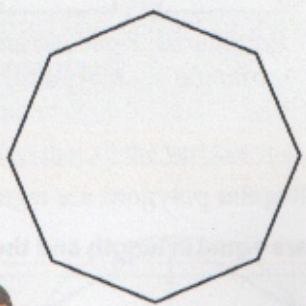
Draw a circle on a piece of origami paper, fold it three times as shown below, and cut it along line AB. Then, open the paper up. What shape will you get?

Circles
Page 150 ⑤



1 Regular Polygons

1 Investigate the properties of the octagon you created.



When we investigate the properties of figures...

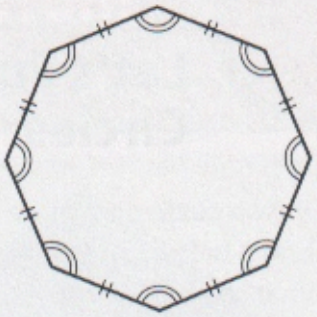


Haruto

Let's investigate the properties of polygons by focusing on their sides and angles.

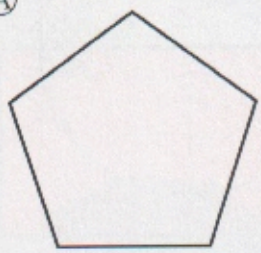
① Measure the length of each side. Also, measure each angle.

All 8 sides of the octagon you created in the previous page were the same length. All 8 angles were also the same measure. Octagons with such properties are called **regular octagons.**

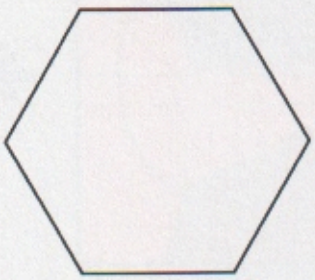


2 Measure the lengths of the sides and angles on the polygons below.

Ⓐ

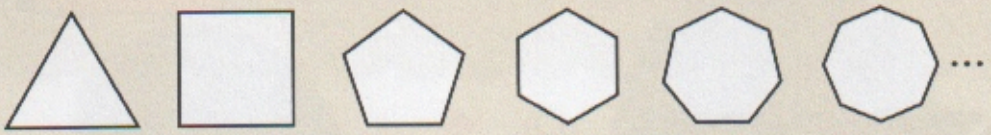


Ⓑ



3 What should we call each of the polygons above?

Polygons with all sides that are equal in length and all angles that are equal in measure are called **regular polygons.**



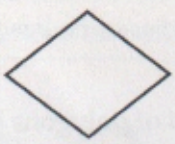
Equilateral triangle | Regular quadrilaterals (Square) | Regular pentagon | Regular hexagon | Regular heptagon | Regular octagon | ...

Regular polygons are regular in shape because **their sides are equal in length and their angles are equal in measure.**

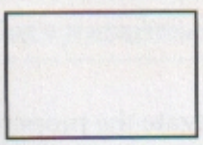


Ami

4 Can we call the polygons on the right regular polygons?
Also, explain the reason for your answer.



Rhombus



Rectangle



Pentagon



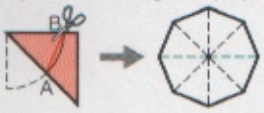
Riku

I wonder how I can draw a regular polygon.

2 Draw a regular octagon.


Misaki

When we made one with a piece of origami paper...

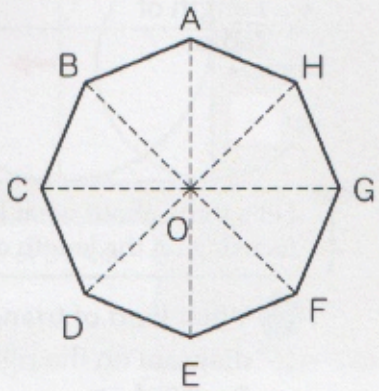


Let's investigate the properties of regular polygons and think about how to draw regular polygons.

1 In the regular octagon on the right, investigate the following.

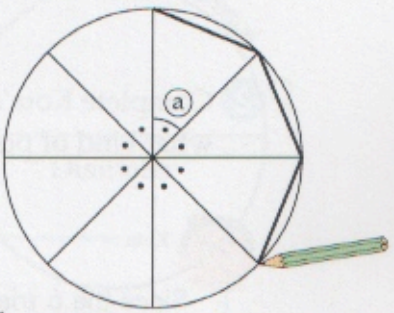
Ⓐ The distances from point O to vertices A, B, ... and H

Ⓑ The measures of the 8 angles around point O



2 In the diagram on the right, draw a circle centered at point O and segment OA as the radius.
Are all the vertices on the circle?

3 Divide the angle around the center of the circle into 8 equal angles as shown on the right to draw radii. Connect the points of intersection of the radii with the circle in order. What must the measure of angle ⓐ be?



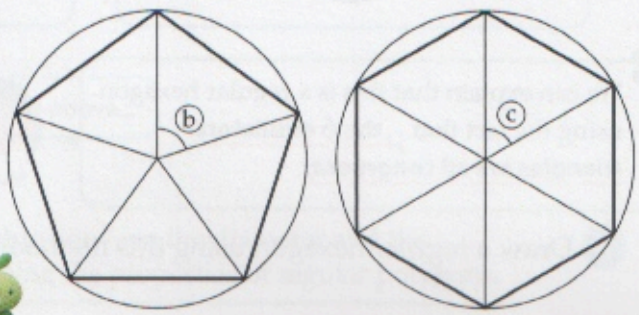
4 Check to see if the figure you have drawn is a regular octagon.

Summary

To draw a regular polygon, divide the angle around the center of a circle into equal parts to draw radii, and set the points of intersection of the radii with the circle as vertices.

This drawing method is based on the fact that all the vertices of a regular polygon are on a circle.

1 Using the method to equally divide the angle around the center of a circle, draw a regular pentagon and a regular hexagon.



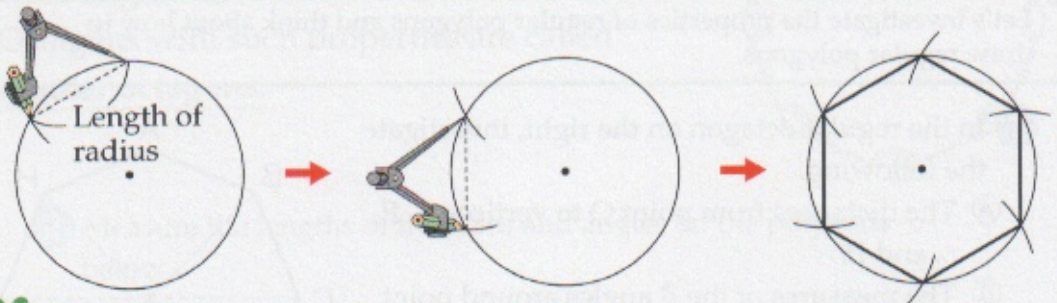
What will the measures of angles ⓑ and ⓒ be?




Kota

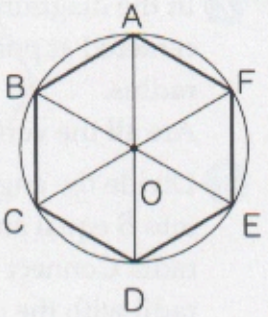
A regular polygon fits exactly into a circle.

3 What kind of polygon is the one drawn as shown below? Explain the reason why.



Let's think about what kind of polygon this is by focusing on the length of the radius.

1 What kind of triangle is triangle OAB in the diagram on the right? What about the other 5 triangles?



2 Complete Kota's and Ami's ideas to tell what kind of polygon it is.



Kota

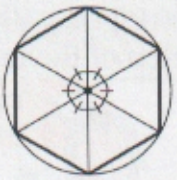
Since the 6 triangles are all congruent equilateral triangles, the lengths of the sides of the hexagon are ...
The measures of the angles are...





Ami

Since the 6 triangles are all congruent, the measures of the angles formed around the center of the circle are ...



We can explain that this is a regular hexagon using the fact that **the 6 equilateral triangles are all congruent.**



Shiho

We were able to draw a regular hexagon using a circle and its radii.



3 Draw a regular hexagon using this method. Make the radius of the circle 5 cm.



You will learn about programming on page 134.

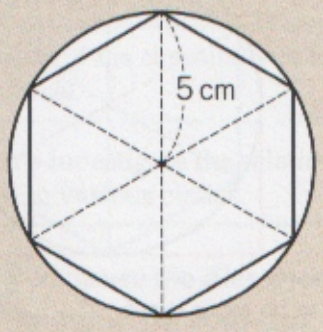


Haruto

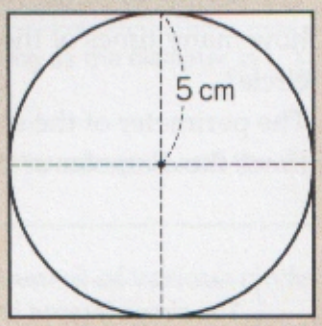
The length around the regular hexagon we drew in 3 is...

2 Length around Circles

Haruto is thinking about the lengths around regular polygons.



The length around a regular hexagon is ☐ cm



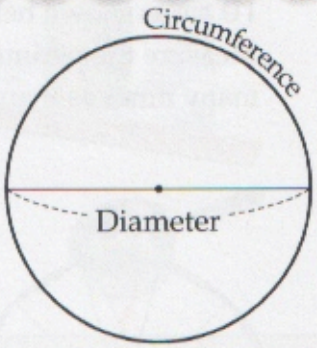
The length around a square is ☐ cm



Haruto

Both of the two circles above have a diameter of ☐ cm.
I wonder what is the length around a circle?

1 The length around a circle is called the **circumference**. Investigate the relationship between diameter and circumference.



1 How should we investigate this relationship?



Riku

We can estimate the circumference from the perimeters of regular polygons drawn inside and outside of a circle ...

If we measure the diameters and the circumferences of circles in many different sizes...



Misaki

(1) Based on Riku's idea, use a regular polygon to find out how many times as long the circumference is as the diameter.



Ami

In Haruto's diagram above...

Let's investigate the relationship between the diameter and the circumference of a circle by using the properties of regular polygons.

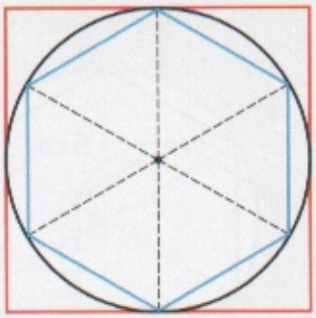
Think about it using a circle with a diameter of 10 cm.

〈Watch〉

2 Look at the diagram on the right and investigate the following.

Ⓐ The perimeter of the regular hexagon is how many times of the diameter of the circle?

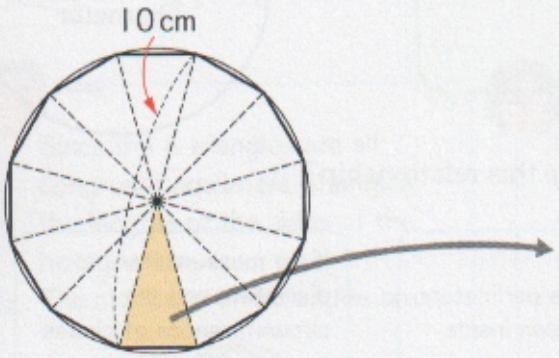
Ⓑ The perimeter of the square is how many times the diameter of the circle?

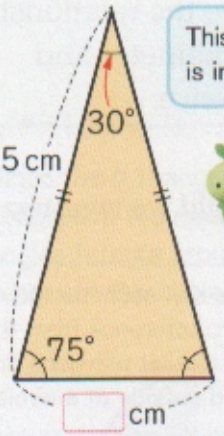


Shiho: The circumference is slightly longer than 3 times the diameter but shorter than 4 times the diameter.

Riku: I want to be more exact. I'll increase the number of vertices of the regular polygon...

3 A regular dodecagon was drawn inside a circle with a diameter of 10 cm as shown below.
Measure the perimeter of the regular dodecagon to find how many times as many the perimeter is as the diameter of the circle.





This isosceles triangle is in actual size.

□ × 12 ÷ 10 = □ (times)

□: Perimeter of regular dodecagon

10: Diameter

Summary

The circumference is about 3.1 times as long as the diameter.

We increased the number of vertices of a regular polygon, making the shape closer to a circle to make an estimate.

Kota: I want to measure circular objects around me to check if this is true.

(2) Using Misaki's idea from page 105, investigate about how many times as long the circumferences are as the diameters on various circles.

I wonder if the circumference is about 3.1 times as long as the diameter in any circle.



Misaki

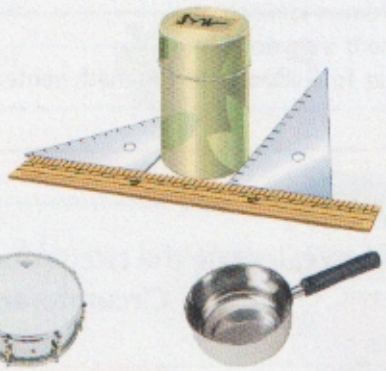
Let's investigate the relationship between the circumference and the diameter using various circles.

4 Measure the diameters and the circumferences of various circles and summarize your data in the table below.

How to measure circumference



How to measure diameter



	Can	Tray				
Circumference (cm)	22					
Diameter (cm)	7					

5 How many times as long are the circumferences as the diameters? Round your answers to the hundredths place.

Write the quotients in the bottom row of the table.

Summary

The rate of the circumference to the diameter is equal in any circle.

Ami



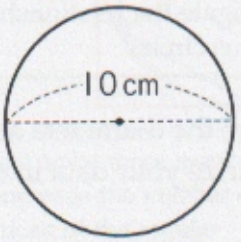
I wonder if we can calculate the circumference of a circle.

The number that expresses how many times as long the circumference is as the diameter is called the rate of circumference, and it is represented with **Pi**, The value of pi is about 3.14.

Pi = Circumference ÷ Diameter

2

Find the circumference when the diameter is 10 cm.



Let's think about a math sentence for calculating the circumference of a circle.

Based on the math sentence: Pi = Circumference ÷ Diameter...



Ami

Summary

We can calculate the circumference using the following formula.

Circumference = Diameter × Pi

Once the diameter is fixed, the circumference is specified accordingly.

1

Find the circumferences of the circles below.

① A circle with the diameter of 12 cm.



② A circle with the diameter of 3 cm.



③ A circle with the radius of 1 cm.

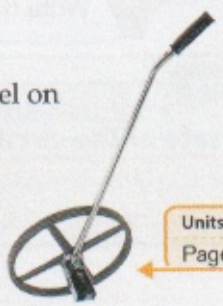


Additional Problems
→ Page 141 U

2

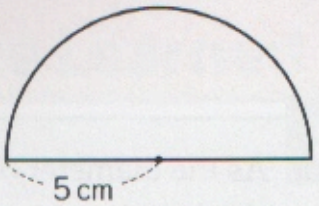
The trundle wheel on the right has a wheel with a circumference of 1 m. What is the diameter of the wheel on this trundle wheel? Round your answer to the nearest tenths place.

□ × 3.14 = 100



Units of Length
Page 151 ⑪

3 The figure on the right is a circle folded in half and cut into the shape. Find the length around this figure.




Haruto: When you think about a circle, pay attention to its circumference and diameter.

Mathlin Newsletter

Calculating Pi

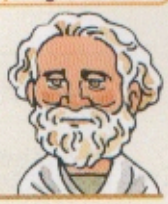
In routine calculations, 3.14 is used as the value of pi. However, the actual value of pi is 3.141592653589..., and it goes on forever. Many people throughout history have tried to calculate the most accurate value of pi.


Write the fractions as decimal numbers and compare them to the value of pi.

About 2,200 years ago:

Archimedes in Greece discovered that the value of pi is greater than $3\frac{10}{71}$ but less than $3\frac{1}{7}$.

Relationships between Fractions and Decimal / Whole Numbers
Page 150 ④



$$3\frac{1}{7} > \text{pi} > 3\frac{10}{71} = \square$$

About 1,500 years ago:

A Chinese mathematician Zu Chongzhi used $\frac{355}{113}$ as the value of pi.



$$\frac{355}{113} = \square$$

About 300 years ago:

A Japanese mathematician Seki Takakazu calculated that pi is less than 3.14159265359.


Even Japanese people were thinking about this. weren't they?



Today:

They say that we know the value of pi to about 22 trillion places after the decimal point (as of 2016).

How the Diameter and the Circumference Change in Relationship to Each Other

3 As the diameter of a circle changes, how does the circumference change?



To investigate relationships, a table...



Kota

Let's explore the relationships by focusing on the relationship between two quantities.

1 Write a math sentence to calculate the circumference, ○ cm, if the diameter is □ cm.



Shiho

Diameter × Pi = Circumference
□ × ...

2 As □ (the diameter) changes 1, 2, 3, ..., 6, what are the corresponding values for ○ (the circumference)? List your answers in the table below.

Diameter □ (cm)	1	2	3	4	5	6	7	8
Circumference ○ (cm)								

3 Is the circumference proportional to the diameter?

Proportional Relationships
Page 151 ⑩

Summary

As □ (the diameter) becomes 2, 3... times, ○ (the circumference) becomes 2, 3... times, too. So, ○ (the circumference) is proportional to □ (the diameter).

4 How many times as long is the circumference of a circle with a diameter of 12 cm as the circumference of a circle with a diameter of 4 cm?



Riku

First, we should find the length of each circumference...

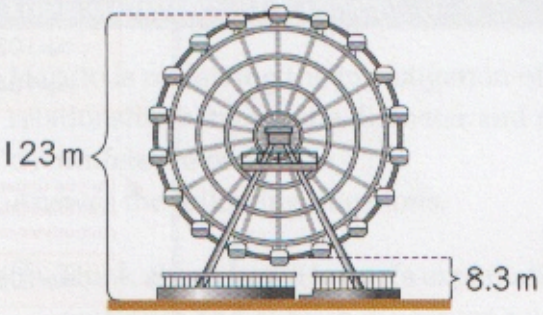
Since the diameter is 3 times as long, the circumference is also...



Misaki

Use What You Have Learned

There is a Ferris wheel as shown below. It is 123 m tall.





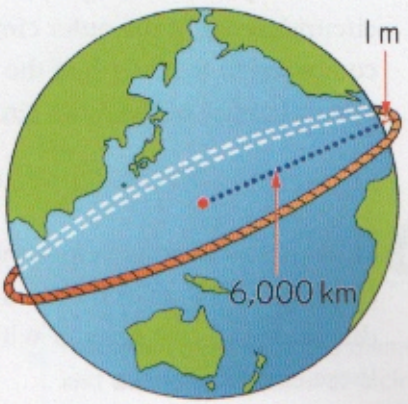
Suita City, Osaka Prefecture

① How many m is the diameter of the rotating part of this Ferris wheel?

② When this Ferris wheel makes one rotation, how far will the people in a passenger car travel?
Round your answer to the nearest tenths place.

③ Suppose the passenger cars of this Ferris wheel travels at 20 m per minute.
How many minutes does it take to make one rotation in a passenger car?
Round your answer to the nearest whole number.

Imagine placing a rope around the Earth 1 m above the ground as shown on the right.
How much longer will the rope be compared to the circumference of the Earth?
Predict first, and then calculate the answer.



Use 6,000 km for the radius of the Earth.

Check Your Understanding

1 Using a circle with a radius of 6 cm, draw a regular pentagon and a regular hexagon.

Can you draw a regular polygon using a circle?

Page 103 2
Page 104 3

2 Find the circumferences of:

① a circle with the diameter of 7 cm

② a circle with the radius of 6 cm

Do you understand how to calculate the circumference?

Page 108 2

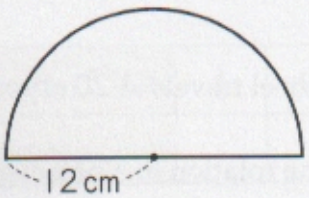
3 What is the radius of a circle if its circumference is 31.4 m?

Do you understand how to calculate the radius?

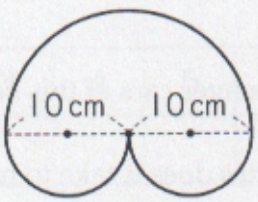
Page 108 2

4 Find the length around the figure below.

①



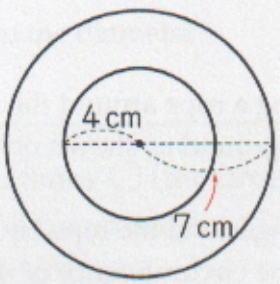
②



Can you calculate the length around a circle-based figure?

Page 108 2

5 In the diagram on the right, how much longer is the length of the circumference of the outer circle compared to the length of the circumference of the inner circle?



Can you calculate the difference between circumferences using the relationship between the diameter and the circumference of a circle?

Page 108 2

6 How many times as long is the circumference of a circle with a diameter of 28 cm as the circumference of a circle with a diameter of 7 cm? Answer without calculating the lengths of these circumferences.

As the diameter of a circle changes, how does the circumference change?

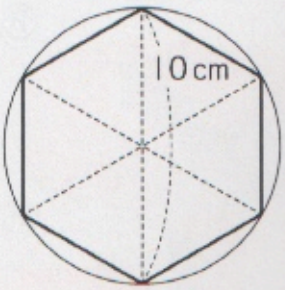
Page 110 3

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Properties of Regular Polygons and Think about the Properties of Circles

Haruto is reviewing the investigation of the relationship between the diameter and the circumference of a circle.
Answer the following questions.

Regular hexagon drawn based on a circle with a diameter of 10 cm



① Think about how Haruto's explanation continues and explain the reason why the circumference of a circle is over 3 times as long as the diameter.


Haruto

The regular hexagon above is a combination of 6 equilateral triangles that are congruent to each other.
Each side of the 6 equilateral triangles is equal to the radius of the circle, 5 cm. The length around the regular hexagon is the sum of the lengths of the 6 sides. So...

② The value of pi is 3.14. Select the correct statement(s) based on this fact from Ⓐ, Ⓑ, and Ⓒ.

Ⓐ The diameter of a circle is always 3.14 m.
Ⓑ The circumference of any circle is 3.14 times as long as the diameter.
Ⓒ The larger a circle, the larger the value of pi.

Look back on what you have learned in "Let's Further Investigate Polygons and Circles" and discuss.


Ami

Regular polygons and circles looked quite different at first, but now I know they are related to each other in many ways.

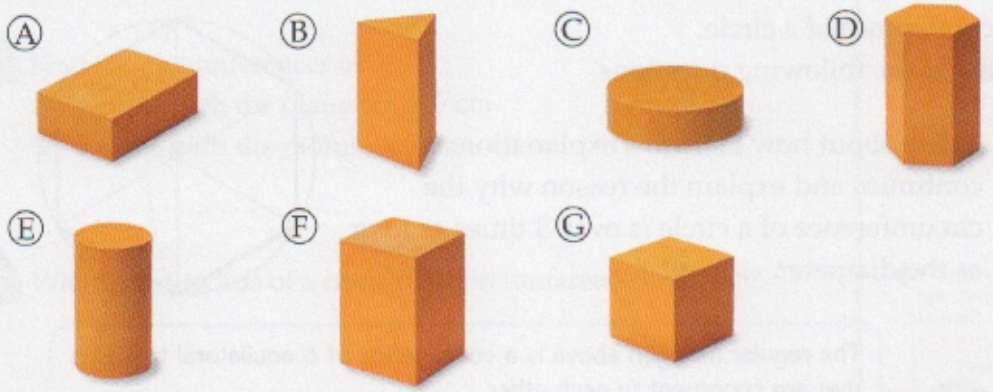

Kota

Now I'm familiar with the relationship between the diameter and the circumference of a circle. I want to think about how to find the area of a circle, too.

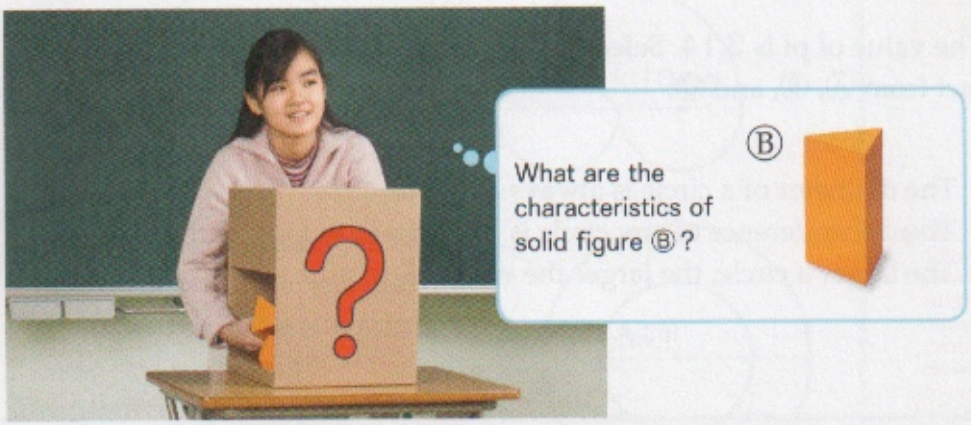

You'll study it in 6th grade.

Can you figure out which solid you have without looking?

A box contains solid figures Ⓐ through Ⓖ.



Tell others which solid figure you want to pull out of the box. Then, try to pick it out by just feeling the objects.



To find solid figure Ⓑ, what characteristics we should focus on? Discuss with your classmates.


Riku

It is long and thin...

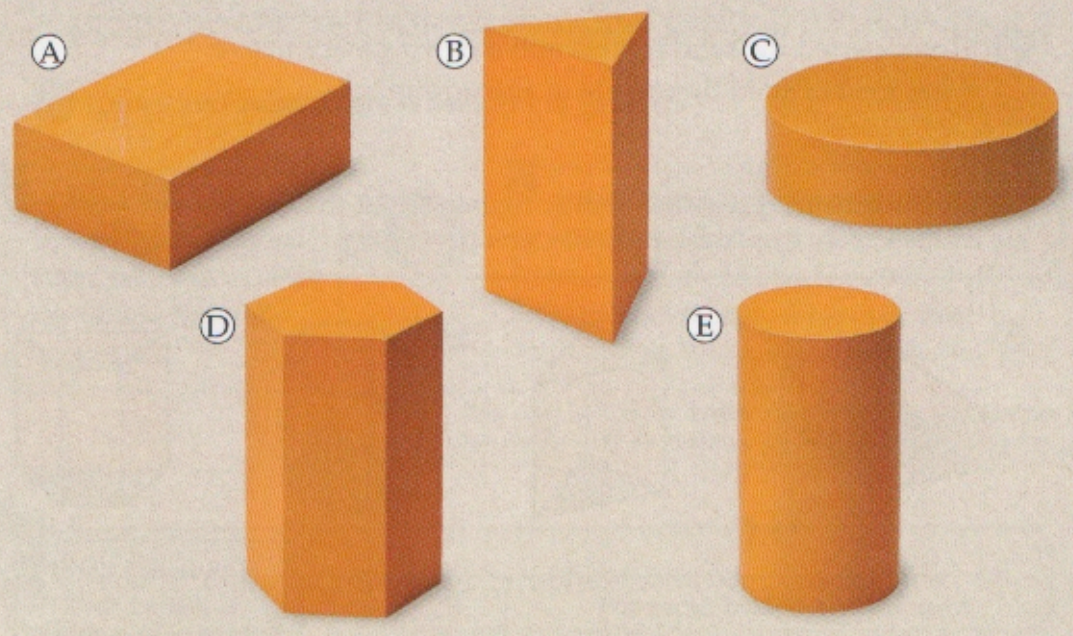

Shiho

It has triangular faces...

Prisms and Cylinders

18 Let's Further Investigate Solid Figures

Students pulled out the following five solid figures from the box.



1 Prisms and Cylinders

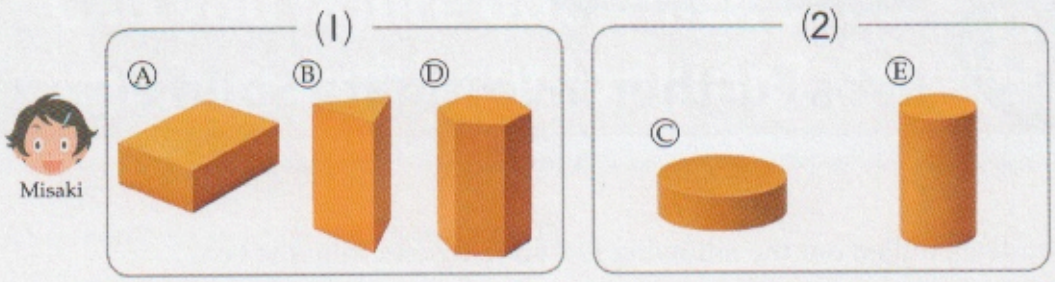
1 Sort the solid figures Ⓐ–Ⓔ into 2 groups by looking at the surfaces that make up the solid.

(1)

Ⓐ



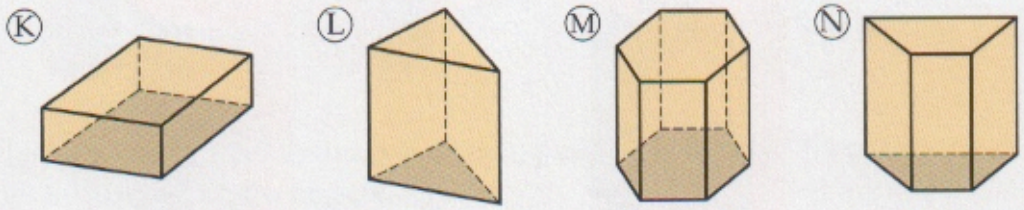
(2)



1 Misaki sorted the solid figures into the two groups above. What kind of solid figures are in each group?

Let's investigate the properties of solid figures Ⓚ–Ⓝ, that are enclosed only by plane figures like group (1) above.





When we investigated cubes and cuboids in 4th grade, we focused on...

Haruto

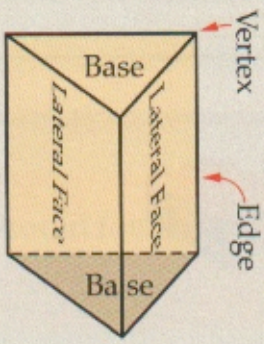
A vaulting horse is shaped like solid figure Ⓝ, isn't it?

2 Which face is parallel to the face on the bottom?

3 Are the faces on the bottom and the top congruent to each other?

4 What kinds of figures are other faces?

Solid figures like Ⓚ–Ⓝ are called **prisms.**
In a prism, the top and the bottom faces that are opposite of each other are called **bases**, and the other quadrilateral faces are called **lateral faces.**



Lateral faces of prisms Ⓚ, Ⓛ, Ⓜ, and Ⓝ are all rectangles.

5 How are the bases and the lateral faces intersecting?

6 What kinds of figures are the bases of Ⓚ, Ⓛ, Ⓜ, and Ⓝ?

If the base of a prism is a triangle, quadrilateral, pentagon, hexagon, ..., it is called a triangular prism, quadrilateral prism, pentagonal prism, hexagonal prism..., respectively.

7 Cubes and cuboids are what kind of prisms?

If you carefully look at the shapes of lateral faces, lateral faces of Ⓚ, Ⓛ, Ⓜ, and Ⓝ are all rectangles.



Ami

If you carefully look at the relationship between bases, each base of Ⓚ, Ⓛ, Ⓜ, and Ⓝ are all parallel and congruent to each other.



Kota



Riku

Why don't we find the numbers of faces, edges, and vertices, just like we did when we studied cubes and cuboids in 4th grade.

2 Investigate the number of lateral faces, vertices, and edges.

Let's find the numbers of lateral faces, vertices, and edges of prisms to further investigate the properties of prisms.

1 Fill the blanks in the table below with numbers.

	Triangular prism	Quadrilateral prism	Pentagonal prism	Hexagonal prism
Lateral Faces				
Vertices				
Edges				

I wonder if there is any relationship between the numbers of lateral faces, vertices, and edges. I'll find it out using a table...



Shiho

2 Look at the table and discuss what you notice.

Look at the table horizontally. When the number of lateral faces increases by 1, the numbers of vertices and edges...



Haruto

Look at the table vertically. The number of vertices is (...) as the number of lateral faces.



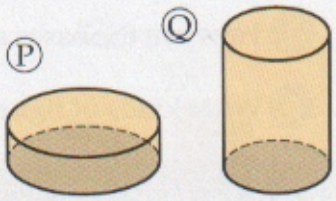
Ami



Misaki

I want to investigate the properties of the solid figures in group (2) on the previous page, too.

3 Investigate the characteristics of solid figures Ⓟ and Ⓠ that are like those in group (2) on page 116.



Let's investigate the properties of solid figures that are enclosed not by plane figures only.

Investigate the properties we focused on when we examined prisms.


Shiho

1 Which face is parallel to the face on the bottom?

2 What kind of shape are the top and the bottom faces? Also, are they congruent to each other?

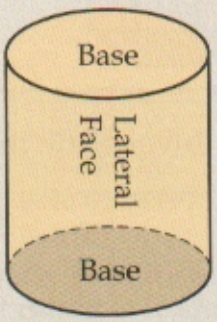
3 Can you describe the shape of the lateral face?


They are similar to prisms in that ...

Solid figures like Ⓟ and Ⓠ are called **cylinders.**

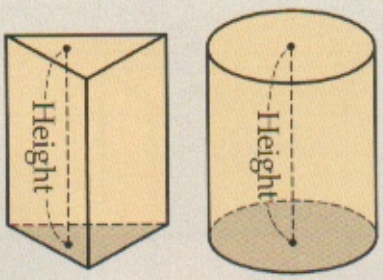
A surface that is not flat is called a curved face.

The lateral face of a cylinder is a curved face.



4 Which part of a prism or a cylinder is considered as its height?

On a line that is perpendicular to the bases of a prism or a cylinder, the length between the two bases is called the **height** of the prism or the cylinder.



The relationship between bases of a cylinder is the same as that of a prism, that is, congruent and parallel.


Riku

Ami


When we investigate prisms and cylinders from the same viewpoint, we can clearly understand the similarities and differences between them.

1 Find objects in our surroundings that are shaped like prisms and cylinders.











Aomori City, Aomori Prefecture



Yokohama City, Kanagawa Prefecture



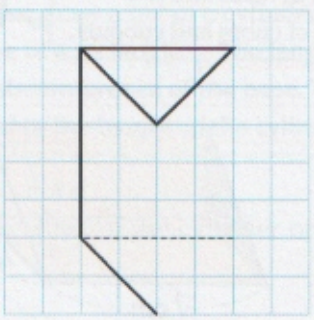




2 Draw a sketch of a prism and a cylinder.

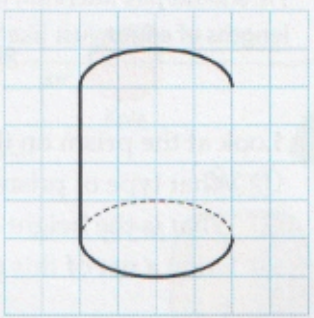
Sketches
Page 151 ⑨

①



Complete the drawings below while paying attention to which edges are parallel to each other.

②

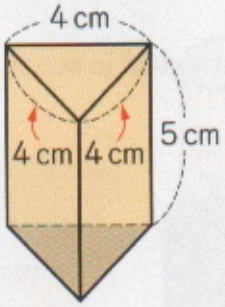




Kota: I wonder if we can draw nets of prisms and cylinders, too.

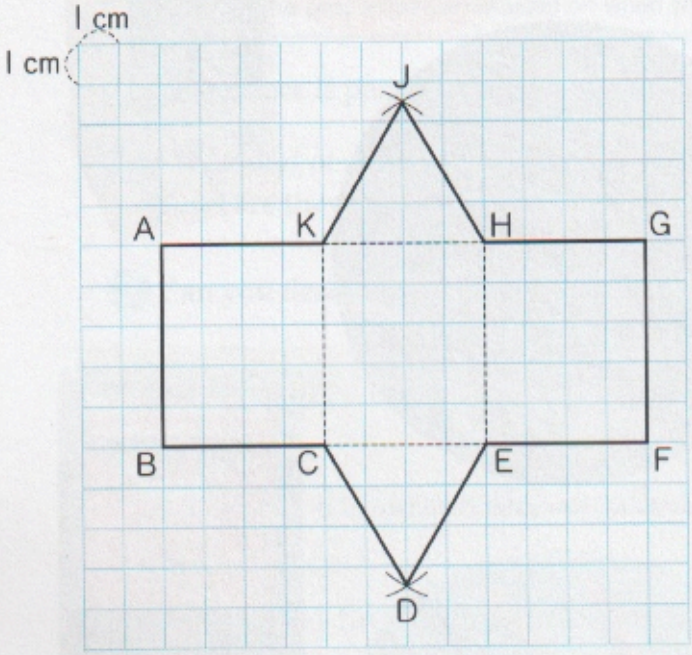
2 Nets of Prisms and Cylinders

1 Let's make a triangular prism on the right using construction paper.



Let's think about how to draw a net of a triangular prism.

Nets
Page 151 ⑨



Look at the net on the left and discuss what you notice.

Shapes of the bases and the lateral faces are...



Shiho

I think there may be other nets.





Haruto

1. In the net above, where do we need to look to determine the height of the prism?
2. When the net above is folded to make the prism, what points will come together with point J?
3. Draw a net like the one above and make a triangular prism.

We should pay attention to **the shapes and connection of faces and the lengths of edges**, just like we did when drawing the nets of cubes and cuboids.



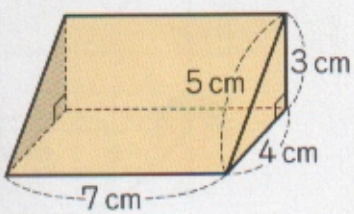
Ami

1 Look at the prism on the right.

① What type of prism is it?

② What is the height of this prism?

③ Draw a net of this prism.



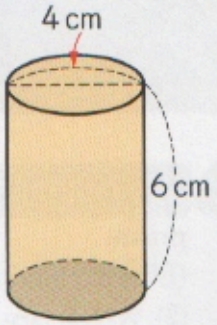
Additional Problems
Page 141 V



Misaki

I wonder if we can draw a net of a cylinder in the same way.

2 Make a cylinder on the right using a construction paper.

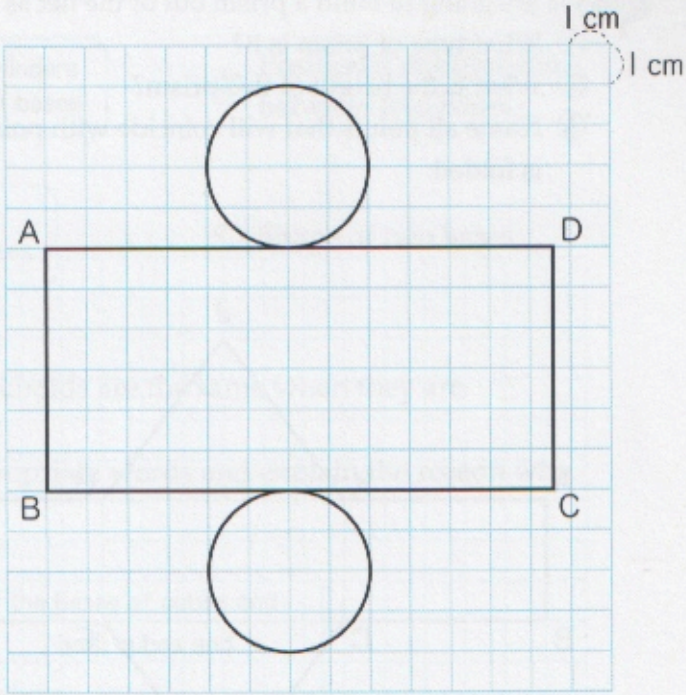


Let's think about how to draw a net of a cylinder.

1 What kind of shape will the lateral face make when a cylinder is cut open?



2 In the net on the right, how can we determine the length of side AD?





Which part of the cylinder is the same length?

3 How long is side AD?

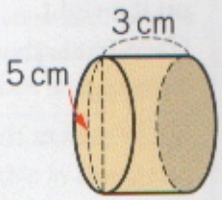
4 Draw the net on the right and make a cylinder.

We can draw a net of a cylinder with a lateral face in the shape of a rectangle. **The length of this rectangle is equal to the circumference of the base. The width of this rectangle is equal to the height of the cylinder.**



Kota

2 Draw a net of the cylinder on the right.



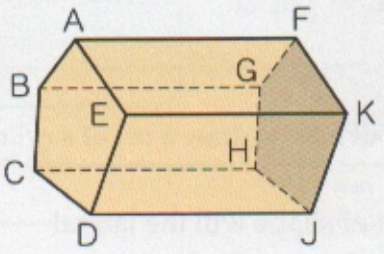
Check Your Understanding

1 The figure on the right is a prism.

① What kind of prism is this?

② Which face is parallel to face ABCDE?

③ Name all edges that are perpendicular to the base.



Do you understand the names and properties of prisms?

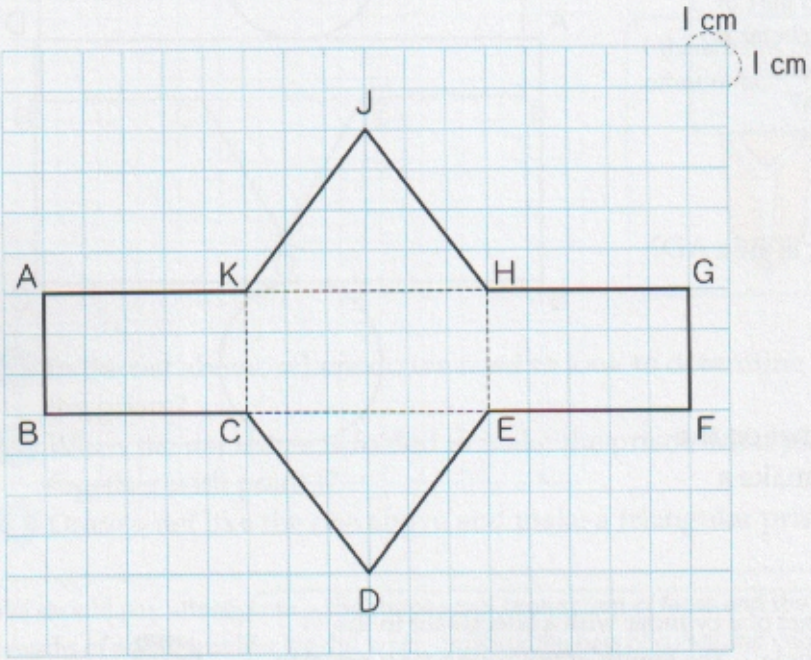
Page 115 1

2 We are going to build a prism out of the net as shown below.

① What type of prism is it?

② What is the height of this prism?

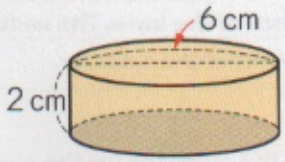
③ Name all points that will coincide with point A when the net is folded.



Can you interpret the net of a prism?

Page 120 1

3 We are going to draw a net of the cylinder on the right.



① What is the radius of the circle at the base?

② What are the width and length of the rectangle for the lateral face?

From a sketch of a cylinder, can you picture a net of the cylinder?

Page 118 3
Page 121 2

Grow Your "Eyes for Math" — Key Viewpoints and Ways of Thinking

Focus on the Relationship between Faces and Investigate Figures

① The three students below are reviewing the properties of prisms and cylinders.
What does each of the students focus on?
Find the answers from Ⓐ, Ⓑ, and Ⓒ.



Shiho: Both prisms and cylinders have two parallel bases.



Haruto: Both prisms and cylinders have two congruent bases.



Ami: Hexagonal prisms have hexagonal bases.

Ⓐ Shapes and sizes of two bases

Ⓑ Positional relationship between two bases

Ⓒ Shape of two bases

② Riku says that cubes and cuboids are the same when they are considered as prisms.
Fill in the ☐ with the appropriate words and explain the reason why.



Riku: Since the shapes of the bases of cubes and cuboids are both ☐, both cubes and cuboids are ☐ prisms.

Look back on what you have learned in "Let's Further Investigate Solid Figures" and discuss.



Kota: When I thought about the net of a cylinder, I understood well that the length of the rectangle for the lateral face is the same as the circumference length of the base.



Misaki: I want to think about how to find the volume of prisms and cylinders, just like we did with cubes and cuboids.



You'll study this in 6th grade.

Challenge Yourself
→Page 147

Focus on the Base Amount

1

Bottles of shampoo with 15% extra free quantity are being sold. The quantity of shampoo including the bonus quantity is 460 mL.
How many mL was the quantity of shampoo without the extra free quantity?




Kota

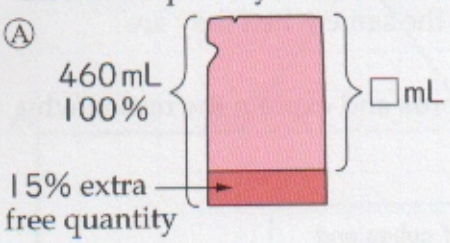
The quantity of shampoo with 15% bonus quantity is 460 mL.


Misaki

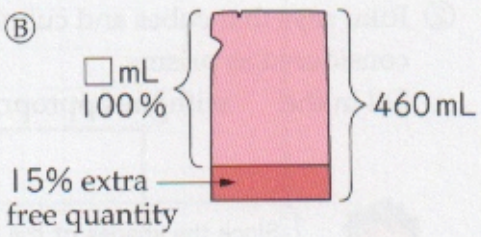
We need to find the quantity of shampoo without the bonus quantity.

1 We will replace the quantity of shampoo without the extra free quantity with □ (mL).
Which diagram correctly represents that the quantity of shampoo including the bonus quantity is 460 mL, Ⓐ or Ⓑ?

Ⓐ



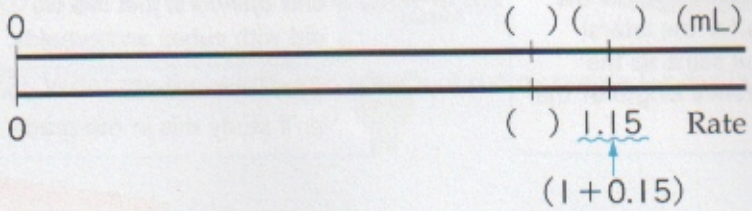
Ⓑ




I wonder what the base amount of 15% would be.

2 If we consider the quantity of shampoo without the bonus quantity as 100%, how many % does the quantity of shampoo including the bonus quantity correspond to?
Express the rate as a decimal number.

3 Express the relationship of quantities in the diagram below.




Fill the ( ) with numbers and □.

4 Explain Shiho's method and find the quantity of shampoo without the bonus quantity.

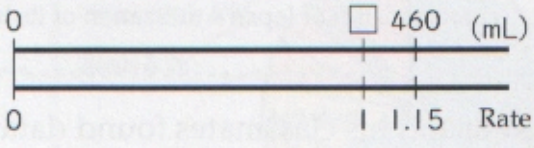


Shiho

□ × 1.15 = 460

□ = 460 ÷ 1.15

= □ Answer □ mL



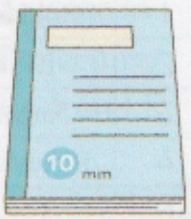


Riku

It is important to consider the base amount.

2

A certain brand of notebooks has been redesigned to weigh less than the previous model by 20%. The size of the notebooks and the number of pages remain the same.
The new model weighs 96 g per notebook.
How much does the previous model weigh?

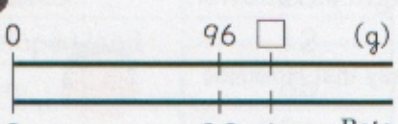


96 g

1 Ami and Haruto expressed the relationship between the quantities as shown below.
Which diagram correctly represents the relationship? The weight of the previous model is represented by □ (g).



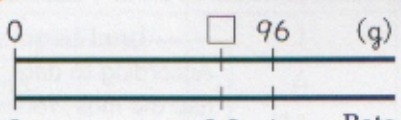
Ami



(1 − 0.2)



Haruto



(1 − 0.2)



What is the base amount of 20%?



Misaki

The weight of the new model, which is less than the previous model by 20%, is □ % of the previous model, so...

Let's Dig Deep into the Land Usage in Japan



In social studies class, students learned about how forests worked and how they were utilized. What are the characteristics of Japan's utilization of its land?

1

Kota and his classmates found data 1 and 2 about forests in Japan.

Data 1 **Rankings of Prefectures with Larger Forest Area**

Rank	Prefecture	Forest Area (ten thousand ha)
1	Hokkaido	554
2	Iwate	117
3	Nagano	107
4	Fukushima	97
5	Gifu	86
11	Kochi	60

(Developed by Tokyo Shoseki based on the Rates of Forested and Artificial Afforestation by Prefecture (as of March 31, 2017, the Ministry of Agriculture, Forestry and Fisheries))

Data 2 **Rankings of Prefectures with Higher Rates of Forested Land**

Rank	Prefecture	Rate (%)	Land area (ten thousand ha)
1	Kochi	85	71
2	Gifu	81	106
3	Nagano	79	136
4	Shimane	78	67
5	Yamanashi	78	45
21	Hokkaido		784

Note: The rate of forested refers to the rate of forest to the total area of land.
Note: Hokkaido's total area of land excludes the Northern Territories.
(Developed by Tokyo Shoseki based on the Rates of Forested and Artificial Afforestation by Prefecture (as of March 31, 2017, the Ministry of Agriculture, Forestry and Fisheries))

1 Kota looked at data 1 and 2 and thought as follows.

> According to data 1, you can say that Hokkaido has the most forest land.



Kota

What do you think about Kota's idea? Explain your idea to others and listen to other students' explanation.

> When we think about the rate of forested land...



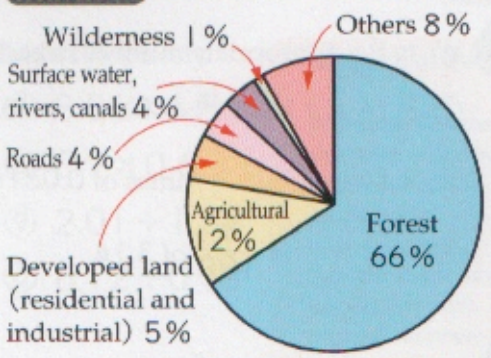
Ami

2 Calculate Hokkaido's rate of forest using data 1 and 2. Round the answer to the second highest place.

2

Shiho found a pie chart that represented the land usage in the whole country.

Data 3 **Land Usage in the Whole Country**



(Developed by Tokyo Shoseki based on the Land White Paper (the Ministry of Land, Infrastructure, Transport and Tourism, 2018))

1 Look at data 3 and fill in the □ with the appropriate number.



Shiho

Forest accounts for about $\frac{\square}{3}$ of the national land.

3

To find out the difference in land usage between the whole country and in different prefectures, Misaki investigated land usage in Hokkaido, which had a large forest area, and Kochi Prefecture, which had a high rate of forested land. She found data 4 and 5.

Data 4 **Land Usage in Hokkaido**

Land use	Area (ten thousand ha)
Forest	555
Agricultural	115
Surface water, rivers, canals	26
Roads	20
Wilderness	20
Developed land	12
Others	86
Total	834

(Developed by Tokyo Shoseki based on the National Land Use Plan (Hokkaido Plan) (the Hokkaido Government, 2017))

Data 5 **Land Usage in Kochi Prefecture**

Land use	Area (ten thousand ha)
Forest	60
Agricultural	3
Surface water, rivers, canals	2
Roads	1
Wilderness	0
Developed land	1
Others	4
Total	71

(Developed by Tokyo Shoseki based on the Kochi Prefecture Basic Land Use Plan (Kochi Prefecture, 2011))

1 Calculate the rate of each type of land usage in Hokkaido and Kochi Prefecture using data 4 and 5 and discuss what you notice.



Misaki

Compared with Kochi Prefecture, the rate of agricultural land in Hokkaido...



Riku

Compared with the whole country, the rates of developed land in both Hokkaido and Kochi Prefecture...

Review of 5th Grade

Answers→ Page 148–149

1 Write the appropriate number in each □.

① $3.14 = 1 \times \square + 0.1 \times \square + 0.01 \times \square$

② 3.14 is □ times of 0.0314.

③ □ is $\frac{1}{100}$ of 3.14.

2 Which of the following numbers are even numbers? Odd numbers? Select all that apply.

2, 3, 5, 9, 13, 14, 15, 18, 19, 20

3 List all common factors of the numbers in each ( ).

① (12, 15) ② (10, 18) ③ (27, 36)

Factors Page 150 ②

4 Find the least common multiple of the numbers in each ( ).

① (4, 7) ② (8, 18) ③ (12, 14)

Multiples Page 150 ②

5 Write the quotients as fractions.

① $1 \div 9$ ② $7 \div 8$ ③ $6 \div 5$

6 Calculate the following.

① $\frac{3}{5}+\frac{2}{3}$ ② $\frac{5}{6}+\frac{2}{21}$ ③ $1\frac{4}{9}+\frac{1}{6}$

④ $3\frac{1}{4}+2\frac{2}{7}$ ⑤ $2\frac{1}{6}+\frac{7}{10}$ ⑥ $\frac{4}{5}-\frac{3}{4}$

⑦ $\frac{7}{9}-\frac{1}{12}$ ⑧ $1\frac{5}{8}-\frac{3}{10}$ ⑨ $\frac{9}{10}-0.3$

7 Express 30 minutes as a fraction of an hour.



1 hour = 60 minutes, so...

Sense of Math

- We have found that whole numbers and decimal numbers have the same structure.
- We focused on the properties of whole numbers and sorted the numbers into groups and solved problems.
- We focused on how to express numbers (fractions) and thought about how to express quotients.
- If we find a common denominator and make fractions have the same base amount, we can calculate with fractions just like we did in 4th grade.

8 Calculate the following. When dividing, divide completely.

① 3.7×4.2 ② 2.6×3.4 ③ 3.56×4.8

④ 60.2×0.52 ⑤ 7.5×2.44 ⑥ 0.7×0.6

⑦ $9.8 \div 3.5$ ⑧ $16.2 \div 3.6$ ⑨ $2.01 \div 1.34$

⑩ $4.4 \div 8.8$ ⑪ $7.5 \div 12.5$ ⑫ $6.12 \div 0.18$

9 Calculate the following. Round the quotients to the second highest place.

① $6.3 \div 2.9$ ② $7.31 \div 1.2$ ③ $18.6 \div 9.7$

10 You can buy 1 kg of salt for 135 yen. How much will 0.8 kg of the salt be?

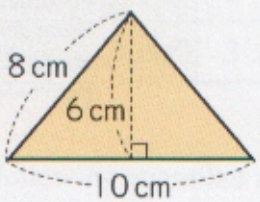
11 We are putting 83.5 kg of sugar into 2.5 kg bags. How many bags can you make and how much sugar will be left over?



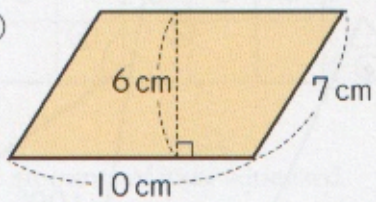
The quotient should be calculated to what place?

12 Find the area of the following figures.

①

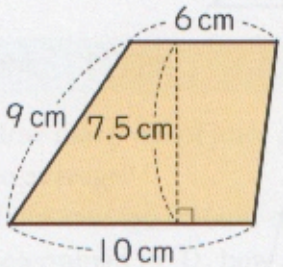


②



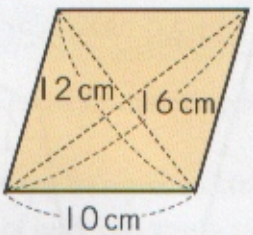
Parallelogram

③



Trapezoid

④



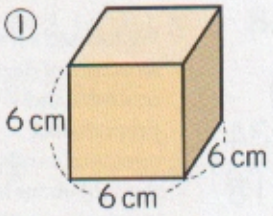
Rhombus

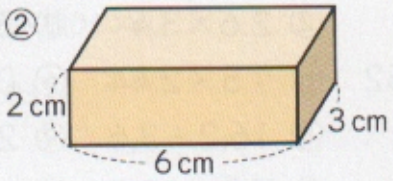
Sense of Math

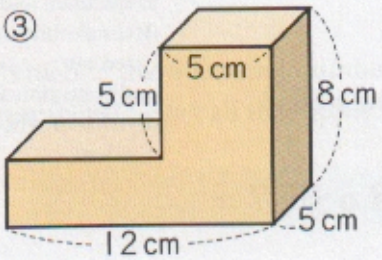
- We focused on the structure of decimal numbers and properties of operations and thought about how to calculate with decimal numbers based on multiplication and division of whole numbers.

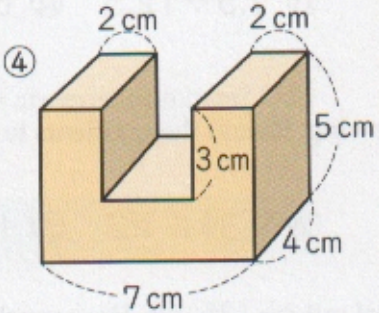
- We made formulas based on ways of calculating the areas of basic figures.

13 What are the volumes of the solid figures below?

① 

② 

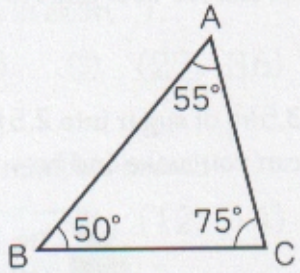
③ 

④ 

Sense of Math

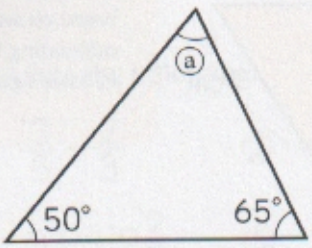
- We expressed volumes as groups of a base amount, 1 cm^3 or 1 m^3, just like we did with areas.
- We thought about complicated solid figures as combinations of cubes and cuboids.

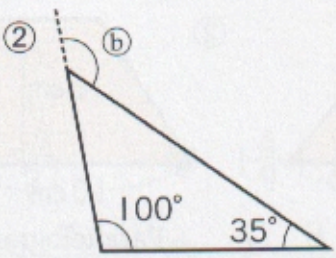
14 We want to draw a triangle that is congruent to the triangle ABC on the right.
In addition to the angle measures given on the figure, what additional information do we need?

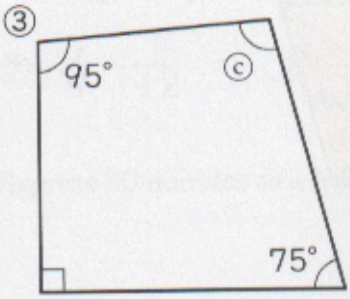


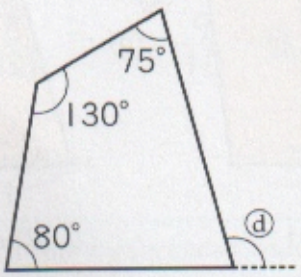
15 What are the measures of angles ⓐ, ⓑ, ⓒ, and ⓓ? Calculate their measures.

- We found the sum of the angles of a quadrilateral by using the sum of the angles of a triangle, didn't we?

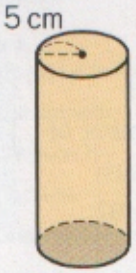
① 

② 

③ 

④ 

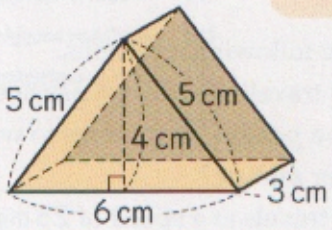
16 The base of a cylinder is a circle with the radius of 5 cm.
What is the circumference of the base of this cylinder?



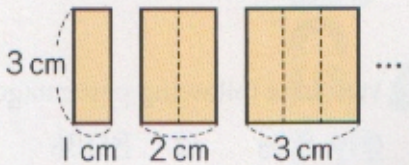
Sense of Math

- We investigated the properties of solid figures by focusing on shapes of their bases and lateral faces and the relationships between the bases.

17 Look at the prism on the right.
① What type of prism is it?
② What is the height of this prism?
③ Draw a net of this prism.



18 There is a rectangle with a length of 3 cm as shown on the right, and we increase the width of a rectangle from 1 cm to 2 cm, 3 cm...
Is the area, ○cm², proportional to the width of the rectangle, □cm? Find the answer by writing the appropriate numbers in the table below.



Width □(cm)	1	2	3	4	5	6	7
Area ○(cm²)	3						

- We focused on the proportional relationship, one of the relationships between two quantities that change in relationship to each other.

19 The following quantities are the amounts of juice squeezed from five apples.

98mL 120mL 113mL 105mL 124mL

- We focused on the amounts obtained by evening out several amounts and estimated the total amount.

① How many mL of juice can be squeezed from one apple on average?
② If the average amount of juice that can be squeezed from one apple as determined in ①, how many mL of juice can you squeeze from 20 apples?

20 The table on the right summarizes the size of two chicken coops and the number of chickens at the school. Which one is more crowded, A or B?

Area of Chicken Coops / Number of Chickens

	Area (m^2)	Chickens
A	10	9
B	15	12

Sense of Math

- We thought about per unit quantity by focusing on the relationship between two quantities (such as the area and the number of chickens; and time and distance).

21 Answer the following questions.

① A car is traveling at 50 km per hour. How many km does it travel in 3 hours?

② It takes a person 15 minutes to walk 1,200 m. How many m does the person walk per minute?

③ A train travels at a speed of 25 m per second. How long does it take for this car to travel 600 m?

22 Write the following percentages as decimal numbers.

① 3% ② 80% ③ 125% ④ 0.5%

- We thought about the rate by focusing on the relationship between the base quantity and the quantity being compared.

23 What % of the amount in the ( ) is each given amount?

① 3.2 m (8 m) ② 370 g (250 g)

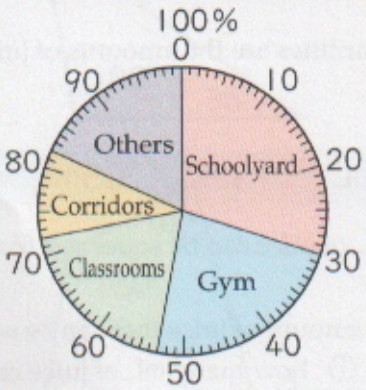
24 Answer the following questions by using the pie chart below.

① What are the rates for each place?

② How many times as many is the number of students who got injured in the schoolyard as the number of students who got injured in the corridors?

③ About what fraction of the total number of students who got injured is the number of students who got injured in the schoolyard and the gym combined?

Breakdown of Places where Students at an Elementary School Got Injured



- We thought about how to express rates clearly and compare them using graphs.

New Mathematics 5B Plus

Additional Problems

Need more practice?
Try these!

10 Let's Extend Addition and Subtraction of Fractions ··· 136
11 Let's Think about Evened Out Amounts ··· 137
12 Let's Think about How to Compare (1) ··· 137
13 Let's Think about How to Find Area ··· 138
14 Let's Think about How to Compare (2) ··· 140
15 Let's Investigate Rates by Representing Them in Graphs ··· 140
16 Let's Investigate How Quantities Change (2) ··· 141
17 Let's Further Investigate Polygons and Circles ··· 141
18 Let's Further Investigate Solid Figures ··· 141
● Answers ··· 142



Let's Try WONDERful Problems!

These are problems that will further extend and deepen the knowledge of what you have studied.
Think about these problems and enjoy the challenge.

10 Let's Extend Addition and Subtraction of Fractions ··· 144
13 Let's Think about How to Find Area ··· 145
18 Let's Further Investigate Solid Figures ··· 147
● Answers ··· 148

Let's Try Programming! ··· 134
Let's Play with Shapes ··· 135

Review Exercises ··· 150
Index ··· 153



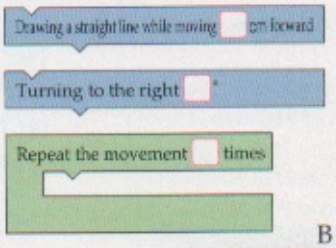
Notice to Teachers and Parents

"New Mathematics 5B Plus" is an optional learning material for students who want or need to work on it.
Not all students need to use "New Mathematics 5B Plus."

Let's Try Programming!

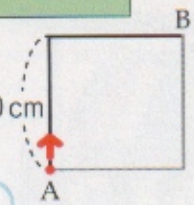
Think about the Process of Drawing a Regular Polygon

To draw a regular polygon using a computer that is capable of doing the three things as listed on the right, what instructions do you need to give to the computer?



1 Think about the process of drawing a square with 10 cm sides.

① What instructions do you need to give to the computer so that a dot starts from vertex A and reaches vertex B in the diagram on the right?





Ami

Each side is 10 cm, so the dot needs to travel 20 cm.

Drawing a straight line while moving 20 cm forward



Kota

When the dot travels 10 cm, it needs to turn to the right 90°, so...

Drawing a straight line while moving 10 cm forward
Turning to the right 90°
Drawing a straight line while moving 10 cm forward

② Think about the instructions to draw a square.



Haruto

If the dot repeats the movement from vertex A to vertex B 2 times...

Repeat the sequence 2 times
- Drawing a straight line while moving 10 cm forward
- Turning to the right 90°
- Drawing a straight line while moving 10 cm forward
- Turning to the right 90°

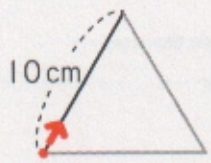


Shiho

Haruto's sequence includes a repetition of a smaller sequence, so...

Repeat the sequence 4 times
- Drawing a straight line while moving 10 cm forward
- Turning to the right 90°

2 Think about the process of drawing an equilateral triangle with 10 cm sides.

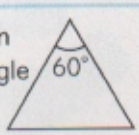


① What must the measure of the angle of turn be?



Riku

Each angle of an equilateral triangle is 60°, so...

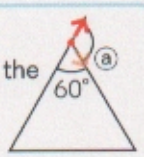




Misaki

The angle of turn is represented by ⓐ in the diagram, so...

180 − 60 = ☐



〈Try programming〉

Let's Play with Shapes

Tessellations

〈Move〉

As shown below, if we change part of a parallelogram, we can make many different tessellations.

1 Try to make different tessellations as shown below.

Which parts of the figure are changed and how?



Additional Problems

Similar Problems Slightly Difficult Problems

10 Let's Extend Addition and Subtraction of Fractions

Answers → Page 142

Page 9 4 →

A ① $\frac{2}{3}+\frac{1}{5}$ ② $\frac{1}{4}+\frac{3}{5}$ ③ $\frac{3}{7}+\frac{4}{3}$ ④ $\frac{3}{2}+\frac{6}{5}$

⑤ $\frac{4}{5}-\frac{1}{6}$ ⑥ $\frac{1}{3}-\frac{2}{7}$ ⑦ $\frac{6}{5}-\frac{3}{8}$ ⑧ $\frac{7}{3}-\frac{3}{2}$

A Find the appropriate number in each □ below.

① $\frac{\square}{7}+\frac{1}{3}=\frac{13}{21}$ ② $\frac{\square}{9}-\frac{3}{7}=\frac{43}{63}$

③ $\frac{3}{4}-\frac{\square}{3}=\frac{1}{12}$ ④ $\frac{\square}{5}+\frac{\square}{3}=\frac{19}{15}$

Page 13 5 →

B ① $\frac{5}{12}+\frac{1}{3}$ ② $\frac{7}{12}+\frac{3}{8}$ ③ $\frac{9}{10}-\frac{2}{5}$ ④ $\frac{11}{6}-\frac{3}{4}$

B Place the four numerals from the ⬚ in the □ to complete math sentences.

① $\frac{\square}{\square}+\frac{\square}{\square}=\frac{7}{8}$

1 3 4 8

② $\frac{\square}{\square}-\frac{\square}{\square}=\frac{1}{8}$

1 2 3 8

Page 14 1 →

C ① $1\frac{1}{4}+3\frac{2}{3}$ ② $\frac{3}{7}+3\frac{1}{3}$ ③ $2\frac{1}{3}+1\frac{1}{6}$ ④ $\frac{7}{10}+3\frac{1}{6}$

C ① $2\frac{3}{4}+1\frac{1}{3}$ ② $2\frac{5}{6}+1\frac{1}{2}$ ③ $\frac{7}{10}+2\frac{1}{3}$ ④ $4\frac{3}{5}+\frac{9}{10}$

Page 14 2 →

D ① $2\frac{2}{3}-1\frac{3}{8}$ ② $4\frac{2}{3}-\frac{2}{5}$ ③ $3\frac{1}{2}-1\frac{1}{6}$ ④ $2\frac{2}{3}-\frac{1}{6}$

D ① $4\frac{1}{3}-1\frac{4}{5}$ ② $3\frac{1}{2}-1\frac{5}{6}$ ③ $1\frac{1}{3}-\frac{7}{8}$ ④ $2\frac{1}{6}-\frac{2}{3}$

Page 15
3 →

E ① $\frac{2}{5}+0.2$ ② $0.75-\frac{1}{4}$ ③ $0.25+\frac{2}{3}$ ④ $\frac{4}{7}-0.3$

E There was 1.5 L of water to share. Person A got $\frac{1}{3}$ L, person B got 0.3 L, and person C got $\frac{2}{5}$ L. How many L of water is left?

11 Let's Think about Evened Out Amounts

Answers→ Page 142

Page 22
2 →

F Find the average of the following lengths.

82 cm 69 cm 92 cm 85 cm 100 cm 70 cm

F Among Ⓐ to Ⓒ, select all the groups of numbers with the average of 80 or greater.

Ⓐ 78, 80, 80, 78, 81, 77

Ⓑ 79, 78, 79, 97, 79, 76, 79

Ⓒ 81, 79, 80, 78, 82

12 Let's Think about How to Compare (1)

Answers→ Page 142–143

Page 33
2 →

G Which eggs are more expensive, the ones that sell for 180 yen for 10 or the ones that sell for 120 yen for 6?

G Orchard A has an area of 40 m² and produced 880 kg of apples.
Orchard B has an area of 50 m² and produced 930 kg of apples.
Orchard C has an area of 100 m² and produced 1,800 kg of apples.
Which orchard had the best harvest of apples?

Page 37
1 →

H Calculate the speed per hour, per minute, and per second of a car that travels 216 km in 3 hours.

H Calculate the speed per hour, per minute, and per second of a car that travels 135 km in 2 hours and a half.

Page 38
2 →

I A car is traveling at a speed of 50 km per hour. How far can the car travel in 2 hours?

I A car is traveling at a speed of 60 km per hour. How far can the car travel in 3.5 hours?

Page 39
3 →

J How many minutes will it take for a bus to travel 3.6 km at a speed of 600 m per minute?

Page 39 → 3

J Tokyo Skytree is an observation tower. Its elevator takes you to the first observatory 350 m above the ground in 50 seconds.

① Assume that the elevator goes up at a constant average speed, and find the speed of the elevator per second.

② In how many seconds after the elevator starts going up does the elevator pass through the level 245 m above the ground?

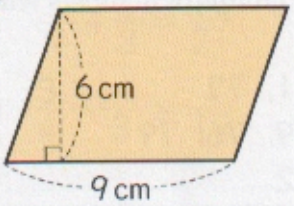
13 Let's Think about How to Find Area

Answers → Page 143

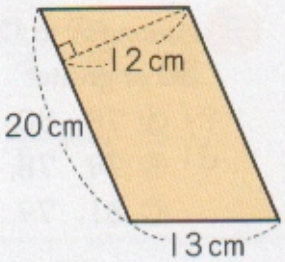
Page 48 → 1

K Find the area of the following parallelograms.

①

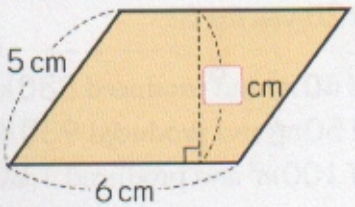


②

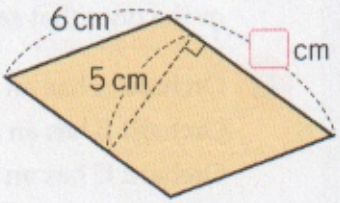


K Find the numbers that go into the □ of the parallelograms with the following areas.

① Area: 24 cm²



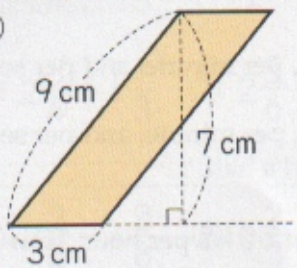
② Area: 35 cm²



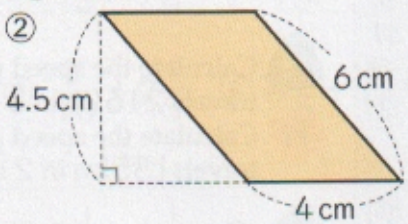
Page 50 → 2

L Find the area of the following parallelograms.

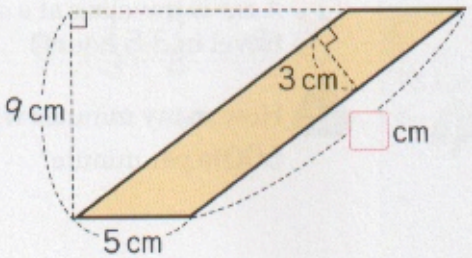
①



②

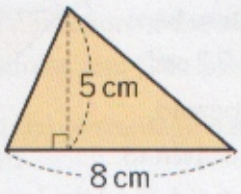


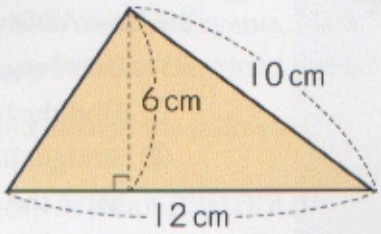
L Find the numbers that go into the □ of the parallelogram on the right.



Page 54
1 →

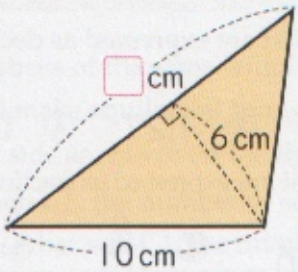
M Find the area of the following triangles.

① 

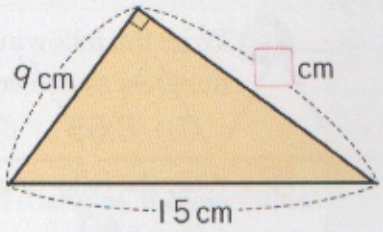
② 

M Find the numbers that go into the □ of the triangles with the following areas.

① Area:42 cm²

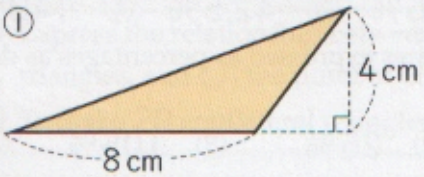


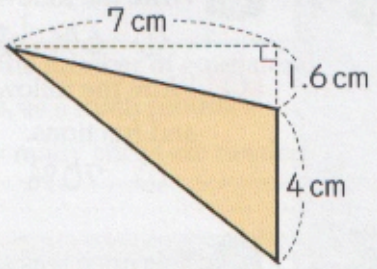
② Area:54 cm²



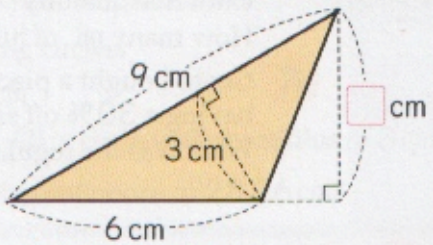
Page 56
2 →

N Find the area of the following triangles.

① 

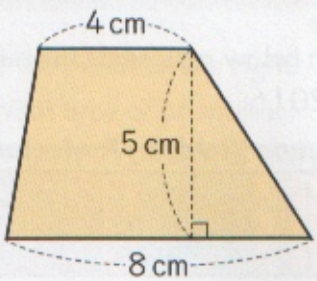
② 

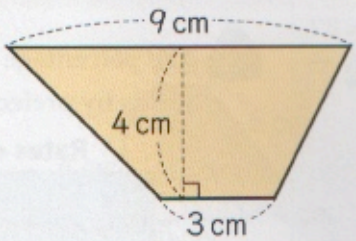
N Find the number that goes into the □ of the triangle on the right.



Page 59
1 →

O Find the area of the following trapezoids.

① 

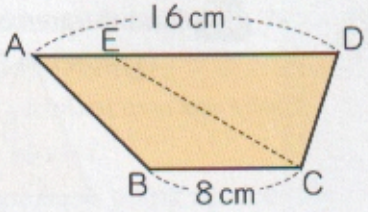
② 

Page 59 → 1

O Look at trapezoid ABCD on the right. Top base AD is 16 cm long, bottom base BC is 8 cm long, and the area is 72 cm².

① Find the height of this trapezoid.

② Straight line CE has been drawn to halve the area of this trapezoid. Find the length of AE.



14 Let's Think about How to Compare (2)

Answers → Page 143

Page 73 → 3

P Write the following rates that are expressed as decimal or whole numbers as percentages.

① 0.55 ② 0.3 ③ 1.27 ④ 0.309 ⑤ 4

P Write the following rates that are expressed as fractions as percentages.

① $\frac{60}{100}$ ② $\frac{7}{100}$ ③ $\frac{9}{10}$ ④ $\frac{4}{5}$ ⑤ $\frac{3}{4}$

Page 73 → 4

Q Write the following rates expressed as percentages as decimal numbers.

① 6% ② 85% ③ 42.5% ④ 140% ⑤ 0.8%

Q Write the following rates expressed as percentages as decimal numbers and fractions.

① 70% ② 25% ③ 105% ④ 12.5%

Page 79 → 2

R This juice was sold in 500 mL bottles, but now it is sold with 20% extra free quantity.
How many mL of juice is in the new bottle?

R Gento bought a piece of clothing for 1,400 yen at a store that was having a 30% off sale.
What was the regular price of this clothing?

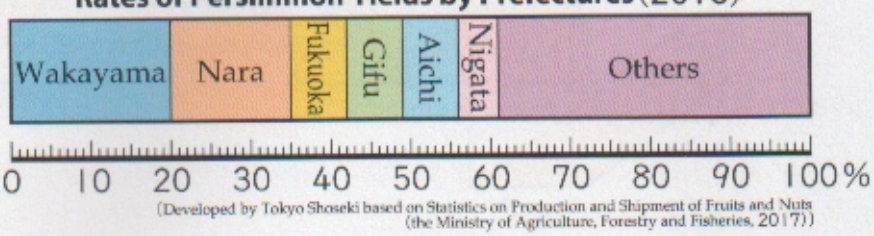
15 Let's Investigate Rates by Representing Them in Graphs

Answers → Page 143

Page 87 → 1

S The percentage bar graph below represents the rates of persimmon yields by prefectures in 2016.

Rates of Persimmon Yields by Prefectures (2016)



(Developed by Tokyo Shoseki based on Statistics on Production and Shipment of Fruits and Nuts (the Ministry of Agriculture, Forestry and Fisheries, 2017))

① How many % does the yield in Nara account for?

② What fraction of the total yield is the yields from Wakayama, Nara, Fukuoka, and Gifu combined? How many % is the combined yields?

Page 87 1 →

S Look at the percentage bar graph in S and answer the following question.

The national yield of persimmons in 2016 was about 230,000 t. About how many t was the yield in Wakayama?

16 Let's Investigate How Quantities Change (2)

Answers→ Page 143

Page 97 1 →

T Using sticks of the same length, we will make equilateral triangles side by side as shown on the right.



① When □, the number of equilateral triangles, increases from 1 to 2..., what will ○, the number of sticks be? Fill in the table below.

Number of equilateral triangles □	1	2	3	4	5
Number of sticks ○					

② Express the relationship between □, the number of equilateral triangles, and ○, the number of sticks, as a math sentence.

③ To make 20 equilateral triangles, how many sticks are needed?

17 Let's Further Investigate Polygons and Circles

Answers→ Page 143

Page 108 1 →

U Find the circumference of the following circles.

① a circle with the diameter of 5 cm

② a circle with the radius of 2 cm ③ a circle with the radius of 8 cm

U What is the radius of a circle with a circumference of 28.26 cm?

18 Let's Further Investigate Solid Figures

Answers→ Page 143

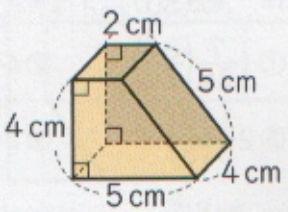
Page 120 1 →

V Look at the prism on the right.

① What type of prism is it?

② What is the height of this prism?

③ Draw a net of this prism.



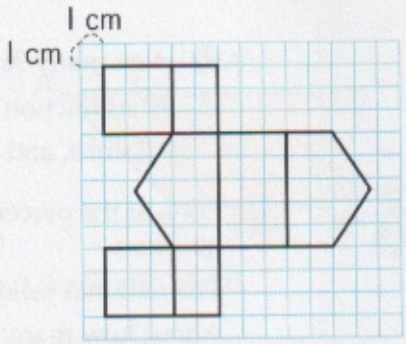
Page 120

1 →

V We are going to make a prism from a net as shown on the right.

① What type of prism is it?

② What is the height of this prism?



Answers

Additional Problems

Page 136–142

10 Let's Extend Addition and Subtraction of Fractions

A ① $\frac{13}{15}$ ② $\frac{17}{20}$

③ $\frac{37}{21}\left(1\frac{16}{21}\right)$ ④ $\frac{27}{10}\left(2\frac{7}{10}\right)$

⑤ $\frac{19}{30}$ ⑥ $\frac{1}{21}$ ⑦ $\frac{33}{40}$ ⑧ $\frac{5}{6}$

A ① 2 ② 10 ③ 2

④ (From left to right) 3, 2

B ① $\frac{3}{4}$ ② $\frac{23}{24}$

③ $\frac{1}{2}$ ④ $\frac{13}{12}\left(1\frac{1}{12}\right)$

B ① $\frac{3}{4}+\frac{1}{8}=\frac{7}{8}$ $\left(\frac{1}{8}+\frac{3}{4}=\frac{7}{8}\right)$

② $\frac{1}{2}-\frac{3}{8}=\frac{1}{8}$

C ① $4\frac{11}{12}\left(\frac{59}{12}\right)$ ② $3\frac{16}{21}\left(\frac{79}{21}\right)$

③ $3\frac{1}{2}\left(\frac{7}{2}\right)$ ④ $3\frac{13}{15}\left(\frac{58}{15}\right)$

C ① $4\frac{1}{12}\left(\frac{49}{12}\right)$ ② $4\frac{1}{3}\left(\frac{13}{3}\right)$

③ $3\frac{1}{30}\left(\frac{91}{30}\right)$ ④ $5\frac{1}{2}\left(\frac{11}{2}\right)$

D ① $1\frac{7}{24}\left(\frac{31}{24}\right)$ ② $4\frac{4}{15}\left(\frac{64}{15}\right)$

③ $2\frac{1}{3}\left(\frac{7}{3}\right)$ ④ $2\frac{1}{2}\left(\frac{5}{2}\right)$

D ① $2\frac{8}{15}\left(\frac{38}{15}\right)$ ② $1\frac{2}{3}\left(\frac{5}{3}\right)$

③ $\frac{11}{24}$ ④ $1\frac{1}{2}\left(\frac{3}{2}\right)$

E ① $\frac{3}{5}(0.6)$ ② $\frac{1}{2}(0.5)$ ③ $\frac{11}{12}$ ④ $\frac{19}{70}$

E $\frac{7}{15}$ L

11 Let's Think about Evened Out Amounts

F 83 cm

F ⑧, ⓒ

12 Let's Think about How to Compare (1)

G The eggs that sell for 120 yen for 6

Ways of Thinking The eggs that sell for 180 yen for 10 : $180 \div 10 = 18$ yen per piece.

The eggs that sell for 120 yen for 6 : $120 \div 6 = 20$ yen per piece.

G Orchard A

Ways of Thinking Orchard A produced $880 \div 40 = 22$ (kg) per 1 m².

Orchard B produced $930 \div 50 = 18.6$ (kg) per 1 m².

Orchard C produced $1{,}800 \div 100 = 18$ (kg) per 1 m².

H 72 km per hour, 1.2 km (1,200 m) per minute, 20 m per minute

H 54 km per hour, 0.9 km (900 m) per minute, 15 m per second

I 100 km

I 210 km

J 6 minutes

J ① 7 m per second ② In 35 seconds

Ways of Thinking ① $350 \div 50 = 7$ (m)

② $245 \div 7 = 35$ (seconds)

13 Let's Think about How to Find Area

K ① 54 cm² ② 240 cm²

K ① 4 ② 7

L ① 21 cm² ② 18 cm²

L 15

Ways of Thinking The area of the parallelogram is $5 \times 9 = 45$ (cm²)

If you consider that the height is 3 cm, the area is $\square \times 3 = 45$

$45 \div 3 = 15$ (cm)

M ① 20 cm² ② 36 cm²

M ① 14 ② 12

N ① 16 cm² ② 14 cm²

N 4.5

Ways of Thinking The area of the triangle is $9 \times 3 \div 2 = 13.5$ (cm²)

If you consider that the base is 6 cm, the area is

$6 \times \square \div 2 = 13.5$

$13.5 \times 2 \div 6 = 4.5$ (cm)

O ① 30 cm² ② 24 cm²

O ① 6 cm ② 4 cm

Ways of Thinking ① If you use $\square$ (cm) for the height

$(16+8) \times \square \div 2 = 72$

$72 \times 2 \div (16+8) = 6$

② Since trapezoids ABCE and ABCD have the same height, the sum of the top and bottom bases of trapezoid ABCE must be half of that of trapezoid ABCD so that the area of trapezoid ABCE can be half of the area of trapezoid ABCD.

The sum of AE and BC is

$(16+8) \div 2 = 12$ (cm)

So, AE is $12 - 8 = 4$ (cm).

14 Let's Think about How to Compare (2)

P ① 55% ② 30% ③ 127%

④ 30.9% ⑤ 400%

P ① 60% ② 7% ③ 90%

④ 80% ⑤ 75%

Q ① 0.06 ② 0.85 ③ 0.425

④ 1.4 ⑤ 0.008

Q ① 0.7, $\frac{7}{10}\left(\frac{70}{100}\right)$

② 0.25, $\frac{1}{4}\left(\frac{25}{100}\right)$

③ 1.05, $\frac{21}{20}\left(\frac{105}{100}\right)$

④ 0.125, $\frac{1}{8}\left(\frac{125}{1{,}000}\right)$

R 600 mL

Ways of Thinking $500 \times (1 + 0.2) = 600$ (mL)

R 2,000 yen

Ways of Thinking $1{,}400 \div (1 - 0.3) = 2{,}000$ (yen)

15 Let's Investigate Rates by Representing Them in Graphs

S ① 15% ② About $\frac{1}{2}$, 49%

S About 46,000 t

Ways of Thinking $230{,}000 \times 0.2 = 46{,}000$ (t)

16 Let's Investigate How Quantities Change (2)

T ①

Number of equilateral triangles □	1	2	3	4	5
Number of sticks ○	3	5	7	9	11

② (Example 1) $1 + 2 \times \square = \bigcirc$

(Example 2) $3 + 2 \times (\square - 1) = \bigcirc$

③ 41 sticks

17 Let's Further Investigate Polygons and Circles

U ① 15.7 cm ② 12.56 cm

③ 50.24 cm

U 4.5 cm

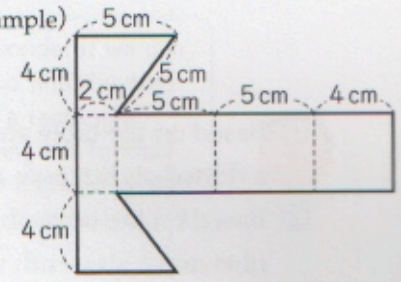
Ways of Thinking The diameter is $28.26 \div 3.14 = 9$ (cm)

The radius is $9 \div 2 = 4.5$ (cm)

18 Let's Further Investigate Solid Figures

V ① Quadrilateral prism ② 4 cm

③ (Example)



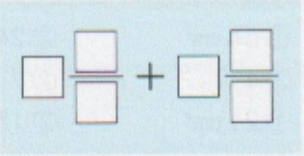
V ① Pentagonal prism ② 3 cm

Let's Try WONDERful Problems!

10 Let's Extend Addition and Subtraction of Fractions

Answers→ Page 148

1 Select 6 out of 9 cards from 1 to 9 to make the addition math sentence on the right.
Note that the fraction has to be a proper fraction, which cannot be simplified.



① Make a math sentence with the largest sum. Also, find the answer.
② Make a math sentence with the smallest sum. Also, find the answer.

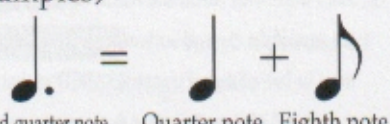
2 Lengths of musical notes are determined like fractions.
Lengths of notes are represented as fractions of a whole note as follows.

Note	Whole note	Half note	Quarter note	Eighth note	Sixteenth note
Length of note	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Some notes are written with a dot.
They are called "dotted notes".
A dotted note is the sum of the note without a dot plus one half of it.

(Examples)



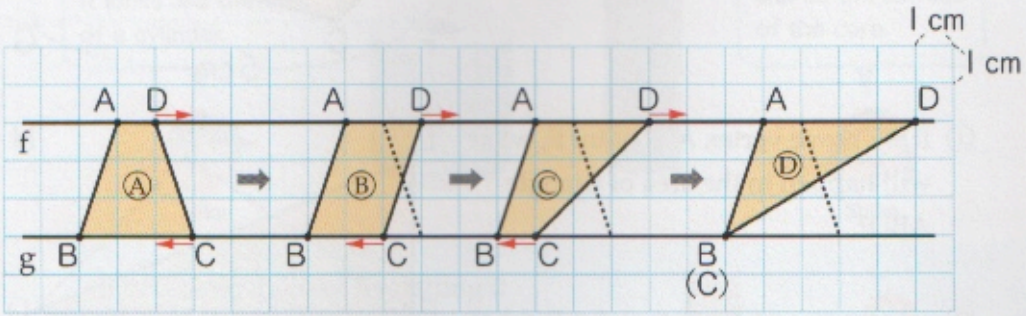
Dotted quarter note = Quarter note + Eighth note

① Based on the table above, express the lengths of a dotted quarter note and a dotted eighth note as fractions.
② Based on the table above, find the difference in length between a half note and a sixteenth note.

13 Let's Think about How to Find Area

Answers→ Page 148

1 If you move vertices C and D of figure Ⓐ on parallel lines F and G by 1 cm in the directions shown by the arrows, Ⓐ changes into Ⓑ, then Ⓒ, and then Ⓓ.



① Write the names of figures Ⓐ through Ⓓ. Also, find the area of each of these figures.

Ⓐ Name( ) Area(□ + □) × 3 ÷ 2 = □ (cm²)

Ⓑ Name( ) Area □ × 3 = □ (cm²)

Ⓒ Name( ) Area(□ + □) × 3 ÷ 2 = □ (cm²)

Ⓓ Name( ) Area □ × 3 ÷ 2 = □ (cm²)

Ⓐ, Ⓑ, Ⓒ, and Ⓓ have the □ area.



Shiho

② Consider parallelogram Ⓑ as a trapezoid where the top base has the same length as the bottom base. Find the area of Ⓑ using the formula for calculating the area of a trapezoid.

(□ + □) × □ ÷ 2 = □ (cm²)

③ Consider triangle Ⓓ as a trapezoid where the top base is 4 cm and the bottom base is 0 cm. Find the area of Ⓑ using the formula for calculating the area of a triangle.

(□ + □) × □ ÷ 2 = □ (cm²)



Compare math sentences you made in ② and ③ with math sentences you completed for Ⓑ and Ⓓ in ①.

I wonder if we can also find the area of a rectangle using the formula for calculating the area of a trapezoid.

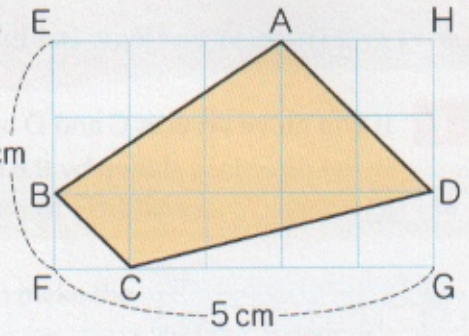


Kota

2 Find the area of quadrilateral ABCD on the right.

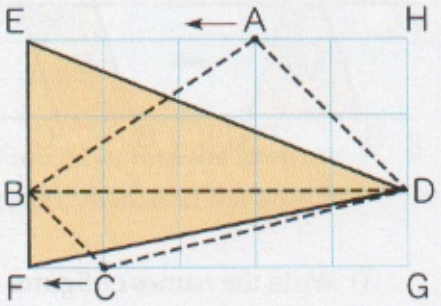

Misaki

We don't know the length of any side ...



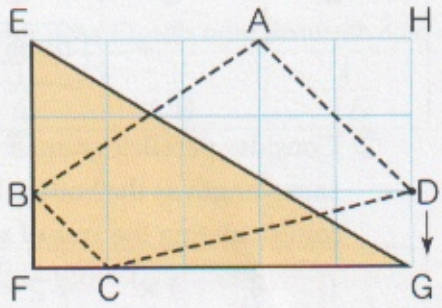
① If we move vertex A to point E, what will happen to the area of triangle ABD?

② If we move vertex C to point F, what will happen to the area of triangle CDB?

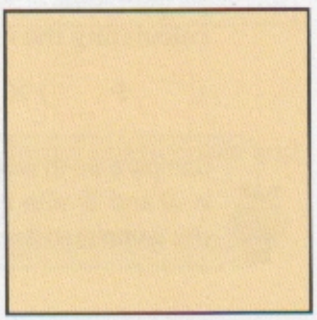


③ Can we say that the area of quadrilateral ABCD is equal to the area of triangle EFD?
Find the area of quadrilateral ABCD.

④ Furthermore, we will move vertex D to point G. Can we say that the area of quadrilateral ABCD is half of the area of rectangle EFGH?



3 Divide a square into 4 parts with equal area.
The four parts do not have to be congruent.



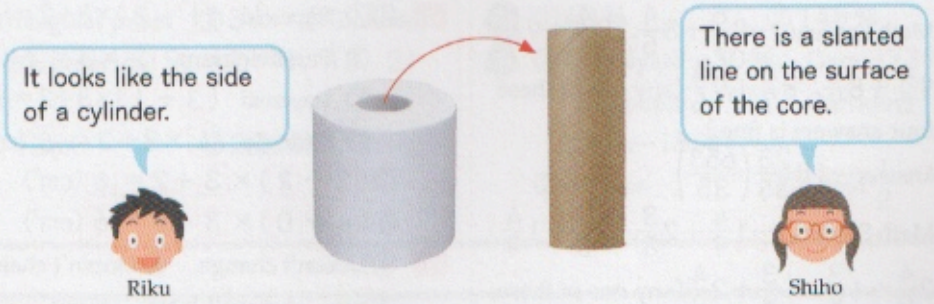


Come up with at least 5 different ways.

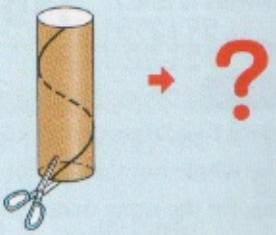
18 Let's Further Investigate Solid Figures

Answers→ Page 148

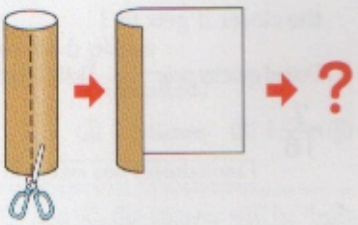
1 The photo below shows a core from a toilet paper roll.



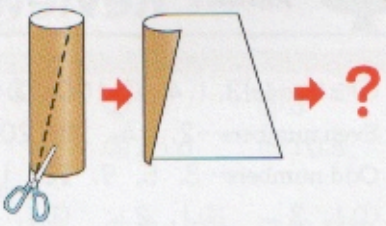
If you open the core by cutting it along the slanted line on the surface, what kind of shape will you get?



① If you cut it along the line that is perpendicular to the base, what kind of shape will you get?
Take a guess.



② If you cut it along the line that is slightly slanted, what kind of shape will you get?
Take a guess.



③ If you open the core by cutting it along the slanted line, what kind of shape will you get?
Take a guess.

Get an empty toilet paper roll, and cut it along the slanted line to see if you guessed right.

Answers — Let's Try WONDERful Problems!

Page 144–147

10 Let's Extend Addition and Subtraction of Fractions

1 ① Math Sentence… $9\frac{6}{7}+8\frac{4}{5}$, $8\frac{4}{5}+9\frac{6}{7}$, $9\frac{4}{5}+8\frac{6}{7}$, $8\frac{6}{7}+9\frac{4}{5}$ (any one of these four answers is fine.)
Answer… $18\frac{23}{35}\left(\frac{653}{35}\right)$

② Math Sentence… $1\frac{4}{9}+2\frac{3}{8}$, $2\frac{3}{8}+1\frac{4}{9}$, $2\frac{4}{9}+1\frac{3}{8}$, $1\frac{3}{8}+2\frac{4}{9}$ (any one of these four answers is fine.)
Answer… $3\frac{59}{72}\left(\frac{275}{72}\right)$

Ways of Thinking

① First, place two largest numbers (8 and 9) in the whole number. As for the fractions, for the same denominator, the smaller the difference between the denominator and the numerator is, the larger the proper fraction becomes and the closer it gets to 1.

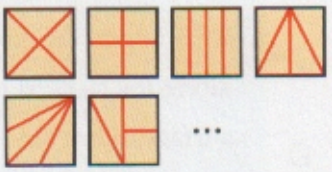
2 ① Dotted quarter note… $\frac{3}{8}$ Dotted eighth note… $\frac{3}{16}$
② $\frac{7}{16}$

13 Let's Think about How to Find Area

1 ①Ⓐ Trapezoid $(1+3)\times3\div2=6$ (cm²)
Ⓑ Parallelogram $2\times3=6$ (cm²)
Ⓒ Trapezoid $(3+1)\times3\div2=6$ (cm²)
Ⓓ Triangle $4\times3\div2=6$ (cm²)
② $(2+2)\times3\div2=6$ (cm²)
③ $(4+0)\times3\div2=6$ (cm²)

2 ① Doesn't change. ② Doesn't change.
③ yes. Area… 7.5 cm² ④ yes.

3 (examples)



18 Let's Further Investigate Solid Figures

1 ① Rectangle ② Parallelogram
③ Parallelogram

Answers — Review of 5th Grade

Page 128–132

1 ①(in this order) 3, 1, 4 ② 100 ③ 0.0314

2 Even numbers… 2, 14, 18, 20
Odd numbers… 3, 5, 9, 13, 15, 19

3 ① 1, 3 ② 1, 2 ③ 1, 3, 9

4 ① 28 ② 72 ③ 84

5 ① $\frac{1}{9}$ ② $\frac{7}{8}$ ③ $\frac{6}{5}\left(1\frac{1}{5}\right)$

6 ① $\frac{19}{15}\left(1\frac{4}{15}\right)$ ② $\frac{13}{14}$
③ $1\frac{11}{18}\left(\frac{29}{18}\right)$ ④ $5\frac{15}{28}\left(\frac{155}{28}\right)$
⑤ $2\frac{13}{15}\left(\frac{43}{15}\right)$ ⑥ $\frac{1}{20}$ ⑦ $\frac{25}{36}$
⑧ $1\frac{13}{40}\left(\frac{53}{40}\right)$ ⑨ $\frac{3}{5}$ (0.6)

7 $\frac{1}{2}$ hour

8 ① 15.54 ② 8.84 ③ 17.088
④ 31.304 ⑤ 18.3 ⑥ 0.42
⑦ 2.8 ⑧ 4.5 ⑨ 1.5
⑩ 0.5 ⑪ 0.6 ⑫ 34

9 ① 2.2 ② 6.1 ③ 1.9

10 108 yen

11 33 bags and 1 kg left over

12 ① 30 cm² ② 60 cm²
③ 60 cm² ④ 96 cm²

13 ① 216 cm³ ② 36 cm³
③ 305 cm³ ④ 104 cm³

14 One of the lengths of side AB, BC, or AC

15 ①Ⓐ 65° ②Ⓑ 135°
③Ⓒ 100° ④Ⓓ 105°

16 31.4 cm

17 ① Triangular prism ② 3 cm ③ (omitted)

18 (in this order) 6, 9, 12, 15, 18, 21
Yes

19 ① 112 mL ② 2,240 mL

20 Coop A

21 ① 150 km ② 80 m per minute
③ 24 seconds

22 ① 0.03 ② 0.8 ③ 1.25 ④ 0.005

23 ① 40% ② 148%

24 ① Schoolyard…30% Gym…23%
Classrooms…19% Corridors…10%
Others…18%
② 3 times ③ About $\frac{1}{2}$

Answers Do You Remember?

Page 19

1 0, 0.65, $\frac{5}{7}$, 1, $1\frac{7}{8}$

2 ①ⓐ 35° ②ⓑ 60°

3 (in this order) 4, 8, 12, 16, 20, 24, 28, 32
Yes

4 ① 8 g ② 0.125 m

5 Ⓑ

Page 43

1 ① $\frac{19}{12}\left(1\frac{7}{12}\right)$ ② $\frac{2}{5}$
③ $3\frac{24}{35}\left(\frac{129}{35}\right)$ ④ $\frac{56}{45}\left(1\frac{11}{45}\right)$
⑤ $\frac{4}{9}$ ⑥ $\frac{1}{3}$
⑦ $3\frac{11}{24}\left(\frac{83}{24}\right)$ ⑧ $\frac{7}{20}$ (0.35)

2 ① 6,210 ② 0.01

3 Trail B…3.36 km, Trail C…2.24 km

4 ① 81 cm² ② 37.5 cm² ③ 24 cm²

Playing with Numbers and Calculations

Ⓐ 1.2 Ⓑ 0.2 Ⓒ 1.8
Ⓓ 0.8 Ⓔ 0.7
Ⓕ $\frac{1}{30}$ Ⓖ $\frac{11}{20}$ Ⓗ $\frac{7}{15}$
Ⓘ $\frac{7}{60}$ Ⓙ $\frac{19}{30}$

Page 65

1 ① 29.756 ② 22.99 ③ 17.992
④ 0.15 ⑤ 9 ⑥ 4.8
⑦ 11.5 ⑧ 0.65

2 ① 36 ② 19

3 $\frac{2}{5}$ hour

4 780 yen for 6 pieces

5 ① 65° ② (omitted)

6 ① 2 m ② 1.5 times ③ 1.2 m

Playing with Numbers and Calculations

In both ① and ②, the answer will be the same as the whole or decimal number you first picked.

Page 83

1 ① 5 ② 10 ③ 3
④ 4 ⑤ 25.6 ⑥ 10.8

2 5 points

3 ①ⓐ 40° ②ⓑ 135° ③ⓒ 70°

4 ① 8 students
② (Example) The students who like carrots but do not like bell peppers.
③Ⓐ 10 Ⓑ 9 Ⓒ 17

5 ① Bar graph ② Broken line graph

Review Exercises

1 Times As Much (5A)

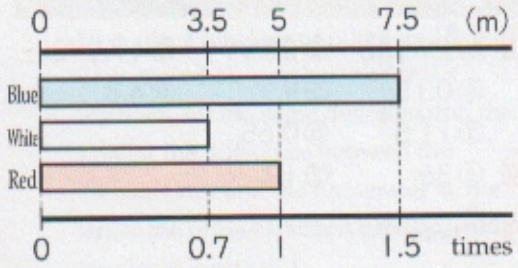
	Length (m)
Red	5
Blue	7.5
White	3.5

Lengths of the blue ribbon and the white ribbon can be expressed as follows based on the length of the red ribbon.

If we consider the length of the red ribbon as 1,
- The length of the blue ribbon corresponds to 1.5. (7.5 ÷ 5 = 1.5)
- The length of the white ribbon corresponds to 0.7. (3.5 ÷ 5 = 0.7)

We can use the word "times" to express these relationships in sentences, a diagram, and math sentences as follows.

- The blue ribbon is 1.5 times as long as the red ribbon.
- The white ribbon is 0.7 times as long as the red ribbon.



Length of red ribbon × 1.5 = Length of blue ribbon (Base amount, times)

Length of red ribbon × 0.7 = Length of white ribbon (Base amount, times)

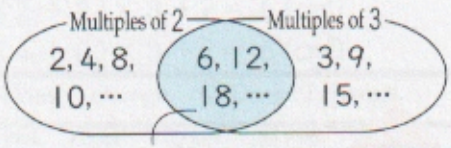
2 Multiples and Factors (5A)

◆ **Multiples** and **Common Multiples**

Multiples of 2 {2, 4, 6, 8, 10, 12, ...}

Multiples of 3 {3, 6, 9, 12, 15, ...}

Common Multiples of 2 and 3 {6, 12, ...}



Common multiples (Multiples of the least **common multiple**, 6)

◆ **Factors** and **Common Factors**

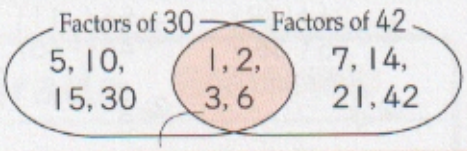
Factors of 30

{1, 2, 3, 5, 6, 10, 15, 30}

Factors of 42

{1, 2, 3, 6, 7, 14, 21, 42}

Common factors of 30 and 42 {1, 2, 3, 6}



Common factors (factors of the greatest **common factor**, 6)

3 Order and Properties of Operations (4th Grade)

◆ Order of Operations
- Calculate from left to right.
- Calculate what is inside the ( ).
- × and ÷ must be calculated before + and −.

◆ Properties of Operations

$(■ + ●) \times ▲ = ■ \times ▲ + ● \times ▲$

$(■ - ●) \times ▲ = ■ \times ▲ - ● \times ▲$

4 Relationships between Fractions and Decimal / Whole Numbers (5A)

◆ Fractions can be expressed as decimal numbers by dividing the numerators by the denominators.

$\frac{1}{3} = 1 \div 3 = 0.333\cdots$ $\quad \frac{2}{5} = 2 \div 5 = 0.4$

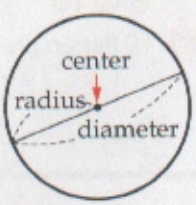
◆ Whole and decimal numbers can be expressed as fractions with denominators of 1, 10, 100, and so on.

$2 = \frac{2}{1}$ $\quad 0.3 = \frac{3}{10}$ $\quad 0.47 = \frac{47}{100}$

5 Circles (3rd Grade)

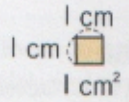
Circle

A round shape that is drawn by making all the distance from the middle point the same

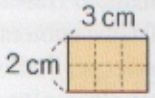


6 How to Express Area and Formulas for Areas (4th Grade)

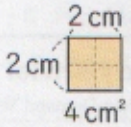
We can express the area by the number of squares with sides of 1 cm, 1 m, etc., that can be placed inside. The area of a square with 1 cm sides is expressed as 1 cm².



Area of Rectangle
= Length × Width
= Width × Length

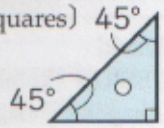


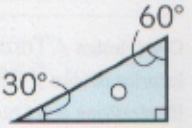
Area of Squares = Side × Side



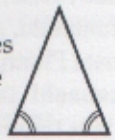
7 Angles of Triangles and Quadrilaterals (2nd–4th Grades)

(Set squares)

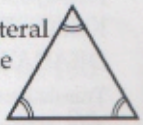




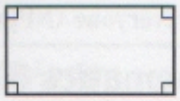
Isosceles triangle



Equilateral triangle



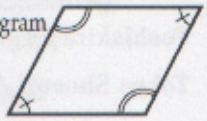
Rectangle



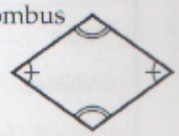
Square



Parallelogram



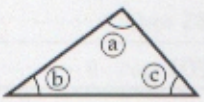
Rhombus



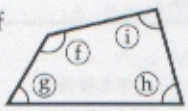
8 Sum of the Angles of a Triangle or a Quadrilateral (5A)

◆ The sum of the 3 angles of any triangle is 180°.

ⓐ + ⓑ + ⓒ = 180°

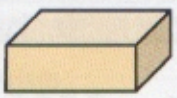


◆ The sum of the 4 angles of any quadrilateral is 360°.



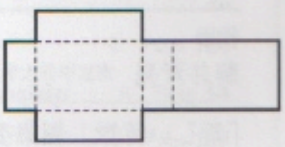
ⓕ + ⓖ + ⓗ + ⓘ = 360°

9 Nets and Sketches (4th Grade)

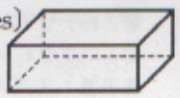


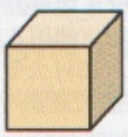
Cuboids

(Nets)



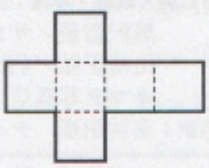
(Sketches)



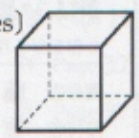


Cubes

(Nets)

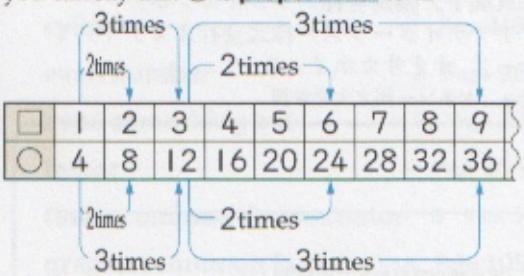


(Sketches)



10 Proportional Relationships (5A)

Suppose there are a pair of quantities, □ and ○, and when □ increases to 2 times, 3 times..., ○ increases to 2 times, 3 times... In this case, you can say that ○ is **proportional** to □.

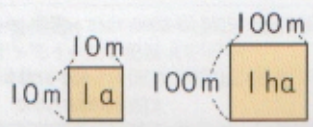


11 Units of Quantity (2nd–4th Grades)

【Length】
1 cm = 10 mm
1 m = 100 cm
1 km = 1,000 m

【Weight】
1 kg = 1,000 g
1 t = 1,000 kg

【Area】
1 m² = 10,000 cm²
1 a = 100 m²
1 ha = 10,000 m²



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This index lists the terms and signs you study in 5th grade.
Use the index to review or confirm your study.



If you use these terms and symbols correctly, you will find it easier to think and explain your ideas to others.

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